PSIWORLD 2012

The Impact of Different Kind of Training on Accuracy Responses in a Novel Mathematical System

Mihaela Tăranu*

*Babes Bolyai University, Developmental Psychology Laboratory, Faculty of Psychology and Educational Science, Cluj Napoca, 400029, Romania

Abstract

When performing numerical calculation people are influenced by the particular perceptual form of the representations of abstract entities. Based on Landy and Goldstone (2007), we were interested to see whether there was a difference on accuracy between three groups of participants that were given different kinds of training stimuli while learning a novel mathematical system. Given a specific modality of spacing the operands—consistent, inconsistent or neutral regarding the classic order of precedence, we asked whether participants would have different performance during testing. We found no significant difference. In the end, the results of the experiment are discussed.

Keywords: Mathematical cognition, novel system, embodied cognition, perceptual learning, symbolic reasoning.

1. Introduction

Genuine embodiment entails a reconsideration of the nature of cognition and of mathematics itself, with corresponding implications for teaching (Lakoff & Núñez, 2000). Although notational mathematics is treated as being an abstract symbol system, it is useless to say that these notations are visually distinctive forms that occur in particular spatial arrangements and physical contexts. Every actual notation has some particular physical presentation and it always contains formally irrelevant physical relations. Often, especially when formal understanding is poor or partial, these relations may be more salient to a participant than the formal abstract relations (Goldstone & Landy, 2008). This issue is of great importance for understanding mathematical reasoning and learning. Although arithmetical notation may be the best-known example of a purely formal symbol system, arithmetic itself contains a variety of non-formal conventions that relate visual aspects of expressions to their formal structure. There is much evidence that people are influenced when performing numerical calculation by the particular perceptual form of the abstract entities’ representations (Campbell, 1994; Zhang & Wang, 2005;

* Corresponding author. Tel.: + 4 0 740 51 20 30.
E-mail address: mihaela.tararu22@yahoo.com

© 2013 The Authors. Published by Elsevier B.V. Open access under CC BY-NC-ND license.
Selection and/or peer-review under responsibility of PSIWORLD 2012
McNeil & Alibali, 2005). Landy & Goldstone (2007a) made an ingenious experiment in which the participants had to learn a novel pair of mathematical operations and discover an order of operation rules governing them. In their experiment, the participants learned two novel operations in isolation, and then had to discover a rule for how to combine them. The participants were instructed that the rule would be a simple order of precedence—one operator was to be bound before the other. The results showed that spacing regularities informed syntactic judgments, but only when that spacing aligns with common mathematic practice by placing higher-order operands together. The authors affirm that the alignment between syntactic structure and spacing orthography is not restricted to the familiar mathematical operations, but is a general part of how people engage with mathematical structures.

We think that in their experiment the results were greatly influenced by their ability to memorize the symbols and the results (the rate in succeeding to reach the criterion was very low probably because those who succeeded had better strategies of memorizing). We tried to avoid this by creating a novel system consisting in three new symbols that had some correspondence with rules that exist in mathematics, but which cannot be found in reality under this form. Our system is different from that built by Kirshner (1989) because the symbols that represent the rules are not intended to be so evident (as presenting M for multiplication) and in contrast with the system build by Landy and Goldstone (2007a), the rules are not arbitrary and the participants do not have to operate only with associations between symbols, operators and results that should be very well memorized.

In this study we were interested in seeing whether there was a difference between three groups of participants that were given different kinds of training stimuli. Considering that the mathematical system was novel, we thought that participants would have different performance during testing if they would have to identify the rule of precedence given a specific modality of spacing the operands—consistent, inconsistent or neutral regarding the classic order of precedence (the operands that are firstly solved are closer to each other than those that are secondly solved).

Perceptual expertise is a very controversial subject. There is psychological evidence that indicates that far transfer of learned principles is often difficult and may only reliably occur when people are explicitly reminded of the relevance of their early experience when confronted with a subsequent related situation (Gick & Holyoak, 1983). This body of evidence stands in contrast to other evidence suggesting that people automatically and unconsciously interpret their world in a manner that is consistent with their earlier experiences (Roediger & Geraci, 2005; Curby & Gauthier, 2009). The perspective we share says that knowledge lays in the perceptual interpretation and motor interactions involving a concrete scenario, with the possibility and power of transfer across contexts (Goldstone, Landy & Son, 2008).

We consider that through different kinds of training we can induce a specific perceptual expertise for each group and the influence for accuracy and reaction of time in solving the experiment trials will be different for those stimuli that were not looking like those which the participants have used in training. Thus, we wanted to see if the experimental group (who received the training that contains all the modalities of spacing—congruent, incongruent or neutral) have better performance than the other two groups (which received stimuli spaced just in one modality—congruent or incongruent).

2. Method

2.1. Participants and procedure

This experiment included 51 students from the Babes Bolyai University, who came for credit course or as volunteers. Participants had to learn a novel calculus system which included three new symbols—, . The participants were randomized in three groups: one experimental (which received in the training part mixed stimuli) and two of control: the group which received in the training part just congruent stimuli and the other group which received just incongruent stimuli (see Table 1).

The experiment had two parts—training and testing. The task was made in Super Lab 4. The expressions were presented in black color on a white background, using Lucida Grande font, 28 points. The participants had to use
the keyboard to report, in the training part— the symbol that had to be firstly solved, and at testing— the correct response of the expressions. The keys $\&$, $\Phi$, $\sharp$, were used for training and $0$, $1$, $2$ for testing.

During the training part, participants completed 45 randomized trial stimuli where the operations that should be firstly solved had to be discovered. In this first part, the meaning of the symbols remained unknown. Each stimulus expression consisted of three operands and two operators. One training stimulus looked like: "$1 \& 2 \Phi ="", "0 \Phi 1 \sharp 2="", "1 \& 2 \sharp 1="". The trials were differently spaced depending on condition, while during testing 30 trials were spaced congruently, 30 incongruently and 30 were neutrally spaced with respect to the precedence rule. The way the stimuli were looking, depending on each condition is presented in Table 1. In the congruent training trials, the symbol that was firstly solved was spaced closer than the symbol that was solved secondly. In the incongruent training trials, the symbol that was firstly solved was wider spaced than the symbol that was secondly solved. In the mixed training trials, the symbols that were firstly solved were sometimes closer, wider or neutral spaced. The meaning of every symbol was revealed and then 90 randomized test stimuli were presented and participants had to solve the expressions taking into consideration the rule of precedence discovered in the first part.

In testing (which was the same for all the participants regardless of the group they were included), the stimuli consisted also of three operands and two operators which were presented in all three modalities—congruent, incongruent and neutral.

Table 1. List of training stimulus depending on condition

<table>
<thead>
<tr>
<th>Training congruent stimuli</th>
<th>Training incongruent stimuli</th>
<th>Training mixed stimuli</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 &amp; 2 \Phi 0=$</td>
<td>$1 &amp; 2 \Phi 1=$</td>
<td>$1 &amp; 2 \Phi 1=$</td>
</tr>
<tr>
<td>$1 \Phi 2 \sharp 0=$</td>
<td>$1 \Phi 2 \sharp 0=$</td>
<td>$1 \Phi 2 \sharp 0=$</td>
</tr>
<tr>
<td>$1 &amp; 2 \sharp 1=$</td>
<td>$1 &amp; 2 \sharp 1=$</td>
<td>$1 &amp; 2 \sharp 1=$</td>
</tr>
</tbody>
</table>

Note. In the training congruent stimuli the operation that must have been solved first is the one with the smaller space; in the incongruent stimuli was the reverse of the congruent ones, in the mixed training stimuli, the participants saw in an equally proportion stimuli spaced congruent, incongruent or neutral regarding the order of operation.

3. Results and discussion

Learning this novel system proved to be very easy as all the 51 participants succeeded in identifying the order of operations. After testing was over they were asked how the probe was and all of them observed the fact that the expressions were sometimes differently spaced but few of them used the space as an indicator of precedence order.

In the training part few of the participants took into consideration the space when solving the expressions. In the experimental group (with all spacing modalities during training) there were 9 participants with maximum performance and the smallest accuracy under 0.5 was 0.44. As expected, the performance of participants who learned the order of precedence more quickly was higher than the performance of other two groups.

In the consistent group, there were 8 participants that had the maximum accuracy of responses, 100% of the items were correctly solved. Only one participant had a lower accuracy than 0.5, the rest of participants had very good performance.

The group with the inconsistent training had the lowest performance; there was no participant with maximum accuracy, the rest of responses being between 0.2 and 0.97.

The fact that, overall, the participants took into consideration the rules and not the spacing can also be seen in the big number of participants from each of the three groups that had accuracy better than 50% on the experimental stage.

From the descriptive statistics (see Fig. 1), we found an interesting tendency on accuracy of the consistency variable. Overall, the highest accuracy was found in consistent trials (m=0.76, MSD=0.19), medium in neutral
trials (m=0.74, MSD=0.20) and the lowest in inconsistent trials (m=0.73, MSD=0.21). This tendency is also sustained by the results obtained by other authors (Landy & Goldstone, 2007, 2007a, 2007b; Tăranu, in press).

In order to see whether there are significant results of expertise variable (given by the type of training participants had) on accuracy depending on different kind of training, we performed a one-way ANOVA. The analysis revealed no significant differences between groups.

This result cannot sustain our hypothesis according to which the group that will receive during training equations spaced differently will have a better performance attest. Even though, we can see tendencies which are in the direction we have anticipated.

One interesting question that arises from the literature is: Can we help students learn the order of precedence by presenting them with $A + X*Y$ before giving them more neutral notation of $A + B * C$? Our results could not give an answer to this question (having the experience with all kind of spacing seemed to give no significant advantage when participants had to solve expressions which were spaced in a specific manner) and further research is required.

The majority of our participants learned quickly the symbols and their meaning and afterwards they were paying greater attention to rules than to any other salient cues. Therefore, probably space is more important in more complex systems or, as in the Goldstone’s novel system, when the operations are meaningless and the responses are non-associative, spacing is a very important cue. This conclusion deserves to be further investigated. Goldstone et al. (2008) says that in skilled participants syntax is processed using formal rule system. Some of our data from an unpublished study proved that even when the participants are very familiar with simple algebraic expressions their responses can be influenced by the way the items are spaced. Therefore, there is no consensus among researchers regarding this matter, future research is necessary in order to prove the way experts and novices integrate the space into their reasoning.

One of the most interesting aspects of the alignment influence advantage/disadvantage is addressed in no experiment (as far as we know), respectively, where does it come from? We tried to address this issue by means of manipulation of the impact of previous experience with a specific kind of spacing. As adults we can more focused on rules than children. We think that in order to reach a more valid answer, this kind of experiments should be adapted to children for the results to be much more trustful (at this age it is more plausible to find he real and most important factors and to control the confounding factors that might interfere)

This study has a limitation that must be considered in any attempt to generalize the findings. We cannot be certain that the training had the effect we expected in learning the novel system. We asked the participants to identify the rule of precedence and they had to type the answer on keyboard. It is very probably that this fact has facilitated the fixation in memory of the rules. Later, in the experimental part, the motor skill (pressing the keyboard) could have been more important for the answer than the spacing cues provided. This fact can also explain why the comparisons were not statistically significant; once the participants learned the rules, it would have been necessary more training and experience with the system in order to see if sometimes the perceptual cues are those that influence the reasoning.
4. Conclusion

This study aimed to contribute to our understanding of how different spacing training within a novel system can influence the accuracy of solving mathematical expressions. No significant results were found, but taking into consideration the limits of our study, the generalization of data could not be made. Studying further the difference between novices and experts on processing the space can have important implication both for mathematical education and for cognitive science.

Acknowledgments

This research was funded by Babes Bolyai University of Cluj Napoca as a performance scholarship nr 30068/56/19.02.2012. The author would like to acknowledge Lecturer Thea Ionescu, PhD, for being the coordinator of this research, the Developmental Psychology Laboratory from the Faculty of Psychology and Educational Science without whose technical support this research would not have been possible and PhD Student Dermina Vasc for all the helpful commentaries she made during the project implementation. The author also want to thank Ioana Tăranu for the help given in editing the manuscript.

References

Tăranu M. (2012). *How does physical space influence the novices’s and expert’s algebraic reasoning?* Manuscript submitted for publication.