Journal of Combinatorial Theory, Series A **93**, 141–152 (2001)
doi:10.1006/jcta.2000.3070, available online at http://www.idealibrary.com on IDEAL

Some New 6-(14, 7, 4) Designs[1]

Z. Eslami and G. B. Khosrovshahi[2]

Department of Mathematics, University of Tehran, and
Institute for Studies in Theoretical Physics and Mathematics (IPM), Tehran, Iran

Communicated by the Managing Editors

Received October 2, 1999

In this paper, we employ trades to produce some new 6-(14, 7, 4) designs with three automorphisms.

1. INTRODUCTION

For given v, k, and t, let $X=\{1, 2, ..., v\}$ and let $P_k(X)$ denote the set of all k-subsets of X. The elements of X and $P_k(X)$ are called points and blocks, respectively. A t-(v, k) *trade* $T=\{T_1, T_2\}$ consists of two disjoint collections of blocks T_1 and T_2 such that for every $A \in P_t(X)$, the number of blocks containing A is the same in both T_1 and T_2. T is called *simple* if there are no repeated blocks in T_1 (T_2). Both T_1 and T_2 must cover the same set of points which is called the *foundation* of the trade and is denoted by *found*(T). The number of blocks in T_1 (T_2) is called the *volume* of T and is denoted by $\mathrm{vol}(T)$. A trade is void if $\mathrm{vol}(T)=0$.

Hwang [8] has shown that for a nonvoid t-(v, k) trade

(1) the minimum foundation size is $k+t+1$,

(2) for $v \geqslant k+t+1$, the minimum volume is 2^t.

She also showed that nonvoid trades with $|found(T)|=k+t+1$ and $\mathrm{vol}(T)=2^t$ exist and have a unique structure. These trades are called *minimal* trades.

Following Graver and Jurkat [7], Graham *et al.* [6] cast these minimal trades in terms of polynomials. Consider the polynomial

$$\Phi(x_1, ..., x_v)=(x_{b_1}-x_{c_1})(x_{b_2}-x_{c_2})\cdots(x_{b_{t+1}}-x_{c_{t+1}})\, x_{b_{t+2}}\cdots x_{b_k}, \qquad *\,(1)$$

[1] This research was partially supported by a grant from IPM.

[2] Correspondence should be addressed at IPM, P.O. Box 19395-5746, Tehran, Iran. Email: rezagbk@karun.ipm.ac.ir.

0097-3165/01 $35.00

in which all the b_i and c_i are distinct. If we multiply the factors out and identify each x_i with i, then the result is clearly a minimal trade.

The *starting index* of a nonzero vector $\omega = (\omega^1, ..., \omega^n)$ in Z^n, is the smallest positive integer i for which $\omega^i \neq 0$. An ordered set of nonzero vectors $\{\omega_1, ..., \omega_k\}$ such that x_i is the starting index of ω_i is *semitriangular* if $1 \leqslant x_1 < \cdots < x_k \leqslant n$. The *starting block* of a trade is well-defined by the lexicographic ordering of the set of blocks, since T and $-T$ have the same starting block.

Two trades $T = \{T_1, T_2\}$ and $T' = \{T'_1, T'_2\}$ are called *isomorphic* if there exists a bijection σ: *found*$(T) \to$ *found*(T') such that $\sigma(T) = \{\sigma(T_1), \sigma(T_2)\} = \{T'_1, T'_2\} = T'$. An isomorphism σ such that $\sigma(T) = T$ is called an *automorphism* of T. Clearly, the set of all automorphisms of T forms a group. T is called *rigid* if its automorphism group is trivial.

For each point $x \in \text{found}(T)$, we consider the set of all blocks containing it. By omitting x from these blocks, we obtain a $(t-1)$-$(v-1, k-1)$ trade and we call it the *derived* trade with respect to x.

A *simple t-(v, k, λ) design* is a set $\mathscr{B}$ of blocks of X such that every t-subset of X occurs exactly λ times in $\mathscr{B}$. $(X, P_k(X))$ is called the *complete* design. A *large set* of t-(v, k, λ) designs, denoted by $LS[N](t, k, v)$, is a partition of the complete design into N disjoint t-(v, k, λ) designs, where $N = \binom{v-t}{k-t}/\lambda$. An $LS[2](t, k, v)$ is called a *halving* of the complete design. A t-(v, k) trade $T = \{T_1, T_2\}$ of volume $\binom{v}{k}/2$ is exactly an $LS[2](t, k, v)$. Clearly, T_1 and T_2 are simple t-$(v, k, \binom{v-t}{k-t}/2)$ designs. The following lemma is obvious.

LEMMA. *Let $T = \{T_1, T_2\}$ be a simple t-(v, k) trade of volume $\binom{v}{k}/2$. Then T is nonrigid if and only if one of the following occurs*:

(i) *T_1 and T_2 are nonrigid designs.*

(ii) *T_1 and T_2 are isomorphic rigid designs.*

In this paper, we are concerned only with simple trades of volume $\binom{v}{k}/2$ consisting of nonrigid designs.

Now, a bit of history: The first simple 6-design was found by and S. S. Magliveras and D. W. Leavitt in 1984 [14]. Then in 1986, D. L. Kreher and S. P. Radziszowski found the smallest possible simple 6-design, that is, a 6-(14, 7, 4) design [13]. They employed a method of Kramer and Mesner [12] which constructs blocks of a design as a union of orbits of a prescribed group on sets. Since then, no other 6-(14, 7, 4) design have been found, so this design produces the only known $LS[2](6, 7, 14)$ which has $(C_{13}\ p)$ as automorphism group and is the Alltop extension [1] of a cyclic $LS[2](5, 6, 13)$. Later on, some recursive machinary appeared in the literature which enables one to construct infinite families of designs from a given 6-(14, 7, 4) design [11, 16]. In [3–5], an algorithm based on the *standard basis* of trades is employed to classify 2-(8, 3) trades, all nonrigid $LS[2](2, 3, 10)$ and 2-(10, 3, 4) designs together with a complete enumeration of rigid ones, and to classify all $LS[2](3, 4, 11)$ consisting of nonrigid designs and hence all such nonrigid designs. In Section 2, a description of this basis together with an exposition of the algorithm, is provided.

In this paper, we employ, with slight modifications, the same approach to obtain some new trades. We first construct three $LS[2](5, 6, 13)$ with $(C_3 \times I_4)\, p$ as their full automorphism group. By extending these trades, we produce two nonisomorphic $LS[2](6, 7, 14)$ with three automorphisms. Derivation then reveals some new $LS[2](5, 6, 13)$: 8 rigid ones and a new large set with three automorphisms.

2. ON A BASIS FOR TRADES

Let $P^v_{t,k} = [p_{A,B}]$ be the $\binom{v}{t} \times \binom{v}{k}$ *inclusion matrix*, where for $A \in P_t(X)$ and B a block, $p_{A,B} = \left\{\begin{smallmatrix} 1 & \text{if } A \subseteq B \\ 0 & \text{otherwise} \end{smallmatrix}\right\}$. For $t < k < v - t$, it is known that the rank of $P^v_{t,k}$ is $\binom{v}{t}$ and hence its kernel, denoted by $N^v_{t,k}$, is a $\mathbb{Z}$-module of dimension $\binom{v}{k} - \binom{v}{t}$. A trade $T = \{T_1, T_2\}$ corresponds to a $\binom{v}{k}$-integral vector F which is a solution of the equation $P^v_{t,k} F = 0$. That is, the set of all t-(v, k) trades is the kernel of $P^v_{t,k}$.

An algorithm to generate a basis for $N^v_{t,k}$ consisting of minimal trades in a semitriangular form can be produced based on the following results from Khosrovshahi and Ajoodani-Namini [10].

Consider the polynomial (*) associated with a minimal trade. Furthermore, assume that, $b_1 < b_2 < \cdots < b_{t+1}$, $b_i < c_i$, $1 \leqslant i \leqslant t+1$, $b_{t+2} < \cdots < b_k$. A block $B = b_1 b_2 \cdots b_k$ is *starting* if its elements satisfy the following set of conditions: $b_h \leqslant \left\{\begin{smallmatrix} v-k-t+2h-2 & 1 \leqslant h \leqslant t+1 \\ v-k+h & t+2 \leqslant h \leqslant k \end{smallmatrix}\right\}$. For each starting block B, we can construct a minimal trade containing B such that $c_i = \min(\{b_i + 1, \ldots, v\} - \{c_{i+1}, \ldots, c_{t+1}, b_{i+1}, \ldots, b_{t+1}\})$. The set of $\binom{v}{k} - \binom{v}{t}$ minimal trades based on $\binom{v}{k} - \binom{v}{t}$ starting blocks forms a semitriangular basis for $N^v_{t,k}$.

Utilizing this basis, Khosrovshahi and Maysoori introduced the *standard basis* as follows [9]. Let $\bar{M} = \left[\begin{smallmatrix} M_1 \\ M_2 \end{smallmatrix}\right]$ be a matrix representation of a semitriangular basis, where M_1 is an invertible square matrix of order $\binom{v}{k} - \binom{v}{t}$. Let $\bar{M}^v_{t,k} = M_2 M_1^{-1}$. Then $\bar{M}$ is columnwise equivalent to

$$M^v_{t,k} = \begin{bmatrix} I \\ \bar{M}^v_{t,k} \end{bmatrix},$$

where I is the identity matrix of order $\binom{v}{k} - \binom{v}{t}$. The columns of $M^v_{t,k}$ form the standard basis for $N^v_{t,k}$. The rows corresponding to I are indexed by the starting blocks and the remaining rows by the nonstarting blocks. In [3], the following block structure is obtained for $M^v_{t,k}$:

$$M^v_{t,k} = \begin{bmatrix} M^{v-1}_{t-1,k-1} & 0 \\ N & M^{v-1}_{t,k} \end{bmatrix}.$$

Hereafter, by a "trade" we mean a trade of volume $\binom{v}{k}/2$. A direct way to produce and classify all t-(v, k) trades is to compute linear combinations of the columns of $M^v_{t,k}$ with coefficients 1 and -1, and then to decide whether the resulting trade is simple. Except for a few small values of the parameters, the dimension of $N^v_{t,k}$ makes it impractical to deal with all the columns of $M^v_{t,k}$. However, considering the recursive structure of $M^v_{t,k}$, the problem turns into classifying $(t-1)$-$(v-1, k-1)$ trades. Suppose $(t-1)$-$(v-1, k-1)$ trades have been classified so that we have one representative for each isomorphism class. Let T be a t-(v, k) trade and let D_1 be its derived trade with respect to the point 1. D_1 is clearly isomorphic to one of the representative $(t-1)$-$(v-1, k-1)$ trades, say D'_1. So, there exists a permutation π such that $D'_1 = \pi D_1$. Therefore, πT (an isomorphic copy of T) will be the extension of D'_1. Hence, to classify t-(v, k) trades, up to isomorphism, it suffices to extend only the representatives of the isomorphism classes of $(t-1)$-$(v-1, k-1)$ trades. The structure of $M^v_{t,k}$ helps us in determining t-(v, k) trades by extending $(t-1)$-$(v-1, k-1)$ trades. Let T' be a $(t-1)$-$(v-1, k-1)$ trade. Then the coefficients of the first $\binom{v-1}{k-1} - \binom{v-1}{t-1}$ columns of $M^v_{t,k}$ are specified by the blocks of T'. To extend T', it suffices to determine the coefficients of the remaining columns of $M^v_{t,k}$ in such a way that the result would be a simple trade. Finally, we check for isomorphism among all extensions.

This recursive construction which ignores isomorphic copies of the derived trades, results in a considerable reduction in the number of extensions that are to be checked further to distinguish isomorphism classes. For example, in classifying 2-(8, 3) trades, this approach produced almost 300,000 extensions, while the total number of distinct simple solutions was over 560,000,000. In the case of 2-(10, 3) trades [4], the number of extensions of the ten 1-(9, 2) trades would be over 200,000,000, so we focused on nonrigid trades and only enumerated the rigid solutions. Therefore, in classifying 3-(11, 4) or 4-(12, 5) trades, our knowledge about the derived trades is restricted to nonrigid ones.

Now, suppose that only nonrigid $(t-1)$-$(v-1, k-1)$ trades have been classified. Let T be a t-(v, k) trade with a nontrivial automorphism π. Suppose that π has at least one fixed point, say 1, and let D_1 be the derived trade of T with respect to the point 1. In this case, π is clearly an automorphism of D_1. Hence, the classification of nonrigid $(t-1)$-$(v-1, k-1)$

trades prescribes all possible D_1 and π that T can admit. Extension is then carried out as before with this further assumption about the automorphisms of the result.

3. RESULTS ON NONRIGID $LS[2](4, 5, 12)$

In Section 1, it is pointed out that each t-(v, k) trade $T=\{T_1, T_2\}$ of volume $\binom{v}{k}/2$ in which T_1 has a nontrivial automorphism π is a nonrigid $LS[2](t, k, v)$. In this section, we apply the approach described in Section 2 together with the information in [5] to produce some $LS[2](4, 5, 12)$ consisting of nonrigid designs. In the next section, we extend these large sets with their automorphisms to obtain new $LS[2](5, 6, 13)$.

Let $T=\{T_1, T_2\}$ be a 4-(12, 5) trade and let π be an automorphism of T_1. As in [2], to examine all possible π that T can admit, we consider a minimum subset of *basic* permutations. For this, we can take π to be of the type $1^n a^m$, that is, π consists of n fixed points and m disjoint cycles of length a, where a is a prime and $n+am=12$. The nonexistance of a derived trade with an automorphism of type $1^n 7^m$ shows $a \in \{2, 3, 5, 11\}$. Let $a=11$ and suitably relabel the points such that $\pi=(1)(2 \cdots BC)$. Let D_i be the derived trade of T with respect to the point i. There are four 3-(11, 4) trades with a total of 11, 22, and 110 automorphisms which possess an automorphism of type $1^0 11^1$. Hence, copies of these trades for which π is an automorphism are possible choices of D_1. Furthermore, since D_1, D_2 and D_3 constitute the coefficients of all the columns of $M_{4,5}^{12}$ and also $\pi 2=3$, we need only determine D_2 and then select simple trades with automorphism π. The case $a=5$ is also treated likewise with $\pi=(1)(23456)(789AB)(C)$ and we obtain 2 and 1267 trades for $a=11$ and $a=5$, respectively. Clearly, these trades can be isomorphic, so we determine the size of the automorphism group of T_1 of these trades by McKay's *nauty* [15]:

$\|\mathrm{Aut}\ T_1\|$	$\#$Trades
5	1262
10	2
11	2
60	2
120	1

Now, isomorphism testing must be carried out within trades of each of these classes. Considering the properties of these trades and their automorphisms, this procedure can be efficiently done. For example, in case of

1262 trades with $|\text{Aut } T_1| = 5$, we need only check 400 permutations to discover some possible isomorphisms. The same approaches can be used to determine the size of the automorphism group of the nonisomorphic trades and so we obtain:

THEOREM 3.1. *Up to isomorphism, the number of nonrigid* $LS[2](4, 5, 12)$ *consisting of designs with an automorphism of basic type* $1^n a^m$ *for* $a = 5, 11$ *is*

|Aut|	5	10	11	20	120	240
# LS	334	545	2	1	1	1

THEOREM 3.2. *Up to isomorphism, the number of nonrigid* 4-(12, 5, 4) *designs with an automorphism of basic type for* $a = 5$ *and* 11 *is*

|Aut|	5	10	11	60	120
# Designs	1213	1	4	1	1

We now consider the case $a = 3$. Although in this case, the only possible automorphism of a 3-(11, 4) trade is of type $1^2 3^3$, it is easy to show that they can not be extended with this automorphism, that is, a 4-(12, 5, 4) design with $1^3 3^3$-type automorphism does not exist. To see this, let f_i and σ_i, $i = 1, 2, 3$, be the fixed points and cycles of an automorphism π of such a design, respectively. For each 4-subset of points, consisting of f_i and three points of one of the cycles, there exists exactly one block in the design in which the fifth point is another fixed point of π. Now consider the 4-subset $f_1 abc$ of the above form and w.l.o.g., suppose that $f_1 f_2 abc$ is a block of the design. For $f_3 abc$, there can not exist a block $f_j f_3 abc$ in the design, a contradiction. Hence, the only possible type is 3^4, i.e. we can take $\pi = (123)(456)(789)(ABC)$. In this case, the information about the derived trades can not be effectively used, so we produced some 4-(12, 5) trades with this automorphism. The computational results, up to isomorphism, are

|Aut|	# Trades	# Designs
3	2, 014	4, 546
6	555	179
12	105	4
24	4	1
48	1	

4. RESULTS ON NONRIGID $LS[2](5, 6, 13)$

It is easy to see that a 5-(13, 6, 4) design can not have an automorphism for $a = 5$. Also none of the 4-(12, 5) trades with automorphisms of the form $a = 11$ are extendable. Hence, $a \in \{2, 3, 13\}$. The case $a = 13$ is completely dealt with in [13]. Utilizing the same approach as in the previous section, we tried to find trades with an automorphism of type $1^1 3^4$, say $\pi = (1)(234)(567)(89A)(BCD)$. By extending the suitable derived trades obtained in Section 3, four trades are produced. Since all these trades accept π as automorphism, checking out a very small number of permutations reveals 3 nonisomorphic solutions. Then, using computer search techniques which employ properties of such a trade, we determined the full automorphism group of these trades. All the three solutions have exactly 3 automorphisms and are therefore nonisomorphic. Furthermore, the two designs in each trade are nonisomorphic; i.e., we have six 5-(13, 6, 4) designs with $(C_3 \times I_4)\ p$ as automorphism group, three of which are presented in the Appendix as Designs #1, #2, and #3. Another two designs with the same automorphism group and 16 rigid ones can be obtained from the results of the next section.

5. RESULTS ON NONRIGID $LS[2](6, 7, 14)$

For a 6-(14, 7) trade, the coefficients of all the 429 columns of $M^{14}_{6,7}$ are specified by blocks of D_1, i.e., the derived trade. So each 5-(13, 6) trade of Section 4 has a unique extension which is already given by Alltop [1]. The three choices of D_1 extend to 2 nonisomorphic trades. The full automorphism group of these trades is $G = \langle (1)(2)(345)(678)(9AB)(CDE) \rangle$, i.e., $(C_3 \times I_4)\ pp$ and they define 4 nonisomorphic designs.

Derivation of these two large sets with respect to the two fixed points gives 4 nonisomorphic $LS[2](5, 6, 13)$, one of which is nonisomorphic with the results of Section 4 and is represented by Design #4 in the Appendix. All the 8 derived trades with respect to the points 3, 6, 9, and C are rigid and nonisomorphic.

APPENDIX

Note: The designs shown in Tables I–IV are nonisomorphic with their complements. $G = \langle (1)(234)(567)(89A)(BCD) \rangle$.

TABLE I

Orbit Representatives of G Generating 5-(13, 6, 4) Design #1

23456	12345C	12346B	123489	12348C	12349B	12356B
12356C	123578	12357A	123589	12358A	12358B	12359A
12359B	12359C	1235AC	123678	123679	12367A	123689
12368A	12368B	12369C	1236AC	12378C	12379A	12379C
1237AB	12389B	12457C	12458C	12459C	12467C	12468C
1246AC	125678	125679	12567C	12568C	12569A	12569B
1256AB	125789	12578A	12579B	1257AB	1257AC	12589C
1258AB	126789	12678B	1267AB	12689A	1268AC	13489B
1348AC	13567B	1356AB	13578B	13589C	1358AB	13678B
13679B	1368AB	14589B	14689B	1468AB	234569	23456A
23456C	234589	23458B	23459A	2345AB	23468B	23469B
23489C	235678	235679	23567B	23567C	23568A	23568B
23569C	2356AB	23578C	23579B	2357AB	2357AC	23589A
2358AC	23679A	23679B	2367AC	23689B	23689C	2368AC
2456AC	24578C	24579C	2457AC	24589C	2458AC	24678C
24679C	24689A	2468AB	1235BD	1236BC	1237BC	1237BD
1238BC	1238CD	1239CD	123ABD	123ACD	123BCD	1256BC
1257BC	12589D	1258BD	1258CD	1259CD	125ABC	125ACD
125BCD	1267BC	1267BD	1267CD	1268AD	1268BD	1268CD
1269AB	1269BC	1269BD	1269CD	1278AC	1278BD	1278CD
1279BC	1279BD	127ACD	1289AB	1289AC	1289AD	1289BC
1289BD	128ABC	129ABD	12ABCD	15678B	15678D	15689A
15689B	15689D	1568AB	1568AC	1568BC	1569AC	1569BD
1569CD	156ABD	156BCD	1589AC	1589BC	158ABD	158ACD
159ABC	159ABD	159BCD	189BCD	2345BC	2345CD	2348BD
2348CD	2357CD	23589D	2358BC	2358CD	2359AC	2359BC
2359BD	235ABD	235BCD	2367BC	2367BD	2367CD	2368CD
2369AB	236ABC	236ABD	236BCD	23789B	2378AB	2378AD
2378BC	2378BD	2379AB	2379CD	237ACD	2389AC	2389AD
2389BD	238ABC	238ABD	239ABC	239ACD	239BCD	25678A
25679A	25679C	2567AB	2567BD	2567CD	25689A	25689B
25689C	2568AC	2568BC	2568BD	2569AD	2569BD	2569CD
256ABC	256ACD	25789A	25789B	25789D	2578AD	2578BC
2578BD	2578CD	2579AC	2579BC	257ABD	257BCD	2589AB
2589BC	258ABD	258ACD	259ABC	259ABD	259ACD	26789B
26789C	26789D	2678AB	2678AC	2678AD	2678BC	2679AC
2679AD	2679BD	267ACD	2689AB	2689CD	268ABD	268BCD
269ABC	269BCD	26ABCD	2789AD	2789CD	278ABC	279ABC
279ABD	279BCD	27ABCD	289ACD	289BCD	28ABCD	56789C
56789D	5678BD	5678CD	5689AD	5689CD	568ABD	568BCD
569ABC	56ABCD	589ABC	589ABD	589ACD	589BCD	

TABLE II

Orbit Representatives of G Generating 5-(13, 6, 4) Design #2

123456	12345C	12346B	123489	12348C	12349B	12356A
12356C	123578	12357A	12357B	123589	12358A	12358B
12359B	12359C	1235AB	123678	123679	12367A	123689
12368B	12369A	12369B	1236AC	123789	12378C	12379C
1237AB	1238AC	12457C	1245AC	12468C	12469C	12478C
12489B	12489C	125678	125679	12567B	12568C	12569A
12569B	1256AC	12578B	12579A	12579C	1257AC	12589A
12589C	1258AB	12678A	12678C	12679C	1267AB	12689B
12689C	1268AB	13489B	13567B	13568B	13569B	13589A
1358AC	13679B	1368AB	14589B	14589C	1458AC	1468AC
234568	234569	23456B	23458A	23458B	23459A	23459B
23459C	2345AC	23489C	235678	23567B	23567C	235689
23568C	23569A	2356AB	23579A	23579B	23579C	2357AC
23589C	2358AC	23678B	23678C	23679C	2367AB	23689A
2368AB	2368AC	24567C	24568C	24569C	24578C	24579C
24678C	2467AC	24689A	24689B	1235CD	1236BC	1236CD
1237BD	1237CD	1238BC	1238BD	1239BC	123ABD	123BCD
1256BC	1256BD	1257CD	1258BC	1258CD	1259BD	1259CD
125ABC	125ABD	1267BC	1267BD	1267CD	1268AD	1269BD
126ABD	126ACD	12789B	1278AC	1278BD	1279BC	1279BD
127ABC	127ACD	1289AB	1289AD	1289CD	128ABC	128ACD
128BCD	129ABC	129BCD	15678B	15678C	15689A	15689B
15689C	15689D	1568AC	1568AD	1569AB	1569CD	156ABC
156ACD	156BCD	1589BC	1589BD	158ABD	158BCD	159ABD
159ACD	15ABCD	189ABC	2345BD	2345CD	2348BC	2348CD
2356BC	2357BC	23589D	2358AD	2358BC	2358BD	2359AB
235ABD	235ACD	235BCD	2367BD	2368BD	2368CD	2369AB
2369BC	2369BD	2369CD	236ACD	23789B	2378AB	2378AC
2378AD	2378CD	2379BD	237ABC	237BCD	2389AB	2389AD
2389BC	2389BD	2389CD	239ABC	239ACD	23ABCD	256789
25678A	25679A	25679B	2567AC	2567CD	25689B	25689D
2568AB	2568AC	2568AD	2568BD	2569AC	2569BC	2569CD
256ABD	256BCD	25789B	25789C	2578AB	2578AD	2578BC
2578CD	2579AD	2579BD	257ABC	257ABD	257BCD	2589AB
2589AC	2589BD	258ACD	259ABC	259ACD	259BCD	26789A
26789D	2678AD	2678BC	2678BD	2679AB	2679AD	2679BC
2679CD	267ABC	267ACD	2689AC	2689BC	2689CD	268ABC
268BCD	269ABD	26ABCD	2789AC	2789BC	2789CD	278ABD
278BCD	279ABD	279ACD	289ABD	28ABCD	29ABCD	56789C
5678BC	5678BD	5678CD	5689AB	5689AD	568ACD	568BCD
569ABC	569ABD	589ABC	589ACD	589BCD	58ABCD	

TABLE III

Orbit Representatives of *G* Generating 5-(13, 6, 4) Design #3

123456	12345C	12346B	123489	12348C	12349B	12356A
12356C	123578	12357A	12357B	123589	12358A	12358B
12359B	12359C	1235AB	123678	123679	12367A	123689
12368B	12369A	12369B	1236AC	123789	12378C	12379C
1237AB	1238AC	12457C	1245AC	12468C	12469C	12478C
12489B	12489C	125678	125679	12567B	12568C	12569A
12569B	1256AC	12578B	12579A	12579C	1257AC	12589A
12589C	1258AB	12678A	12678C	12679C	1267AB	12689B
12689C	1268AB	13489B	13567B	13568B	13569B	13589A
1358AC	13679B	1368AB	14589B	14589C	1458AC	1468AC
234567	23456A	23456B	234589	23458B	23458C	23459A
23459C	2345AB	23468B	23469B	23489A	235679	23567C
235689	23568A	23568B	23568C	23569C	2356AB	23578A
23578C	23579A	23579B	2357AC	23589B	23678B	23679A
23679B	2367AC	23689C	2368AC	24567C	24569C	2456AC
24578C	2458AC	2467AC	24689A	2468AC	1235CD	1236BC
1236CD	1237BD	1237CD	1238BC	1238BD	1239BC	123ABD
123BCD	1256BC	1256BD	1257CD	1258BC	1258CD	1259BD
1259CD	125ABC	125ABD	1267BC	1267BD	1267CD	1268AD
1269BD	126ABD	126ACD	12789B	1278AC	1278BD	1279BC
1279BD	127ABC	127ACD	1289AB	1289AD	1289CD	128ABC
128ACD	128BCD	129ABC	129BCD	15678B	15678C	15689A
15689B	15689C	15689D	1568AC	1568AD	1569AB	1569CD
156ABC	156ACD	156BCD	1589BC	1589BD	158ABD	158BCD
159ABD	159ACD	15ABCD	189ABC	2345BC	2348BD	2348CD
234BCD	2356BD	2357BC	2357BD	2358AD	2358CD	2359AC
2359AD	2359BD	235ABC	235ACD	235BCD	2367BC	2367BD
2368AD	2369AB	2369CD	236ABC	236ABD	236BCD	2378AB
2378BC	2378CD	2379AB	2379BC	2379CD	237ACD	2389AB
2389AC	2389BC	2389BD	2389CD	238ABC	238ABD	239ABD
239ACD	256789	25678A	25679D	2567AB	2567AC	2567AD
2567BC	25689A	2568AB	2568BC	2568BD	2568CD	2569AB
2569AD	2569BC	2569CD	256ACD	25789B	25789C	25789D
2578AC	2578BD	2579AB	2579BC	2579CD	257ABD	257BCD
2589AC	2589AD	2589BC	2589BD	258ABD	258ACD	259ABC
25ABCD	26789B	26789C	2678AB	2678AD	2678BC	2678CD
2679AC	2679AD	2679BD	2679CD	2689AB	2689AC	2689BD
268ACD	268BCD	269ABC	269BCD	26ABCD	2789AC	2789AD
278ABC	278ABD	278BCD	279ABD	279ACD	27ABCD	289BCD
29ABCD	56789A	56789B	5678BD	5678CD	567BCD	5689AC
5689BD	5689CD	568ABC	568ABD	569ABD	569BCD	589ABC
589ACD	59ABCD	89ABCD				

TABLE IV

Orbit Representatives of G Generating 5-(13, 6, 4) Design #4

123459	12345A	12345C	12346B	12348C	12349B	123567
123569	12356B	123579	12357A	12357C	123589	12358A
12358B	12358C	123678	12367C	123689	12368A	12369B
12369C	1236AB	1236AC	12378A	12378B	12379C	1237AB
12389A	12456C	12457C	12468C	12469C	12479C	12489B
12489C	1248AC	12567A	12567B	125689	12568A	12568C
12569A	12569C	125789	12578B	12579B	1257AC	12589C
1258AB	126789	12679A	1267AB	1267AC	12689B	1268AC
1348AC	13568B	13569B	1356AB	13579B	1357AB	13589B
1358AC	13679B	1368AC	14589C	14689B	1468AB	234567
234568	234569	23456C	23458A	23459B	23459C	2345AB
23469B	23489A	23489B	23489C	235678	23568B	23568C
23569A	2356AB	2356AC	235789	23579A	23579B	2357AC
23589C	2358AB	2358AC	23678B	23678C	23679A	23679B
23679C	2367AB	23689A	2368AB	24567C	24569C	24578C
24579C	24589B	2458AC	2467AC	24689A	2468AB	1235BC
1235BD	1236CD	1237BD	1238BC	1238BD	1238CD	1239BC
123ACD	123BCD	1256BD	1256CD	1257BC	1258CD	1259AC
1259BD	1259CD	125ABC	125ABD	125ACD	1267BC	1267CD
1268AD	1268BC	1268BD	1269BC	1269BD	126ABC	126ABD
12789B	1278AC	1278BC	1278CD	1279BD	127BCD	1289AB
1289AD	1289CD	128ABC	128ABD	129ACD	129BCD	15678B
15678C	15678D	15689A	15689D	1568AB	1568BC	1569AD
1569BC	156ABD	156ACD	156BCD	1589AB	1589AD	1589BC
1589BD	158ABC	158ACD	158BCD	159ABC	189BCD	2345BD
2345CD	2348BC	234BCD	2356BC	2357BC	2357BD	2357CD
23589D	2358BD	2359AB	2359AD	2359BC	2359CD	235ACD
2367BC	2368AD	2368BD	2368CD	2369BD	2369CD	236BCD
23789B	2378AC	2378AD	2378CD	2379BD	2379CD	237ABC
237ABD	2389BC	2389BD	238ABC	238ACD	239ABC	239ABD
23ABCD	25678A	25678B	25678C	25679A	25679B	25679C
25679D	2567BD	2567CD	25689B	25689D	2568AC	2568AD
2569AB	2569BC	256ABC	256ACD	256BCD	25789A	2578AB
2578AD	2578BC	2578CD	2579AC	2579CD	257ABC	257ABD
2589AC	2589AD	2589BC	2589BD	258ABD	258BCD	259ABD
259BCD	26789C	26789D	2678AB	2678AC	2678BD	2679AD
2679BC	267ABD	267ACD	2689AB	2689AC	2689BC	2689CD
268BCD	269ABD	269ACD	26ABCD	2789AB	2789AD	2789BD
278BCD	279ABC	279ACD	279BCD	27ABCD	289ABC	289ACD
28ABCD	56789A	56789B	5678CD	567BCD	5689AC	5689BD
5689CD	568ABC	568ABD	568BCD	569ABC	569ABD	569ACD
589ACD	59ABCD	89ABCD				

ACKNOWLEDGMENT

During the summer of 1999, while the first author was visiting University of Bayreuth, using the group $(C_3 \times I_4)\,p$, Professor R. Laue produced six isomorphic copies of Design #4 of the Appendix by DISCRETA (a computer package for the generation of discrete structures).

REFERENCES

1. W. O. Alltop, Extending t-designs, *J. Combin. Theory Ser. A* **12** (1975), 177–186.
2. C. J. Colbourn, S. S. Magliveras, and D. R. Stinson, Steiner triple systems of order 19 with nontrivial automorphism group, *Math. Comp.* **59** (1992), 283–295.
3. Z. Eslami, G. B. Khosrovshahi, and B. Tayfeh-Rezaie, On classification of 2-(8, 3) and 2-(9, 3) trades, *J. Combin. Math. Combin. Comput.*, in press.
4. Z. Eslami, G. B. Khosrovshahi, and B. Tayfeh-Rezaie, On halvings of the 2-(10, 3, 8) design, *J. Statist. Plann. Inference* **86** (2000), 411–419.
5. Z. Eslami and G. B. Khosrovshahi, A complete classification of 3-(11, 4, 4) design with a nontrivial automorphism group, *J. Combin. Des.*, in press.
6. R. L. Graham, S.-Y. R. Li, and W.-C. Li, On the structure of t-designs, *SIAM J. Algebraic Discrete Methods* **1** (1980), 8–14.
7. J. E. Graver and W. B. Jurkat, The modular structure of integral designs, *J. Combin. Theory Ser. A* **15** (1973), 75–90.
8. H. L. Hwang, On the structure of (v, k, t) trades, *J. Statist. Plann. Inference* **13** (1986), 179–191.
9. G. B. Khosrovshahi and Ch. Maysoori, On the bases for trades, *Linear Algebra Appl.* **226-228** (1990), 731–748.
10. G. B. Khosrovshahi and S. Ajoodani-Namini, A new basis for trades, *SIAM J. Discrete Math.* **3** (1990), 364–372.
11. G. B. Khosrovshahi and S. Ajoodani-Namini, Combining t-designs, *J. Combin. Theory Ser. A* **58** (1991), 26–34.
12. E. S. Kramer and D. M. Mesner, t-designs on hypergraphs, *Discrete Math.* **15** (1976), 263–296.
13. D. L. Kreher and S. P. Radziszowski, The existence of simple 6-(14, 7, 4) designs, *J. Combin. Theory Ser. A* **43** (1986), 237–243.
14. S. S. Magliveras and D. W. Leavitt, Simple 6-(33, 8, 36)-designs from $P\Gamma L_2(32)$, *in* "Computational Group Theory" (M. D. Atkinson, Ed.), pp. 337–352, Academic Press, San Diego, 1984.
15. B. D. McKay, "Nauty User's Guide (Version 1.5)," Technical Report TR-CS-90-02, Computer Science Department, Australian National University, 1990.
16. L. Teirlinck, Locally trivial t-designs and t-designs without repeated blocks, *Discuss. Math.* **77** (1989), 345–356.