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Soviet Mathematics and Dialectics in the Stalin Era

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Stalinist philosophers and other champions of Marxist theory argued against the modern trend in the mathematization of science. They thought that the unrestricted mathematization of the natural sciences and humanities invited "mathematical idealism," an archenemy of materialistic dialectics. In their opinion, Georg Cantor's set theory was based on speculation unrelated to the real world. Such leading mathematicians as A. N. Kolmogorov and P. S. Aleksandrov had different ideas: they welcomed the modern trend in the accelerated mathematization of science, and they considered set theory the most advanced and promising branch of mathematics. Despite Stalin's unceasing war on free expression in science, Soviet mathematicians made significant contributions to modern mathematical knowledge. © 2000 Academic Press

Сталинские философы и другие теоретики марксизма вели упорную и постоянную борьбу против "преувеличенной математизации" науки. Они считали, что неограниченная математизация естествознания и гуманитарных наук ведёт к "математическому идеализму," широко распространенному на Западе. В теории множеств Георга Кантора они видели продукт умозрения не основанного на реальности. Ведущие математики, такие как А.Н. Колмогоров и П.С. Александров, полагали, что прогрес науки зависит от степени совершенства и распространения математических методов. Они считали, что теория множеств является самой прогрессивной отраслью современной математики. Вопреки жёсткой сталинской политике против автономии науки, советская математика достигла замечательных успехов. © 2000 Academic Press

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MARXIST CRITICISM IN THE 1930s

The Stalinist era, inaugurated by the massive collectivization of agriculture, five-year plans for the industrial development of the country, and a total war on independent thought, produced sweeping actions on the ideological front. These actions did not bypass the community of mathematicians. The Mathematical Section of the Communist Academy assumed the main role in criticizing the ideological lethargy and philosophical aloofness of the leading mathematicians and in pointing out the "correct" path to be followed in the future. The Communist Academy (until 1923 named the Socialist Academy) was founded in Moscow in 1918 for the purpose of clarifying and advancing theoretical and practical aspects of Marxist theory. In 1930, Ernst Kol'man, a Czech expatriate, philosopher of science, and spokesman for Stalinist Marxism, used blunt language to inform mathematicians about their obligations on the ideological front:

> A discussion of radical changes in philosophy took place recently in these very quarters. Now all of us understand clearly the main point of that discussion: under the dictatorship of the proletariat

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> neither philosophy nor any other discipline can exist in isolation from politics and Party leadership. We all know that any effort to view a theory or a science as an autonomous and self-contained discipline would mean objectively to oppose the general Party orientation and the principles of the dictatorship of the proletariat. Mathematics cannot claim to be an exception to this rule; like all social and natural sciences in our country, it must be a Party science .... The *partiinost'* of mathematics, no less than that of philosophy, is unconditionally obligatory. Mathematics must be closely tied to socialist construction not only externally but by its entire structure and its entire content as well. It cannot be separated either from the philosophy of dialectical materialism or from the policies of our Party. [42, 53]

The norm of *partiinost'*—of strict adherence to the philosophical precepts of dialectical materialism as interpreted by government authorities—became the watchword of the day. Marxist vigilantes labored intensively to expose violators of the spirit of *partiinost'* and to subject transgressors to public criticism. Several years after his death, V. A. Steklov, a former Vice President of the Academy of Sciences of the USSR, did not escape the inquisition conducted by the guardians of *partiinost'*. In *Galileo*, a small volume published in 1923, Steklov asserted that the Roman clergy knew that Galileo's defense of Copernicus was fully justified on scientific grounds and that they turned against it solely under the pressure of "party discipline"—loyalty to the Church and its intellectual spirit [75, 98–99]. Obviously alluding to Marxist ideologues, he warned that science, in order to advance, must rise above and be fully independent of party loyalties. In 1934, at an early peak of Stalinist pressure for *partiinost'* in science, M. Ia. Vygodskii, a Marxist historian of mathematics, bitterly attacked Steklov's comments. He charged that Steklov's disparaging assessment of *partiinost'* was only a thinly veiled attack on Bolshevik views on the commanding position of Party loyalty in the scientific community [79, 5–6]. In the neutrality of science Vygodskii could only see a strategy or cover for anti-Soviet activity.

A few years earlier, in 1930, S. A. Ianovskaia, a mathematician, a new member of the Communist Academy, and a Bolshevik since 1918, took it upon herself to describe the key issues and dilemmas of mathematics in the age of accelerated socialist construction. She pointed out that Russia had long maintained a tradition of isolating mathematics from the natural sciences—that, for example, Russia had not produced a single eminent mathematician who also worked actively in physics. "Russia did not have her Gausses and Kleins, mathematicians who were interested in the theoretical questions of natural science and who worked in physics" [31, 91]. She was especially exasperated by the Moscow mathematical school which, in her opinion, took great pride in its complete isolation from the scientific and technical needs of the day. The entire community of mathematicians, she said, was dominated by detachment from politics and by neutrality in philosophy. The word "dialectics," she noted, did not appear even once in scholarly discussions organized by the mathematical societies of Moscow and Leningrad, whose membership included most of the country's leading mathematicians. She saw the source of trouble not only in the deliberate isolation of mathematicians from the multiform processes of socialist construction but also in the pronounced philosophical disunity among the leading Marxist mathematicians. She reasoned that a relentless critique of the idealistic foundations of logicism, formalism, and intuitionism could provide a rallying point for the forces devoted to achieving unity in Marxist circles and for mounting a campaign against academic isolationism [31].

Adding a new source of urgency to Ianovskaia's arguments, A. A. Maksimov asserted that such orientations as formalism in mathematics brought philosophical idealism to modern physics [58, 36]. In the mathematical "conquest" of "entire branches" of physics, particularly those of more recent origin, Maksimov saw the main source of crisis in modern science.

Kol'man added further to the debate, attacking skeptics who concluded that, because of its concern with formal procedures of quantitative measurement rather than with substantive analysis, mathematics stood above dialectics in nature. He wrote in 1932 that Engels deserved credit for the discovery of "dialectical laws in arithmetic and, primarily, in algebra and higher mathematics," which helped him to oppose the efforts of "bourgeois philosophers" to reduce mathematics to "formal logic" and to treat it as a product of "pure thought." Kol'man criticized Friedrich Adler, the author of *Marksizm i estestvoznanie* [*Marxism and the Natural Sciences*], who asserted that it was possible to create "an algebra of dialectics," but that all efforts to lay the foundations for a "dialectics of algebra" were destined to fail [41, 167].

In 1931, the Communist Academy sponsored the publication of a symposium on the relationship of mathematics to dialectical materialism [64]. The unsigned introductory chapter presented a stinging critique of the Moscow school, especially of N. N. Luzin, its founder and indisputable leader. It claimed that Luzin's apparent detachment from issues of a philosophical nature was but a mask concealing his outright dedication to an idealistic philosophy that reduced the entire world of reality to personal experience. It drew its "evidence" not from Luzin's writings but from Henri Lebesgue's introductory remarks to a book Luzin published in the renowned monographic series established by Emile Borel. While the symposium directly attacked the impractical interests of the Moscow school, it did not ignore the Leningrad school, even though its criticism in this case did not take the form of personal castigation. Generally acclaimed for its dedication to applied mathematics, this school was also well known for its long and distinguished tradition in probability theory, a theory not fully acceptable to the Marxists. Indeed, one contributor to the symposium voiced open dissatisfaction with the primary concern of the Leningrad school with "fluctuations of random variables," a mathematical technique which, he thought, was more applicable to the "anarchic behavior" of the capitalist economy than to the "planned economy" of the Soviet system [74, 29].

Other articles in the symposium dealt with mathematics and politics, the role of dialectical-materialistic methodology in mathematics, the idealistic orientations of the foundations of mathematics, and the methods of teaching mathematics. Kol'man, one of the more militant champions of materialistic mathematics, was in favor of total planning in mathematical research. He called for a concerted effort to create a unified Marxist orientation in the foundations of mathematics with mathematicians waging war on all "outdated" theories. First of all, he said, mathematics must be totally integrated into socialist construction, dialectical materialism, and Soviet politics [42, 68]. As their immediate task, mathematicians must clarify the dialectic of continuity and discreteness as categories of mathematical analysis, link probability theory with other branches of mathematics, reorient mathematics to make it more responsive to practical needs, synchronize logical and historical treatments of mathematical theories, and wage a widely based attack on logicism, intuitionism, and other "idealistic" orientations in the foundations of mathematics [44, 206–211; 16, 215–229; 53, 378–388].

In a paper presented at the Second International Congress of Historians of Science, held in London in 1931, Kol'man argued that modern mathematics was dominated by two general orientations, one emphasizing the study of continuities in natural processes and the other concentrating on the study of discrete phenomena. He thought that differential calculus dealt exclusively with continuity and that set theory, particularly as advanced by Luzin and his followers, recognized only the discrete nature of phenomena [16, 217]. In his opinion, the two orientations, left to their own resources, and continuing to subscribe to an idealistic philosophy, could not achieve a unified mathematical orientation. Although he thought that Marxist theory, relying on the dialectical method, was well equipped to handle the problem of unity, he neither made nor suggested moves in that direction. Kol'man thought that the young and gifted Soviet mathematicians A. O. Gel'fond and L. G. Shnirel'man were engaged in bringing mathematics closer to dialectical materialism.

S. A. Ianovskaia, in full agreement with Kol'man, argued that the real cause of crisis in the foundations of mathematics must be sought in the one-sided preoccupation of scholars with the formal aspects of their science and their near-total disregard of substantive content drawn from real life [25, 250]. She thought that the foundations of mathematics could perform a socially useful role only by establishing harmonious relations between the theoretical postulates of dialectical materialism and the practical needs of socialist construction.

The publication of Marx's "mathematical manuscripts" in 1933 marked an important event in the relations of dialectical materialism to mathematics. The task of interpreting the papers went to Ianovskaia, who made use of this opportunity to establish herself as the chief Marxist spokesperson for modern developments in mathematics. She presented Marx's papers as a model application of dialectical materialism to mathematics and as a stimulus for future work on a broader and more modern Marxist orientation in the foundations of mathematics. In her opinion, Marx attacked "all efforts" to base the foundations of mathematics on absolute—ahistorical—foundations and on a symbolism that reflected philosophical idealism [28, 112–115; 60].

Marx's mathematical papers prepared Ianovskaia for a general critique of "mathematical idealism" from a position of dialectical materialism. She noted that idealistic philosophy was particularly attracted to mathematics and that some Soviet mathematicians demonstrated idealistic inclinations [24, 48–49; 26, 41]. She claimed that mathematics deals with phenomena that are empirical, quantitative, material, historical, and dialectical, although she provided no examples illustrating the work of dialectics in the growth of mathematical knowledge.

Marx's "mathematical manuscripts" advanced two sweeping ideas: they asserted categorically that even the most abstract mathematical theories carried indelible imprints of the social values and styles of thought of their time; and they created the distinct impression that mathematics was, in a sense, an historical discipline sensitive to changes in its own symbolism, substantive foci, and underlying philosophies. At the time of the publication of Marx's mathematical manuscripts, there was considerable discussion about the primary significance of the historical approach to "Marxist mathematics," but no significant changes could be detected. Although the Soviet Union produced rich literature on the history of mathematics from ancient Mesopotamian and Egyptian times to the most recent developments, only a small fragment was written from Marxist theoretical positions [36, 1028–1029]. No general or systematic study of the role of Marxism in mathematics was undertaken during the Stalinist phase of Soviet history.

To give stronger expression to the empirical nature and historical dynamics of mathematical wisdom, T. I. Rainov, a specialist in the sociological and philosophical aspects of science, relied on selected statements by distinguished representatives of 19th-century scientific thought. He cited Bernhard Riemann's claim that "geometrical propositions cannot be derived from a general notion of quantity," and that the distinctive feature of space "can be derived only from experience." He was equally impressed with Hermann Helmholtz's statement that the foundations of geometry rest not on "imaginary principles but on facts of experience" [70, 91–92]. Rainov criticized V. F. Kagan's interpretation of Lobachevskii's and Bolyai's non-Euclidean geometry as "a product of the free creativity of the human intellect" and a triumph for the idea of mathematics as a discipline characterized by strict formalism totally free of ties with actual experience [70, 87–91]. Rainov did not mention that in 1926—on the occasion of the celebration of the 100th anniversary of Lobachevskii's non-Euclidean geometry—Kagan was no longer willing to defend a formalist position in the foundations of mathematics.

In the 1920s, Marxist theorists displayed near-perfect consistency in avoiding criticism directed at individual mathematicians [78]. In the 1930s, by contrast, such attacks were not only common but also widely publicized. They were part of a Stalinist policy aimed at weakening the intelligentsia by means of a frontal attack on independent thought, automatically regarded as a direct challenge to the basic premises of Marxist philosophy.

The Leningrad Mathematical Society, an affiliate of Leningrad University, was in a major crisis at the time of Stalin's rise to power. With its members evenly split into "right" and "left" groups, and with only a few "neutrals," the society was an easy target for Marxist groups engaged in a sustained effort to break the spirit of the scientific intelligentsia [19, 70–72]. The crisis was so deep that the society ceased to exist in 1930 without making a formal announcement. The Communist Academy quickly organized the Society of Mathematicians–Materialists, whose members, mostly young persons, were noted more for their Marxist enthusiasm than for their command of mathematical knowledge. The new society did not escape a series of immediate internal purges; its president was quietly dismissed as a follower of N. I. Bukharin's wing of the Marxist orientation.

At Moscow University, the first major victim of the official war on the scientific intelligentsia was D. F. Egorov, a Soviet pioneer in differential geometry and measure theory [20]. Because of his statements favoring academic autonomy, he was accused of open enmity toward the Soviet system and in 1929 was dismissed as director of the Institute of Mathematics and Mechanics at Moscow University [13, 35–36; 65, 8]. After spending some time in a political prison, he died in a Kazan hospital. V. F. Kagan, the leading expert on non-Euclidean geometry and on the foundations of geometry, was, as noted, publicly criticized for his favorable comment on the formalist orientation in the foundations of mathematics. A. Ia. Khinchin was reminded of his indiscretion in making favorable comments about intuitionism in the 1920s. He was also criticized for omitting Marxist bibliographical sources for his article on "Motion" in the *Bal'shaia sovetskaia entsiklopediia* [*Great Soviet Encyclopedia*]. Ianovskaia went so far as to castigate O. Iu. Schmidt, a Marxist veteran and chief editor of the *Great Soviet Encyclopedia*, for his article on algebra in the same encyclopedia displaying, in her opinion, clear leanings toward Machian neopositivism [27, 308; 97, 314]. Intuitionism, in her opinion, was not an isolated phenomenon: based on intuitive laxity and "free logic," it reminded her of irrationalism and phenomenology in philosophy, vitalism in biology, and subjectivism in “the sociological school” [25, 223]. She had even less faith in Russell’s logicism and Hilbert’s formalism.

At the beginning of the 1930s, Kol’man defined “Marxist mathematics” as a unique body of abstract operations produced with the help of the dialectical method, deeply involved in socialist construction, and subjected to rigorous and centrally integrated planning. He proposed that the national five-year plans for economic development become the prime force in directing the work of mathematics. In his later writing, he was much more involved in criticizing Western mathematical thought than in elaborating the structure of Marxist mathematics.

In 1938, Kol’man chose to make his position clearer by condemning the rebirth of “Pythagoreanism” in modern thought. By “Pythagoreanism” he meant the intellectual striving to make the notion of number the ultimate base and the essence of all reality. In an article published in *Pod znamenem marksizma* [*Under the Banner of Marxism*] in 1938, he aimed his criticism at Arthur Eddington’s determined effort to identify “mathematical theory,” rather than experiment, as the true source of scientific knowledge. To Eddington, according to Kol’man, experiment counted only so far as it provided corroboration for mathematical propositions [45, 144]. Instead of making mathematics a tool of physics, Eddington, in Kol’man’s view, made physics a branch of mathematics. For Kol’man, Eddington was a typical representative of “mathematical idealism.” Kol’man lamented the continued presence of adherents of this orientation among Soviet physicists, typified by F. A. Fock, the most distinguished Soviet contributor to the mathematical apparatus of quantum mechanics [18, 139].

Ideological uncertainties, the sparsity of Marxist experts, and the very dynamic situation in mathematics made it difficult for Marxist writers to undertake more comprehensive and systematic studies in the history of mathematics. They could do no better than offer isolated essays on such scattered topics as Plato as a mathematician, the notion of number and its evolution, the problems of the history of mathematics from the point of view of Marxist dialectics, and the underlying philosophy of Georg Cantor’s contributions. If these essays had a common denominator, it was the effort to assemble arguments in favor of a dialectical approach to the history of mathematics and to make this discipline an indispensable component of mathematical studies.

MATHEMATICIANS AND MARXISM

There were two categories of established mathematicians who accommodated themselves to Marxism. The first category consisted of young mathematicians with established scientific reputations, who participated, on a limited scale, in various activities sponsored by Marxist organizations. The most prominent members of this group were A. O. Gel’fond, who worked in group theory, L. S. Pontryagin, whose duality theorem presented in the American *Annals of Mathematics* in 1934 helped to establish a set-theoretical orientation in algebra as well as his own international reputation [68], and L. G. Shnirel’man, who applied topological methods to the study of the calculus of variation. They played no significant role in building a bridge between the community of mathematicians and the Communist Academy or between Marxist and non-Marxist philosophies of mathematics. In fact, Gel’fond and Shnirel’man took serious exception to some of the basic claims presented by Kol’man in his newly published book on the subject matter and method of mathematics [43; 21; 34; 9].

The second category consisted of an increasing number of leading mathematicians who tried to make adjustments to Marxist philosophy without participating in Communist political activities. It was not unusual for the publications of the Communist Academy and other Marxist organizations to carry articles by such well-known mathematicians as P. S. Aleksandrov, V. Glivenko, A. N. Kolmogorov, and V. I. Gnedenko. These scholars made favorable references to dialectical materialism, without specific references to philosophical and ideological issues. They contributed papers to such Marxist journals as the *Front nauki i tekhniki* [*Front of Science and Technology*] but made little or no effort to add to the theoretical strength of dialectical materialism as a philosophical and ideological orientation.

In a report presented at the Second All-Union Congress of Mathematicians, held in Leningrad in 1934, Aleksandrov asserted that set theory represented a major advance in mathematics and that most mathematicians in the Soviet Union as well as abroad worked within its framework which included not only the general and special theories of sets but also the theory of functions of a real variable (embracing, among other fields, the theory of integration and the theory of trigonometric series), set-theoretical topology, and functional analysis. Aleksandrov argued that "traditional mathematics," represented by the calculi of Euler and Lagrange and their subsequent refinements, presented a "method or apparatus" for the solution of previously formulated problems in science and technology [7, 32]. In his opinion, "new mathematics," based on set theory, made it possible to open new paths to knowledge on "systems of relations" in the material world, leading to higher levels of abstraction, the foundations of the modern philosophy of nature. What Aleksandrov actually meant was that while traditional mathematics, dominated by differential and integral calculus, handled problems formulated by physics, modern physics often handled problems formulated by mathematics and ruled by set theory.

Aleksandrov also stated that mathematicians must avoid two common errors in interpreting set theory: one error leading to excessively theoretical constructions deprived of contact with concrete reality, and the second error made by undue emphasis on "crude applicability," a result of the oversimplification of theoretical elaborations. He did not state it directly, but it was clear that, in his opinion, the first type of transgression was common among Western mathematicians, while the second type was deeply ingrained in the thinking of Soviet Marxists. Aleksandrov pleased Marxist writers by relating set theory directly to the study of "the material world;" he satisfied mathematicians by recognizing the scientific power and superior achievements and promises of Cantor's creation [7, 32–33].

V. I. Glivenko worked on the general aspects of set theory, the construction of implicit functions, and mathematical statistics. He also explored the mathematical background of intuitionist logic and went a long way in defending Marxist approaches to mathematics. Finding formalism and logicism totally unacceptable, he praised intuitionism for introducing new—non-Aristotelian—logic in constructing theoretical objects of mathematical scrutiny; but he also shared the Marxist view of intuitionism as "an idealism that has become aggressive in its striving to control science in all its problems and methods" [22, 59]. Glivenko agreed with Kolmogorov's earlier claim that intuitionism opened new paths for promising exploration in areas where mathematics and logic met. Following Kolmogorov's arguments, he favored a kind of intuitionism that did not contradict the classical paths of mathematics. Operating on two fronts, Glivenko sided with Marxist critics of the idealistic background of the leading Western orientations in the foundations of mathematics and brought the thinking of Soviet mathematicians closer to the theoretical structure of Brouwer's intuitionism.

A. N. Kolmogorov's article on mathematics in volume eight (1938) of the *Great Soviet Encyclopedia* provided an excellent example of the new alliance of mathematics and dialectical materialism [47]. The world-renowned mathematician stated directly that the "dialectical development of mathematics" followed three general paths: the creation of algorithms for new theories; the development of new algorithms solving problems beyond the reach of old algorithms; and the construction of new algorithms for an expedient solution to problems only partially accessible to old algorithms. In other words, the "dialectical development of mathematics" consists of developing mathematical procedures for new scientific theories, advancing new mathematical procedures for old scientific theories, and making old procedures more efficient and comprehensive. What was "dialectical" about all this? Kolmogorov gave no detailed answer; he merely noted that "mathematics fears no contradictions: a new system of axioms can contradict an old system . . . and the rules of a new algorithm can contradict the rules of an old algorithm" [47, 394]. He did note, however, that the modern axiomatic method did not violate the rule that "every mathematical theory corresponds to the axioms of reality" and is therefore empirical in its origin [47, 393]. He cited Marx in support of his claim that mathematical analysis could play only a secondary role in the social sciences, preoccupied with dominant idiosyncratic features of individual stages of socioeconomic development [47, 384]. The tone of Kolmogorov's argumentation made him acceptable to the Marxists, but it did not entitle him to call himself a Marxist. His article, however, acquired a reputation as the most complete "Marxist" interpretation of mathematics available at the time. Almost 10 years later, Aleksandrov described the article as "the only effort in Russian literature to present the broad philosophical base of the modern mathematical world view [8, 32; 34, 8–9]. He did not identify it as a Marxist essay; in 1957, however, A. D. Aleksandrov referred to it as "the first detailed Marxist explanation of mathematics" [1, 6].

Kolmogorov's nominal acceptance of dialectical materialism in the encyclopedia article did not prevent him from defending genetics in 1938, the main target of Lysenko's war on modern biology, as a true science based on the mathematical theory of probability. Kolmogorov made nominal concessions to Marxist thought in order to achieve concrete results in protecting mathematics from the debilitating excesses of ideological and political attacks. His allegiance to Marxist ideology was fortuitous and superficial; his major achievements in mathematics were profound in a truly scientific sense. As a person belonging to a family with deep aristocratic roots, he had to be particularly skillful in making adjustments to the proletarian bias of Bolshevik ideology.

In 1936, Kolmogorov had joined a group of Marxist writers and mathematicians in publishing a collection of essays on mathematics as a unique body of knowledge and a cultural phenomenon of immense complexity [73]. His brief article, originally published in the *Front of Science and Technology* in 1934, stated that the rising level of abstraction made it possible for modern mathematics to tackle a broader range of "real phenomena" and, at the same time, to be less rigid and "schematic" than classical mathematics [49, 25–28]. The 19th century, in his opinion, recorded two major developments in mathematics: vast methodological improvements in two 17th-century legacies, infinitesimal calculus and analytical geometry, and the opening of new problems to mathematical inquiry by Georg

Cantor's set theory. Whereas classical analysis concentrated exclusively on continuities in nature, the real strength of set theory was in opening discontinuities in nature and society to mathematical treatment. In algebra and set theory, he saw the future stronghold of mathematics. Kolmogorov made no effort to discuss the uneasy relations between set theory and Marxist thought. Contrary to the reigning Marxist view, he anticipated a promising future for the axiomatic method in mathematics.

Ianovskaia followed a completely different line of reasoning. She presented many examples of Western idealistic "aberrations" to illustrate the "decadence" of "bourgeois science" and to indicate the modes of thinking Soviet mathematicians must avoid [32, 37–43]. She attacked David Hilbert's claim that the consistency of axioms displayed by arithmetic represented axiomatic consistency of mathematics in general as a retreat into metaphysical thinking with no links to concrete reality. Nor was she ready to accept Henri Poincaré's argument that the verb "to exist" could mean nothing else but elimination of inconsistencies. On this occasion, however, she refrained from criticizing the philosophical background of Cantor's mathematical legacy.

Mathematics was helped by its very nature—by the bewildering complexity of its endless structures and the celestial remoteness of its new axioms—to navigate a course that removed it from daily involvement in ideological warfare. The resounding international recognition of the achievements of Soviet mathematicians in such abstract branches of mathematics as the axiomatics of probability theory, point-set topology, and number theory placed this science in the forefront of national treasures and protected it from reckless attacks by zealous ideologues. The leading Marxist mathematicians often relied on philosophical rhetoric as an appeasement technique to protect the best interests of their discipline. Ianovskaia continued her defense of dialectical materialism in the house of mathematics, but she also became a staunch supporter of burgeoning efforts to give mathematical logic and the foundations of mathematics a broader footing and a more prominent place in the family of sciences [30, 45].

Built into the general Marxist orientation in the philosophy of mathematics were a number of assumptions. First, every mathematical axiom had an empirical base and was therefore grounded in the real world. Second, all Western orientations in the foundations of mathematics—dominated by formalism, logicism, and intuitionism—were manifestations of idealistic philosophy. Third, the function of mathematics was not only to formulate theorems and fashion methods but also to work out procedures for applying mathematical theories to various branches of scientific knowledge and engineering. A mathematician could live at peace with philosophers by a mere endorsement of these general principles. But he could also try to work out his own modifications of the dictates of philosophical orthodoxy.

The community of mathematicians boasted a nucleus of leading scholars who accepted Marxist philosophy in principle, but who favored changes in dialectical materialism to make it more adaptable to the substance and the spirit of modern science. No leading mathematician accepted the strategy of Marxist philosophers which demanded a recasting of the theoretical achievements of modern science to make them acceptable to Marxist orthodoxy. Mathematics did not produce a Lysenko, that is, a dominant figure in the field who sacrificed the interests of science to the interests of dialectical materialism. In their search for satisfactory modifications of Marxist views, mathematicians crystallized a philosophy that met the Marxist demands in principle and yet served the scientific spirit of their discipline. This philosophy consisted of four principles.

First, all mathematical axioms and theorems have an empirical base, but it is equally true that the more removed they are from their empirical base—that is, the more abstract they are—the wider the scope of their theoretical and practical applicability. The search for higher levels of mathematical abstraction is equivalent to the search for a wider applicability of mathematical methods; this made topology, mathematical logic, and number theory the most practical of all mathematical disciplines, at least in their inherent potentialities. Three prominent mathematicians, Aleksandrov, Gnedenko, and V. V. Stepanov, stated in a joint article: “Set-theoretical mathematics, characterized by generality and ... abstract methods, belongs to sciences of the highest theoretical engagement that seem to have only the most tenuous ties with the reality of the practical world. However, this is not so: at present, the ideas of this theory are dominant in a number of mathematical branches directly linked with the natural sciences. Through functional analysis, probability theory, and the theory of differential equations, it exercises a strong influence on modern physics, and, through physics, on technology” [10, 42].

Second, Western orientations in the foundations of mathematics are steeped in philosophical idealism and should be criticized on that ground. However, they are not equally idealistic and some—particularly intuitionism—contain kernels of thought that should be carefully explored and integrated into the Marxist foundations of mathematics [21, 53, 58–59]. Formalism and logicism were far removed from Marxist interests because they concentrated exclusively on “internal” determinants of mathematical thought—the former on the axiomatization of mathematical concepts, and the latter on the rigor of logic of axiomatic structures. Intuitionism, by contrast, focused on thought that allowed for “external” (social) input in the genesis of mathematical thought [11, 10]. This externalism made intuitionism open to compromises with Marxist theory.

Third, there is a pressing need for “applied mathematics” to broaden and expedite the flow of mathematical methods to the natural sciences and technology. Work in applied mathematics, however, should be clearly related to work in pure mathematics, the mathematician’s main concern. No mathematician should be given practical assignments outside his or her main interest in pure mathematics. Mathematicians need to be constantly reminded that their task is to study “forms” of “real” phenomena, and that the same mathematical “forms” can profitably be applied to different domains of reality.

Fourth, mathematics advances by the inner logic of its developmental momentum or by responses to technological problems. Kolmogorov admitted that Soviet mathematics was by tradition lax in responding to the technological needs of society. He responded favorably to Stalinist pressures for more extensive involvement of mathematics in the ongoing industrialization of the country [50, 39–42]. He warned, however, that the growth of practical assignments should not impede the internally generated impulse for raising mathematics to higher levels of abstraction.

This strategy found strong support in the community of mathematicians. It endorsed the Marxist philosophy of mathematics but not until it conveyed its own interpretation, the only choice it had at its disposal. If there was criticism, it came from outside the ranks of mathematicians. The physicist Abram Ioffe was one such external critic. Otherwise known and respected as a tactful defender of the interests of the scientific community, he complained in 1942 that the leading Soviet mathematicians concentrated on the most abstract branches of mathematics—number theory, group theory, set theory, topology, and algebra—preventing them from making significant contributions to the technical progress of the country [33, 151]. In a tactical concession to Stalinist ideologues, he also said that Soviet mathematicians had made little effort to meet the needs of modern physics. Ioffe's complaints produced no serious reverberations. He made his critical statements when he became a member of the Communist Party, and he obviously echoed the views of government authorities.

In 1936, at the time of the most intense Stalinist pressure in favor of applied science, Kolmogorov made a series of statements on "pure" and "applied" mathematics, obviously designed, in their total effect, to appeal to both mathematicians and Marxist theorists [50, 41–42]. To satisfy mathematicians, he noted that it was virtually impossible to draw a line separating "pure" from "applied" mathematics. Today, he said, some of the "purest"—that is, the most abstract—branches of mathematics are the "basic apparatus" of entire branches of natural science: number theory plays a major role in crystallography, and topology provides the main methods in the study of chemical equilibria. Obviously, he was concerned with the application of mathematics to various branches of natural science. To satisfy both mathematicians and Marxist philosophers, he noted that the country was much in need, not of a shift of emphasis from "pure" to "applied" mathematics, but of an expansion and intensification of the flow of the most abstract mathematical knowledge to modern technology. The technical application of mathematics, traditionally underdeveloped, should be raised to the heights reached by "pure" mathematics.

S. N. Bernstein was an outstanding representative of mathematicians who did not believe in the possibility of a *modus vivendi* in relations between mathematics and dialectical materialism. Widely known for his effort to advance an axiomatic arrangement of the theory of probability as well as for his work on differential equations and on what he and his school identified as constructive function theory, he worked for many years as a professor of mathematics at various Kharkov institutions of higher education. In 1929, in a recognition of his achievements, he was elected a full member of the USSR Academy of Sciences. During the late 1920s and early 1930s, he argued publicly that the official campaign to introduce dialectical materialism in mathematics threatened to turn this science into an intellectual wasteland. He made no secret of his deep conviction that mathematics and dialectical materialism had nothing in common. In contrast to Marxist interpreters, he argued that mathematics was fully independent of political commitments and that it stood above social-class interests [12, 59–61; 13, 38]. Social-class consciousness, he argued, could not explain why Lagrange, a marquis, created an atheistic cosmic view, and why Galois, a revolutionary, produced one of the most abstract theories in mathematics. Bernstein was one of the rare mathematicians who defended N. N. Luzin at a time when he was targeted for strident attacks by Marxist journalists and organized professional and political groups.

With the exception of minor attacks by local Communist vigilantes, the authorities chose not to undertake retaliatory measures against Bernstein and did not contest his election to membership in the USSR Academy of Sciences located in Leningrad. In 1934, the Academy moved to Moscow where Bernstein spent the rest of his long life working at the Institute of Mathematics. There, he headed the section on probability theory. After the early 1930s, he made no public statements critical of dialectical materialism and no detours from his professional duties. No doubt, he was silenced by Stalin's rapidly growing reliance on extreme forms of suppression of heterodox thought. In 1942, he received a Stalin Prize for distinguished contributions to science, and between 1952 and 1954, the Academy of Sciences published his collected works in two volumes. In 1955, the Paris Academy of Sciences elected him a foreign member.

THE N. N. LUZIN AFFAIR

In the mid-1930s, N. N. Luzin, a leading member of the Soviet community of mathematicians and a member of the USSR Academy of Sciences, became a target of vicious and bitter Marxist attacks that lasted several months [35; 55]. According to Aleksandrov, a majority of Moscow mathematicians were, directly or indirectly, Luzin's students [5, 135]. Luzin's main contribution, Aleksandrov noted, was "in creating a totally new orientation built on set theory and its applications" [9, 76]. Luzin's troubles with the guardians of Marxist ideology in mathematics came at the time Stalin had unveiled his plan to make dialectical materialism both the official philosophy of Soviet science and a monolithic system of Marxist epistemology. In 1931, Ianovskaia made a denunciatory announcement that Luzin had publicly admitted that he and his school had no interest whatever in the practical applicability of their theories [31, 89]. She also added that A. Ia. Khinchin, a noted expert in probability theory, had made a public announcement endorsing Luzin's statement. Perhaps because of his great prestige in the scientific community, Luzin was a logical choice to become a rallying point—or a model—for passive resistance to the ongoing effort to subject science to Marxist philosophy and ethics. Initially, no action was undertaken to discipline Luzin. In 1936, however, the Communist Party ordered the academic authorities to appoint a special commission to make a thorough study of his alleged indiscretions and to make a recommendation as to whether he should be expelled from the Academy of Sciences. The commission recommended that Luzin not be penalized by demotion, but that, instead, he be given a chance to redeem himself. Should Luzin refuse to discontinue his "unpatriotic behavior," the commission recommended that proceedings be started for his dismissal from the Academy membership [35, 113]. The commission did allege, however, that his scholarship was very unsatisfatory and that his loyalty to the Soviet state was questionable. To give their decision more weight, his accusers charged him with having presented as his own many ideas originated by his students and with having gone out of his way to undervalue the scientific achievements of the Soviet Union [67, 7–10; 69, 123–125; 35, 102–113].

Marxist philosophers of mathematics lost no time in mounting a scorching attack on the alleged anti-Marxism of Luzin's general intellectual orientation. They concentrated on Luzin's ties with the Paris school in set theory headed by Lebesgue, Borel, and Baire. Luzin, according to Marxist writers, borrowed from this school not only a general mathematical outlook on set theory but also a strong dose of "subjective idealism" of a metaphysical nature [62, 85]. In 1930, Lebesgue wrote the "preface" to Luzin's newly published *Leçons sur les ensembles analytiques et leurs applications* commending the author's contribution to the unity of mathematics and philosophy [54]. Marxist critics took Lebesgue's praise as confirmation of Luzin's affinity with the philosophical and foundational views of the general orientation of effectivism, a cross between formalism and intuitionism and a stance usually associated with the Paris school in set theory. They especially resented the international publicity given to “idealism” as the alleged philosophical foundation of the leading Soviet mathematical school. Used almost exclusively in the Soviet Union, the label “effectivism” described a movement started by Emile Borel’s rejection of Zermelo’s axiom of choice and the theorem of well-ordered sets based on it [40, 210].

V. Molodshii, a Marxist commentator on ideologically sensitive developments in modern mathematics, wrote a book on effectivism as a fundamental orientation supported by Luzin. Effectivism, he thought, rested on two basic principles. First, it was built on the assumption that the subject of mathematics had no counterpart in the real world—in the objectively existing external universe; it was treated exclusively as a pure product of the human mind. This stood in direct opposition to the Leninist principle that all mathematics consists of true reflections of a world independent of the human mind. Second, it was guided by the principle that the intuition of particularly gifted individuals, rather than the mechanisms of logic, was the basic force behind the accumulation of mathematical knowledge. In other words, effectivism shared the “sins” of Brouwer’s and Weyl’s intuitionism, roundly attacked by spokespersons for Marxist mathematics [62, 56–57].

Molodshii did not dissect the inner mechanism of effectivist thought, but merely categorized it as idealistic and antagonistic to Marxist theory. He used the same verbal armor in attacking effectivism and Luzin that other Marxist writers used in combatting formalism, logicism, and intuitionism. Unable to find direct links between Luzin’s philosophy and effectivism, he relied on implication, innuendo, and guilt by association. In a special article published in 1936, Molodshii tried to present a picture of Luzin as an unwavering enemy of the Soviet political system [63, 8–18]. The article presented Luzin as a sentimental and ideological ally of the “old” Moscow Mathematical School, founded by N. V. Bugaev at the end of the nineteenth century, which combined work in mathematics with a defense of the reactionary political views of the Black Hundreds [63, 9]. Molodoshii also admitted that the Bolshevik attack on Egorov that resulted in his dismissal as director of the Institute of Mathematics and Mechanics (mentioned above) was actually an attack on the lingering residues of the “old school.” Egorov was generally known as a scholar who had exercised a profound influence on Luzin’s early work in mathematics [20].

The Great Soviet Encyclopedia acted quickly and resolutely in subjecting Luzin to the most severe punishment it could enforce: it did not include an article on his life and work. The general article on mathematics, which had a special section on Russian and Soviet mathematics, made only a parenthetical reference to Luzin’s work on projective sets. The same encyclopedia, in the article on set theory, counted Luzin among the representatives of effectivism, labeling it a “variation of idealism,” totally unacceptable to Marxism [6, 591]. It was clear that the editors of the encyclopedia were ordered to portray Luzin as an academician whose contributions to mathematics were more than neutralized by his unpardonable digressions in philosophy and alleged anti-Sovietism.

“Luzinshchina” became a new label for a crime against the state and a new warning to academic recalcitrants [77, 8–9]. For his part, however, Luzin continued to avoid collective philosophical ventures by mixed teams of mathematicians and Marxist theorists and to stay strictly within the realm of his science. No one—not even the Marxist critics—challenged his rights to be called the indisputable leader of one of the most powerful mathematical schools in the Soviet Union. The blistering attack on Luzin was only in part a punishment for behavior considered injurious to the new political system; it was, above all else, a stern and unambiguous reminder to all scholars that political loyalty and ideological orthodoxy had now become carefully scrutinized aspects of scholarship. Most leading Soviet mathematicians remained loyal to Luzin and used every opportunity to record their respect for him as a great mathematician, an illustrious teacher, and the pride of the Soviet scientific community. They referred to the vast domain of his influence as Luzitania, the mathematicians' promised land.

Many Communist organizations called special meetings to rehash Luzin's "crimes" and to garner new examples of his alleged unpatriotic deeds and unprofessional behavior. Rancorous attacks by younger and little-known mathematicians, including a number of Luzin's students, provided a lamentable index of the Stalinist campaign to break the spirit of the scientific community. Overflowing with attacks on Luzin, the press was prohibited from publishing serious counterarguments either of a scientific or of a philosophical–ideological nature. Luzin was neither asked nor allowed to defend himself. The attack on him was engineered by government authorities with the aim of weakening the dominant role of the set-theoretical orientation in the community of Soviet mathematicians. Set theory was officially considered the most formidable foe of Marxist epistemology as well as a mathematical orientation farthest removed from Stalinist socialist construction.

L. S. Pontryagin, active in the field of topology, regarded Luzin's position in the community of mathematicians as an example of unearned authority in science, a result of the unwanted practice of favoring established scholars over the younger and lesser lights in the field [69, 124]. He criticized the journal *Matematicheskii Sbornik* [Mathematical Collection] for rejecting a paper submitted by a young mathematician who took three pages to explain a problem that took Luzin 160 pages [69, 124]. Pontryagin's claim that the Soviet community of mathematicians was generally dominated by the authority of established contributors to the field was an endorsement of the then current government policy favoring younger scientists exposed to Marxist indoctrination as part of their formal education and less prepared to tackle the problems of the onrushing streams of mathematical thought.

At a special meeting at Moscow University in 1936, Kolmogorov, the brightest star in Soviet mathematics, was called upon to make his contribution to the anti-Luzin campaign. In an effort to share the "guilt" with Luzin, he noted that he too had committed "unpardonable errors"—such as publishing his papers in foreign journals before they were published in Russian [69, 124]. He said that some of Luzin's indiscretions were considered normal behavior by the community of mathematicians, which was not always provided with guidelines for expected behavior. Proper advice and guidance would have helped Luzin to avoid at least some of his "indiscretions." Kolmogorov reminded his listeners of the impressive proportions of Luzin's leadership in Soviet mathematics.

Among many other grim consequences, the Luzin affair brought an end to the publication of Soviet mathematicians' scholarly papers in foreign journals, thereby ending valuable contact with the international community of mathematicians. Soviet mathematicians were also discouraged from visiting Western centers of learning and from maintaining personal contact with foreign colleagues. Visits by foreign mathematicians, previously quite common, came to an abrupt end. The policy of isolation from the foreign world adversely affected the entire Soviet community of scientists.

STALINISM AND MATHEMATICS AFTER 1940

World War II generated intense government efforts to bolster Russian nationalism as a unique source of Soviet patriotism. There were two distinct phases in making nationalism an ideological force of primary significance. During the first phase, emphasis was strictly on the substantive breadth and historical depth of mathematical thought in Russia. During the second phase, dominated by anticosmopolitanism during the waning years of the Stalinist era, the study of the national history of mathematics placed a great emphasis not only on the "progressive" character of Russian mathematics but also on the "decadence" of its Western counterpart. It was dominated by a trenchant ethnocentric bias.

During the first phase, ending in 1947, the name of Lobachevskii figured prominently in the new climate of nationalist fervor. Earlier, the two major Soviet schools in mathematics—the Leningrad school, which stressed number theory and probability theory, and the Moscow school, built on multiple ramifications of set theory—gave no attention to Lobachevskii's legacy. In 1937, the Soviet Union celebrated the 20th anniversary of the October Revolution, and, to mark the occasion, the popular Russian journal, *Priroda* [*Nature*] devoted an entire issue to the achievements of Soviet science. The essay on mathematics, written by R. O. Kuz'min, offered a laconic but lucid overview of main trends in the growth of Soviet mathematics [51, 9–190]. Kuz'min, however, made no effort to include a statement on Soviet studies focusing on Lobachevskii's non-Euclidean geometry. Perhaps his reticence revealed the true situation in the field, which could show only a few scattered essays on this matter. Marxist writers were uncertain about their approach to Lobachevskii's legacy in the spirit of dialectical materialism. Developments in the main body of mathematics did not cross paths with non-Euclidean geometry. B. G. Kuznetsov published an article on Lobachevskii in *Under the Banner of Marxism* that dealt mainly with the negative attitude of early Russian mathematicians toward Lobachevskii's daring ideas [52].

The situation changed dramatically during World War II. With lightning speed, Lobachevskii was made the leading light in a growing historical literature. Both Marxist writers and non-Marxist mathematicians supplied vivid details on Lobachevskii's life and work. Both groups followed the English mathematician, William Kingdon Clifford, who identified Lobachevskii as "the Copernicus of Geometry" [15, 553].

As treated by mathematicians, Lobachevskii received credit not only for lodging geometry in an empirical substratum but also for serving as an imaginative pioneer of mathematical trends leading to advances in mathematical analysis, function theory, the theory of relativity, relativistic mechanics, the foundations of mathematics, and the general theory of space. The publication of Lobachevskii's collected works (volume one appeared in 1946), of Modzalevskii's unusually rich collection of biographical material related to Lobachevskii, and of Kagan's biography of Lobachevskii recaptured one of the brightest pages in the annals of Russian scientific thought [56; 61; 37].

Aleksandrov praised Lobachevskii for his pioneering role in the advancement of the axiomatic method in mathematics, which he considered a major contribution to modern science [4, 42]. He went so far as to acknowledge Lobachevskii's contribution to mathematical trends leading to the triumph of set theory [4, 43]. Kolmogorov wrote of Lobachevskii's general influence on the branches of 19th-century mathematics that, in his opinion, made use of a geometry not based on Euclidean principles. Lobachevskii, in Kolmogorov's opinion, was the first scholar to make constructive use of the axiomatic method [48, 124].

Mathematicians worked assiduously to meet the demands for historical studies of their science within the Russian national context. In 1946, Gnedenko published the first history of Russian mathematics [23], and in 1947, B. N. Delone published a book on the St. Petersburg school in number theory [17]. Also in 1947, Moscow University published a memorable collection of essays, written by such eminent mathematicians as Kolmogorov, Aleksandrov, Bernstein, and Pontryagin, on the contributions of Russian mathematicians to “the development of world culture” [71]. While satisfying the elementary needs of early postwar nationalism, the essays were careful and objective accounts of Russian achievements in mathematics. None expressed national fervor, and none underrated Western mathematical thought on scientific and ideological grounds.

The Marxist group, represented by Kol’man and Maksimov, concentrated on Lobachevskii not only as a major contributor to both Russian and world science but also as a powerful exponent of the materialistic tradition of Russian scientific thought [80, 21; 81, 95]. In his philosophy they detected embryonic dialectics. They made no sweeping attacks on Western interpretations of non-Euclidean geometry as an expression of “mathematical idealism.”

The second phase of aroused nationalism began in 1947 and was dominated by government-inspired ethnocentric attacks on cosmopolitanism. As officially defined, cosmopolitanism implied a servile attitude toward Western values and scientific ideas steeped in philosophies incompatible with dialectical materialism. Marxist attacks concentrated on two unacceptable features of Western mathematics: renewed “Pythagoreanism,” which stressed “form” rather than “content” of mathematical operations, and the exclusive dependence on formal–logical analysis in the construction and arrangement of mathematical concepts [40]. To Marxist writers, geometry was a reflection of the spatial laws of nature, not a construction of the human mind as maintained by the reigning “neo-Platonism” dominating Western thought.

Ianovskaia, as always representing the Marxist view, could see no good reason for linking Lobachevskii’s non-Euclidean geometry with the formalist advocacy of axiomatic method. She regarded the formalist orientation in the foundations of mathematics, which favored the axiomatization of mathematics, as an idealistic delusion [29, 34]. She also argued that Henri Poincaré and Felix Klein treated Lobachevskii’s geometry as a “logical exercise” and that Marxist writers recognized “the supremacy of experiment and practical life over logic” [29, 33; 72, 17; 59, 214–215]. Lobachevskii, she wrote, did not deny the importance of logical rigor in mathematical constructions, but he found it essential to search for “practical” support in astronomical observations. He looked for proofs not in the subjective activity of the human mind but in the objective existence of the external world. Lobachevskii’s non-Euclidean geometry was elevated to a position of major force in the struggle against the kinds of “idealism” represented by Kant’s apriorism, John Stuart Mill’s “subjective empiricism,” and Poincaré’s “conventionalism” [29, 30–33].

Whether they attacked the “idealism” of Western mathematics or traced the “materialistic” roots and manifestations of the Russian mathematical tradition, Marxist writers were interested primarily in protecting the ideological sanctity of dialectical materialism. Their task was to check the spread of Western philosophical and foundational principles of mathematics in the Soviet Union. Under various guises and with adroit compromises, many Western ideas were firmly entrenched in the Soviet community of mathematicians and were gradually becoming a growing and dynamic reality.

The publication of a collection of essays—entitled *Sto dvadtsat' piat' let neevklidovoi geometrii Lobachevskogo* [*One Hundred and Twenty-Five Years of Non-Euclidean Geometry*]—in 1952 was the first work on Lobachevskii that brought together Marxist philosophers and mathematicians [66]. The two groups were far apart in their views: while the chief representative of the philosophers presented the standard Marxist interpretation of Lobachevskii's epistemological views as an expression of materialistic philosophy, the mathematicians, a majority of the contributors, avoided philosophical involvement of any kind and stayed strictly in the domain of scientific analysis.

Khinchin was among the rare mathematicians who took part in the ongoing philosophical discussion by contributing a critical paper on the idealistic underpinnings of probability theory in the West to the symposium on "the philosophical questions of modern physics" held in Moscow in 1952, but his article was clearly the least belligerent in the thick volume. He concentrated on the Machian epistemology of Richard von Mises's frequency theory of probability, which, he thought, was based on "an agnostic rejection of statistical regularities expressing the objective nature of phenomena" [38, 529]. He could mention only one Soviet scientist—M. A. Leontovich—who "openly favored von Mises' theory"; moreover this scientist was not a mathematician but a physicist, who made respectable contributions to his science [38, 528].

Efforts to make mathematics a Marxist discipline encountered three major difficulties. First, the study of the social roots of mathematics did not advance beyond an embryonic stage: it did not produce a single comprehensive and systematic study applying Marxist social theory to the evolution of mathematical thought. The published papers were noted more for their programmatic pronouncements on the promises of Marxist approaches to mathematics than for their strict adherence to the accepted standards of historical scholarship. For inexplicable reasons, Marxist scholars demonstrated only a marginal interest in the evolution of mathematics in Russia.

Second, Marxist mathematics encountered serious problems in its effort to study the role of dialectical processes in the growth of mathematical knowledge. Whereas the first problem was related to external conditions influencing the growth of mathematics, the second problem dealt with the internal dynamics of mathematical theory. The history of mathematics, as Marxist writers saw it, was the history of dialectical reconciliations of such opposites as infinity and finitude, continuity and discreteness, differentiation and integration, and Euclidean and non-Euclidean spaces. Marxist experts concentrated their attention on dialectics as a philosophical attitude; they made little use of dialectics as a method of understanding concrete situations in the internal dynamics of mathematics. While recognizing the dialectical reconcilability of opposite principles explaining the work of nature, they failed to recognize the reconcilability of opposite systems of philosophical thought. They, for example, advocated the full rejection of all orientations in the foundations of mathematics based on "idealistic" philosophies.

Third, Marxist theorists placed more emphasis on the foundations of mathematics than they could handle. Their approach to this branch of mathematics was negative: it concentrated exclusively on exposing the alleged weaknesses of formalism, logicism, intuitionism, and effectivism. Marxist criticism of the philosophical foundations of set theory was especially elaborate and persistent during the 1930s [21; 24]. Marxist commentators identified a special variety of philosophical idealism behind every foundational orientation.

Intuitionism, for example, was identified as an offshoot of "subjective idealism" and effectivism as a brand of Machian neopositivism. Marxist critics made no effort to crystallize a comprehensive and integrated replacement for the condemned orientations.

In contrast to the philosophers, mathematicians did not recognize the possibility of—or the need for—a Marxist mathematics. Unlike the philosophers, they continued to look to the West for suggestive ideas in reorienting the foundations of mathematics and in expanding the scope of mathematical logic. Since Marxism was the only philosophy sanctioned in the USSR, they tried to bring borrowed Western principles closer to the general positions of Leninist epistemology. Often their recognition of dialectical materialism was a tactical concession rather than a substantive accommodation.

Stalinist writers paid only fleeting attention to the cognitive processes leading to the formation of mathematical concepts and theories; they concentrated almost exclusively on ideological messages and general technological potentials of established mathematical tools. Biographical studies generally tended to follow the paths of conventional historiography. In their discussions of historical themes, Marxist scholars were interested more in buttressing the philosophical metaphors of Marxist ideology than in elucidating the inner logic leading to the growth of new ideas and procedures.

In 1947, the USSR Academy of Sciences observed the thirtieth anniversary of the Soviet Union by publishing, along with other massive reports, *Matematika v SSSR za tridtsal let* [*Thirty Years of Soviet Mathematics*], a record of national achievement in the main branches of mathematical knowledge. The lead chapter, written by Ianovskaia, highlighted Soviet achievements in mathematical logic and the foundations of mathematics. Ianovskaia lamented the low national productivity in these two closely related disciplines and pleaded for their future expansion [30, 45]. In recognition of the pressing need for the concerted study of logical and foundational aspects of mathematics, three major Western works were translated into Russian: *The Foundations of Geometry* by David Hilbert, *The Foundations of Theoretical Logic* by Hilbert and Wilhelm Ackermann, and *The Introduction to Logic* by Alfred Tarski. Marxist critics were quick to record their objections to the general orientation of these works: they criticized the sole concern with "forms" of logical principles without relation to material "contents," and they rejected the idea suggested—especially by Tarski—of making formal logic a universal methodology [76, 336–337]. Negative philosophical criticism did not prevent these studies from playing a major role in strengthening the position of mathematical logic and the foundations of mathematics in the field of exact sciences, particularly in the age of post-Stalinist thaw.

A MARXIST DETOUR

In 1951, A. D. Aleksandrov (as distinct from P. S. Aleksandrov) published two articles in *Priroda* which concentrated on the relevance of Lenin's philosophical views for mathematics and the alleged weaknesses of "mathematical idealism" [2; 3]. He was the first mathematician who was a member of both the Academy of Sciences and the Communist Party and a deeply involved student and ardent defender of the Marxist philosophy of science. It was his work in the general theory of curves and surfaces, which covered both regular surfaces of classical differential geometry and such unsmooth surfaces as polyhedra and arbitrary convex sets, that earned him membership in the Academy. And it was his willingness to undertake a Marxist philosophical analysis of mathematics on a more systematic scale that catapulted him to the position of a leading critic of idealism in mathematical thought.

In the *Priroda* articles, Aleksandrov ranged far and wide in comparing the main characteristics of Marxist and "idealistic" philosophies of mathematics as contradictory orientations. In an obvious concession to Stalinist ideologues, he staged a savage attack on all the main Western orientations in the foundations of mathematics [3, 3–6]. In previous years, such leading mathematicians as Kolmogorov and Khinchin had expressed views on Brouwer's intuitionism as a philosophy containing ideas acceptable to Marxists, at least in principle. Now Aleksandrov could offer only a sweeping rejection of intuitionism as an extreme subjective orientation, completely devoid of relations with material reality [3, 6]. He did not elaborate on his criticism and did not bring patriotic virtues into the debate. In Kolmogorov's treatment of axioms be detected a lone digression from the otherwise impeccable "materialistic" orientation [3, 6]. Kolmogorov "erred" in absolutizing axioms and treating them as closed systems, not subject to modifying influences of practical experience and theoretical conflicts [3, 7].

One cannot help but feel that Aleksandrov's defense of Leninism–Stalinism in mathematics was only an adroit method for modulating Marxist views to make them less discordant with the ideas held by the community of mathematicians. Aleksandrov argued, for example, that new mathematical knowledge must pass not only the test of practical application (as orthodox Marxists were prone to emphasize) but also the test of logical integration into the existing body of theoretical knowledge and methodological procedures. New mathematical knowledge, he stated, was not necessarily a product of the scientific community's response to socially originated needs, for it could also be generated by the internal dynamics of existing abstractions. He wrote:

> The cases of imaginary numbers and of non-Euclidean geometry show that the internal needs of mathematics—the needs for resolving abstract mathematical problems—may lead to significant conclusions and that the development of mathematics . . . cannot be reduced to simple and direct reflections of nature because it includes formulations of far-reaching abstractions and theories. [2, 8]

Aleksandrov also reminded Marxist worshippers of "applied science" that many mathematical ideas could find a practical application only after they had gone through several stages of internally stimulated development [2, 8]. Lobachevskii's non-Euclidean geometry, he wrote, was not a response to a practical problem of social significance; its origin was in the need of mathematical thought "to establish logical dependence or independence of the parallel postulate from the other postulates of Euclid's geometry" [2, 189]. He reminded the Marxists that the higher the level of abstractions in individual branches of mathematics, the broader the sweep of their practical application.

Although these were elementary questions, they occupied high positions on the scale of ideological values and formed the core of the Marxist philosophy of mathematical knowledge. Aleksandrov played a major role in the war on the idealism of Western mathematics, but he also took pains to emphasize the need for keeping separate tabs on the scientific and philosophical aspects of mathematical theories. It would be a mistake, he wrote, to assume that set theory brought forth only "difficulties and idealistic fallacies." It also brought grand new successes in mathematics: "without it, modern analysis, geometry, and algebra would be unthinkable" [3, 11].

The group represented by Aleksandrov was exceedingly small and obviously not fully in tune with Stalin's anticosmopolitanism. It accepted the war on Western idealism as a sacred duty of the day, but it also stressed the need for enforcing a clear separation of philosophical criticism from scientific approaches. This group served more as a harbinger of the post-Stalinist thaw than as an expression of the Stalinist ethos. Aleksandrov's essays in *Priroda* were early signs of the waning fervor of anticosmopolitanism and of the Stalinist orthodoxy in Marxist thought.

CONCLUSION

The Stalinist government was deeply involved in a paradoxical situation: on the one hand, it encouraged Marxist writers to wage a total war on all major orientations in the foundations of mathematics and on set theory; on the other hand, it allowed and helped Cantor's creation to establish a reigning trend in the growth of Soviet mathematics. The government encouraged attacks on the philosophical foundations of the capital works of Cantor, Hilbert, and Brouwer, but it did not object to the translation of their major studies into Russian. The Soviet government's attitude toward mathematics oscillated between two poles: the recognition of mathematics as one of the nation's highest and most original intellectual achievements, and the ideological conviction that the philosophical foundations of modern mathematics, as they developed in the West, represented a serious challenge to Marxist thought and Soviet ideology.

Marxist theorists considered dialectics the key process responsible for the progress of scientific knowledge. In their theory, dialectics signified a functional "synthesis" of contradictions in scientific thought; without "contradictions," they stated, no scientific progress was possible. Dialectics was seen as a guarantee of continuity—but not of gradualness—in the progress of mathematics; in dialectics, the past could never be fully obliterated. Marxist writers of the Stalinist era found nothing in "mathematical idealism" that could profitably be integrated into a new dialectical synthesis of the ontological and epistemological foundations of mathematics. They were accustomed to conducting a total—rather than a dialectical—war on dominant strains in modern mathematical thought. In fighting "mathematical idealism," they sought no help from dialectics. The robust health and rich diversity of Soviet mathematics was in no way indebted to Marxist criticism and guidance.

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