How compatible are the 9th grade mathematics written exams with mathematical power assessment criteria

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Abstract

Assessment and Evaluation has a significant role in mathematics education. One of the essential objectives of this process is to determine students’ level of mathematical knowledge and process abilities. Mathematical Power (MP) concept was also incorporated into new curriculum objectives in Turkey in 2004. In this research, we aimed to determine whether 9th grade mathematics written exams in Turkey are in accordance with the MP concept. The research application was performed by analyzing written mathematics exams of seven randomly chosen high schools located in Ankara. The assessment items and their distribution were analyzed based on MP assessment criteria. The results tell us that the assessment and evaluation (A&E) area in Turkey needs to be reformed to allow assessment and development of higher level abilities which are covered under the MP concept.

Keywords: Mathematics education; new mathematics curriculum; assessment and evaluation in mathematics; mathematical power; problem solving.

1. Introduction

For a long time in education, learning and instruction (L&I) processes have been heavily affected by narrow-scoped, elimination purposed assessment and evaluation (A&E) practices, such as course exams or college entrance exams. During this period, L&I could not be improved fast enough because of the poor A&E system. The paradigm of this period can be classified as A&E-dependent L&I. However, this period seems to have come to an end towards the end of the 20th Century. Instead, a new period has started with a new paradigm which focuses more on L&I processes to prepare individuals for real life and career and not just for some narrow-scoped exams. In this new period the purpose and focus of instruction is to raise individuals who are able to reach and construct information, construct why-and-how reasoning, make analysis/connections, solve problems and learn actively by themselves rather than memorizing information/concepts and studying to pass specific exams. On the other hand, although slow, A&E systems also started to be reformed in line with this new paradigm.
Parallel to this change of paradigm in Turkey, new primary and secondary education curriculums were launched beginning with the academic year 2004-2005, with the aim and hope of solving the chronic problems of previous curriculums and A&E systems. However, in some recent researches it is stressed that there exist many problems and deficiencies (Şahin, 2007). Lack of teachers and modern educational tools, material and technological hardware and A&E practices and applications are among the leading problem areas. Especially, it proved to be difficult to change the long-time habits of a significant number of teachers who are not willing or incapable of adapting. Therefore, a complete transformation into the new curriculum has not yet been achieved and requires more time.

A&E has always been a key driver in education systems. It effects and shapes the teaching and learning practices. If A&E approach and practices are narrow-scoped, elimination-purposed and result-oriented rather than real life-scoped, improvement-purposed and process-oriented, then the related instructional processes cannot be kept away from the negative effects of A&E. This was more or less the situation in Turkey before the new curriculum change. For students in Turkey, there are some nation-wide milestone exams such as high school and college entrance exams (SBS and ÖSS) which had a significant effect on the entire primary and secondary L&I system. These national exams and school course exams generally have limited scope and assess students’ memorization-based, narrow-scoped information rather than their actual capabilities, interests and potentials. This limited scope of exams is seen as the fundamental reason for failure in the full-scale implementation and adaptation of new curriculum objectives (Şahin, 2007). Thus in order to accomplish the transformation and achieve new curriculum objectives, not only should L&I processes be changed but A&E should also be changed to allow for complete implementation of the new curriculum.

Therefore, it is an inevitable fact that A&E process has a significant role in mathematics education. One of the essential objectives of this process is to determine the students’ level of mathematical knowledge and mental process abilities. Since the adoption and implementation of Mathematical Power (MP) concept by National Council of Teachers of Mathematics (NCTM) in 1990s, MP assessment has been increasingly incorporated into mathematics instruction and A&E standards (NCTM, 1991; NAGB, 2002). MP concept was also incorporated into the new mathematics curriculum objectives in Turkey in 2004.

The MP concept is an overall mathematical proficiency that can be described as an individual’s mathematical ability in conceptual understanding, procedural knowledge, connections, reasoning, conjecturing, problem solving, mathematical thinking and communication. Assessment of these abilities requires diverse and rich assessment methodology, tools, items and relatively longer assessment time. To achieve this, A&E system should be reformed concordant with MP concept.

It is observed that the MP concept is being widely practiced in Euro-Atlantic countries primarily in the USA. Contrary to this, MP-oriented A&E system and tools have not yet been developed with sufficient awareness and widespread practice in Turkey. In particular, the assessment items should be constructed in such a way that they should assess not only the conceptual understanding and procedural knowledge (which depends more on short-termed, memorization-based studying), but also mathematical thinking and wider mental process abilities (reasoning, problem solving, etc.). This can be accomplished by extensively using open-ended items that require different formatted (numerical, prose, mixed, multiple choice, etc.) responses. Building upon this motivation, this research focused on to find out how well the current A&E approaches & practices, specifically math examinations, are in accordance with MP A&E criteria.

2. Methodology

The aim of this research was to determine how compatible the 9th grade mathematics written exams were with the MP assessment criteria. These traditional exams have been being extensively used as an assessment tool in high school education in Turkey. The research application was performed by analyzing written mathematics exams of seven randomly chosen high schools located in Cankaya District of Ankara Province in Turkey in the 2005-2006 academic year. In this research, we used a descriptive research model (Karasar, 1986). The written exams, rubrics and answer sheets were collected from mathematics teachers of the seven schools (Table-1). The assessment items in these exams and their distribution were analyzed based on MP assessment objectives and criteria.
3. Findings

When the exam items were analyzed, it was observed that the items were prepared to assess basic mathematical subject information. They were also scored based only on the solution results but not the solution procedures which reflect students’ problem solving abilities. In addition, there were no items which can distinguish individual ability differences and which can determine MP level. Even though the items were named as “problems”, they were actually not like real problems but like typical exercises. Furthermore, we found no items which direct the students for problem solving, higher conceptual understanding, applying procedural knowledge, making connections with other subjects / daily life / other disciplines, reasoning, conjecturing, mathematical thinking, and communication. As an example, some selected items are presented in Table 2 to show the abovementioned deficiencies.

Table 2. Sample Items From An Examination

- If \( f(10) = 62 \) in the equation \( f(x + 1) = f(x) + 2x \) then \( f(1) \) is equal to what?

- If \( f(x) = (x + 2) / x \) then find \( f(2x) \) in terms of \( f(x) \)

- If \( f(2x - 2) / (x + 4) = 4x + 2 \) then find the function \( f'(x) \)

- For the functions \( f \) and \( g \), if \( f(x + 1) = 2x + 6 \) and \( g(x - 2) = -2x + 4 \) then find the value of the composite function \((fg)(x)\)

- If \( f(1) = 4 \) in the equation \( f(x) - f(x - 1) = 2x \) then find the value of \( f(16) \)

- In the real numbers set, an operation is defined as \( a \Delta b = a+b \) for \( a>b \); and \( a \Delta b = a.b \) for \( a \leq b \); then what is the value of the operation \((3 \Delta 4) \Delta 5 = ?\)

- In the set \( \mathbb{R} - \{-2\} \), an operation is defined as \( x * y = 2x + 2y + xy + 2 \); then what is the inverse of 4 based on * operation?

In addition to the items themselves, student responses and teacher scores for each item were examined. Sample student responses and teacher scores for responses are presented in Table 3.

Table 3. Sample Student Responses And Teacher Scores For Exam Items

- For the functions \( f \) and \( g \); if \( g(x) = 4x + 2 \) and \((fg)(x) = 2x - 1\) then find \( f(x) \).

- If \( g(x) = x - 4 \) and \((g^{-1} \circ f^{-1})(x) = 2x + 2 \) then find \( f(x) \).

- Write the simplest form of argument \((p \implies q) \lor (p \land q)\)

- If \( f(x) = (x - 2) / x \) then find the function \( f(2x) \) in terms of \( f(x) \).
Similar to the items, student responses were also computation and result-focused. They did not reflect deeper mathematical thinking. Also, for both teachers and students procedural knowledge and communications abilities were of little concern. A significant finding is that 60% of teachers (12 out of 20) had prepared rubrics for scoring, the others scored the exams without any rubric or even predefined written criteria (Table 1). This also clearly shows that a significant number of the assessments were completed based only on results and teachers’ subjective momentary judgments but not solution steps in student responses. We also examined question phrases of each item to compare them with MP-oriented question phrases (Table 4). Table 4 also proves the poor construction of the exam items when compared to MP-based item question phrases. It is thought that one of the main reasons behind these limited/poor question phrases is annual university entrance exam items that are also poor and limited (Ortaş, 2007).

Table 4. Comparison Between Analyzed Item Question Phrases and MP-Based Item Question Phrases

<table>
<thead>
<tr>
<th>Question phrases of analyzed exam items</th>
<th>MP-based item question phrases</th>
</tr>
</thead>
<tbody>
<tr>
<td>☐ Find the answer of the following question / operation / equation.</td>
<td>☐ How did you reach your result? Please show and explain the steps.</td>
</tr>
<tr>
<td>☐ Find the result / solution.</td>
<td>☐ Why? How?</td>
</tr>
<tr>
<td>☐ Solve the following equation.</td>
<td>☐ Prove / Justify your solution.</td>
</tr>
<tr>
<td>☐ What does the following equation equals to?</td>
<td>☐ Explain your solution method.</td>
</tr>
<tr>
<td>☐ What is the simplest form of this operation?</td>
<td>☐ How can you be sure about your solution method.</td>
</tr>
<tr>
<td>☐ What is the result of this operation?</td>
<td>☐ Support your solution by using graphical / verbal explanation.</td>
</tr>
<tr>
<td>☐ What is the value of this operation?</td>
<td>☐ Can you form a rule.</td>
</tr>
<tr>
<td>☐ How did you reach your result? Please show and explain the steps.</td>
<td>☐ Please give an example.</td>
</tr>
<tr>
<td>☐ Prove / Justify your solution.</td>
<td>☐ Which problem did you choose? And how did you do it?</td>
</tr>
<tr>
<td>☐ Explain your solution method.</td>
<td>☐ Do you think there is a different way to solve this question / problem? If yes try the different way.</td>
</tr>
<tr>
<td>☐ How can you be sure about your solution method.</td>
<td>☐ Create a mathematical model of this problem?</td>
</tr>
<tr>
<td>☐ Support your solution by using graphical / verbal explanation.</td>
<td>☐ Make connections with other sciences.</td>
</tr>
<tr>
<td>☐ Can you form a rule.</td>
<td>☐ Make connections with daily life.</td>
</tr>
<tr>
<td>☐ Please give an example.</td>
<td>☐ How can you convince us about your solution?</td>
</tr>
</tbody>
</table>

We also compared the (traditional) analyzed exams with another sample MP-focused exam to see how rich the analyzed exam items are in terms of MP-based assessment criteria and objectives. This sample exam was selected for comparison because it included 40 MP-based assessment items which had been prepared and applied in another research to assess the MP level of students. We compared the percentage of each assessment criteria and objectives used in analyzed exams to the percentages in the sample exam. Figure 1 clearly shows that the analyzed exam items are poor in MP-based assessment criteria and objectives when compared to a MP-rich sample exam. It can also be seen at Figure 1 that the analyzed exams are only rich in simpler assessment criteria, such as computational skills and low level subject information/conceptual understanding.
4. Conclusions

As a result, it was found that the items were designed to assess only basic computational skills and low level mathematics subject information/conceptual understanding. Even though some of them were named as problems, the items were more similar to routine exercises but not to real problems. There was almost no item type which could distinguish the students’ high level ability differences. Furthermore, there was almost no item type which addressed MP-based high level abilities. Thus the traditional exams used fail to lead students to higher order mathematical thinking and abilities. The results obtained present a picture that tells us that the A&E area within mathematics education system in Turkey needs to be reviewed and reformed to allow assessment and development of higher level abilities which are covered under the MP concept.

The results also indicate that the new curriculum standards and objectives have not yet been understood and put into practice in the A&E area at 9th grade level. Likewise it was discovered by Nichols ve Sugrue (1999) that MP criteria had not been incorporated in A&E practices at schools in USA. In order for the new curriculum to achieve its objectives in Turkey, there should be a planned and extensive teacher orientation and training on the new curriculum (ERG, 2005; Butakn and Özgen, 2007). The teacher training should also be continuous practice to orient teachers, keep them informed and especially create a strong motivation in the new curriculum objectives (Butakn ve Özgen, 2007).

On the other hand, it is thought that a major reason behind the poor nature of A&E applications in high schools is the strong negative effect of college entrance exams. This exam should also be reformed to be in line with the new curriculum and also with MP concept. The discrete and consequent instruction and assessment of each MP abilities is deemed important and highly recommended (Baker and Czarnocha, 2002).

Consequently, we propose that such a change in the A&E system will automatically push other areas of mathematics education, primarily the instructional system, towards a more MP orientated shape leading to higher achievement in mathematics education.

References

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