



## Preface

This is the fifth special issue on Global Optimization, Control, and Games of the journal *Computers and Mathematics with Applications*. It continues the tradition of the first four special issues on Global Optimization, Control, and Games of this journal which appeared as Volume 21, Issues 6/7, 1991; Volume 25, Issues 10/11, 1993; Volume 37, Issues 4/5, 1999; and Volume 44, Issue 7, 2002, respectively.

The papers in this issue cover almost all directions in optimization and control with applications, including nonconvex nonsmooth global optimization and control, multiobjective programming, dynamic programming, stochastic problems and nonlinear filtering, fuzzy problems, computational methods and random number generation, and important methodological issues.

The first paper presents a powerful direct method in multicriteria analysis and optimal design called parameter space investigation (PSI) method with the use of uniformly distributed sequences in multidimensional domains. Pareto optimal solutions are found in models of important engineering applications. The method is implemented in the MOVI (multicriteria optimization and vector identification) software that can be executed on a standard PC.

The second paper presents a MAPLE code of the Beta algorithm for full global optimization of nonconvex Hölder continuous functions over general compact sets in  $R^n$ . The set-monotonic algorithm contains a block for problems with equality constraints, and operates within the unit cube  $[0, 1]^n$  universal for all problems. The solution yields the unique global minimum value and the entire exact set of all global minimizers, in the limit, or their approximations in a finite number of iterations. The solution set can be visualized in plane projections and sections on the screen of computer.

The third paper presents the measure based integral minimization method for nonconvex global optimization, complete with necessary and sufficient conditions for global optimality without convexity nor differentiability assumptions. The algorithm is monotonic, and it is implemented with a properly designed Monte Carlo method. The mean value and variance conditions are studied and used to set stopping criteria, and interesting test examples illustrate the results.

The fourth paper is devoted to the concept of robustness of sets and functions and presents important results about accessibility and approximability of minimizers in iterative processes over metric spaces. It is worth noting that robustness is a necessary condition for validity of all nonconvex optimization methods even if it is not mentioned explicitly (the Slater condition is insufficient for nonconvex problems).

The fifth paper presents a study of sufficiency and duality in nonsmooth multiobjective optimization making use of the Clarke generalized subgradient, the Geoffrion restricted efficiency, and some generalized functions. It will be interesting for the reader to compare gradient methods developed in this paper with nongradient approaches presented elsewhere in this issue.

The next paper presents a new notion of distance uniformity and a fast algorithm for generating uniformly distributed small samples of points in  $R^n$  by using the combination of a popular

random number generator with a distance rejection operator. Such samples improve the results obtained with standard Monte Carlo simulation, and can also be used for nonconvex nonsmooth multiobjective global optimization, for evaluation of multiple integrals, and for stochastic applications.

The seventh paper studies sensitivity of multiobjective differential programs with equality constraints, in regard to set-valued maps of efficient points. This represents a departure from standard considerations of a scalar utility function, or of just one of efficient points determined by an iterative method. Interesting results are obtained for  $T$ -optimal solutions in Banach spaces.

In the eighth paper, nonconvex global optimization is applied to financial problems in imperfect markets with nonconvex transaction costs and the presence of arbitrage. The concepts of pseudo-arbitrage and efficiency are introduced and analyzed in both scalar and vector (nonscalarized) framework. Several sensitivity results are presented to obtain a significant transaction costs reduction. The study can be used for optimal portfolio design in imperfect markets and for other applications.

The next paper studies the valuation of boundary-linked assets and their derivatives in continuous-time markets. This leads to a stochastic boundary value problem, usually non Markovian, that is solved by using a wavelet-collocation method for a finite difference Milstein approximation. Monte Carlo simulations are used for numerical solutions presented to illustrate the method.

In the tenth paper, a new method for optimal nonlinear filtering is developed for noisy observations modeled by a persistent fractional Brownian motion with Hurst index greater than  $1/2$ . The method is based on multiple stochastic fractional integral expansions truncated to produce an approximation to the optimal filter. An interesting example is presented where the correct trend is revealed in the case when the magnitudes of the signal and noise are of the same order.

The next paper is devoted to modeling and optimization of problems with fuzzy coefficients. This requires a modification of traditional mathematical programming methods in order to maximally cut off dominated alternatives with subsequent contraction of the decision uncertainty region, based on reducing the problem to models of multiobjective optimization in a fuzzy environment. Three different techniques for fuzzy preference modeling are discussed in the paper which can be applied to solve multiobjective engineering and management problems in the presence of uncertainty. It would be interesting for the reader to see how crisp methods presented in other papers of this issue should be modified for real-life fuzzy applications.

The twelfth paper presents an important application of fuzzy multiobjective optimization to optimal allocation of resources. The principle of guaranteed result is used for obtaining harmonious solutions of associated maximin problems. The corresponding adaptive interactive decision making systems (AIDMS) in C++ are developed for practical implementation of the results for use in power engineering problems.

The next paper presents a parallel hybrid algorithm for global optimization of problems in multidimensional scaling (MDS) used for visualization of multidimensional data. Global optimization is performed by combining evolutionary global search with "genetic operators" and a local descent (conjugate gradient local search, or Powell's local search). It will be interesting for the reader to compare this algorithm with other algorithms for nonconvex global optimization presented in other papers of this issue.

In the fourteenth paper, a special preference structure of equitability is derived, and a scalarization approach is proposed to finding equitably efficient solutions of general multiple objective programs. "While Pareto efficiency assumes that the criteria are incomparable, equitability is based on the assumption that the criteria are not only comparable (measured on a common scale) but also anonymous (impartial). The latter . . . models equitable allocation of resources."

The issue ends with a paper of rather theoretical nature which presents a discussion of some 50 years old problems. Generalizing a proof given in the literature for Bellman's optimality principle (for an optimal trajectory, every remaining part thereof is itself optimal), it is shown that the principle implies total optimality, that is, the optimality of every part of an optimal

trajectory. Then, a modification is proposed for the derivation of the main functional equations of dynamic programming to demonstrate that they are valid also in the case of nonoptimal remaining trajectories under certain contiguity condition (necessary and sufficient) that is defined and analyzed in the paper. The contiguity condition requires that a nonoptimal remaining part and the optimal semi-trajectory, both started at the same intermediate point of an optimal trajectory, be touching to the second or higher order, i.e., lie in a weak neighborhood of one another (cf. the curvature in the Weierstrass form of the Euler-Lagrange equations). Then the Bellman equations of dynamic programming are valid, and usually have a solution, irrespective of the principle of optimality.

The issue presents recent developments in the area of nonconvex global optimization and related fields of mathematical research and algorithmic development from all over the globe including five continents and nine countries: Australia, Brazil, Canada, China, India, Lithuania, Russia, Spain, and U.S.A. The issue is addressed to mathematicians, computer scientists and programmers, engineers, economists, educators, and researchers in all areas of science and industry who have particular interests in obtaining globally optimal solutions. Many papers present results and algorithms ready for computer implementation. I would like to thank all authors for their constant interest in the issue. I am greatly indebted to many scientists who helped with the reviewing of papers and who contributed to the quality of the issue.

Efim A. Galperin  
*Guest Editor*