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Quasi-supersymmetric G^3 unification from intersecting D6-branes on Type IIA orientifolds

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Abstract

We construct three quasi-supersymmetric G^3 GUT models with S_3 symmetry and gauge coupling unification from intersecting D6-branes on Type IIA orientifolds. The Standard Model fermions and Higgs doublets can be embedded into the bifundamental representations in these models, and there is no any other unnecessary massless representation. Especially in Model I with gauge group $U(4)^3$, we just have three-family SM fermions and three pairs of Higgs particles. The G^3 gauge symmetry in these models can be broken down to the Standard Model gauge symmetry by introducing light open string states. And 1 TeV scale supersymmetry breaking soft masses imply the reasonable intermediate string scale.

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1. Introduction

Since 1984, there has been a lot of work and effort devoted to the string model building or string phenomenology, whose goal is to obtain the Standard Model (SM) or Minimal Supersymmetric Standard Model (MSSM) as an effective theory of the string-based models. And these models are mainly built in the weakly coupled heterotic string theory with $E_8 \times E_8$ gauge group [1,2], because it naturally obtains the

Grand Unified Theory (GUT) through the elegant E_8 breaking chain: $E_8 \supset E_6 \supset SO(10) \supset SU(5)$. Even now, this is an interesting subject because of the model buildings in M-theory on S^1/Z_2 [3–7].

In recent years, the emergence of M-theory opened up many new avenues for the consistent string model buildings. Especially, we can construct the open string models that are non-perturbative from the dual heterotic string description due to the advent of D-branes [8]. The technique of conformal field theory in describing D-branes and orientifold planes on orbifolds has played a key role in the construction of consistent 4-dimensional supersymmetric $N = 1$ chiral models on Type II orientifolds. There are two kinds of the-

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ories which have chiral fermions from the D-brane constructions: one from D-branes located at orbifold singularities where the chiral fermions appear on the worldvolume of D-branes [9–15] and the other one from intersecting D-branes on Type II orientifold where the open string spectrum contains chiral fermions localized at the D-brane intersections [16].

For the second kind of scenarios, a lot of non-supersymmetric three-family Standard-like models and GUT models were explored in the beginning [17–30]. However, there are uncancelled Neveu–Schwarz–Neveu–Schwarz (NSNS) tadpoles and may exist the gauge hierarchy problem. On the other hand, since the first supersymmetric model with intersecting D6-branes on $T^6/Z_2 \times Z_2$ was constructed in Refs. [31,32], the supersymmetric Standard-like models, $SU(5)$ and Pati–Salam models have been discussed in detail later [33,34], as well as the phenomenology [35–37]. Moreover, the supersymmetric Pati–Salam models based on Z_4 and $Z_4 \times Z_2$ orientifolds with intersecting D6-branes were also constructed [38, 39]. In these models, the left–right symmetric gauge structure was obtained by brane recombinations, so the final models do not have the explicit toroidal orientifold construction, where the conformal field theory can be applied for the calculation of the full spectrum and couplings.

Looking back on these model buildings, we may find that people took such philosophy: directly construct the familiar models, such as Standard-like models, $SU(5)$ and Pati–Salam models, etc., from the intersecting D-branes on type II orientifolds since these models have been understood very well from the traditional phenomenological analysis. Unfortunately, no GUT model with gauge coupling unification has been built up due to the strong constraint of RR-tadpole cancellation and supersymmetry (SUSY) preservation. In this Letter, we take a completely different philosophy: constructing the “natural” 4-dimensional $N = 1$ GUT models from the intersecting D6-branes on Type IIA orientifolds where the “natural” means:

- (1) Gauge coupling unification;
- (2) The Standard Model gauge group is the subgroup of the gauge symmetry at string scale, and three families of quarks and leptons and a pair of the SM Higgs doublets are included in the massless open string spectrum;

- (3) The gauge symmetry at string scale can be broken down to the Standard Model gauge symmetry via Higgs mechanism or Wilson line;
- (4) RR-tadpole cancellation. And the observable D6-branes preserve the same 4-dimensional $N = 1$ supersymmetry as the orbifold background.

Adding S_3 symmetry on the observable D6-branes and complex structure moduli, we obtain three models with above four properties from $T^6/(Z_2 \times Z_2)$ orientifolds with intersecting D6-branes. In these models, three stacks of physical D6-branes, which form the observable sector, preserve the same 4-dimensional $N = 1$ supersymmetry as the orbifold background. To cancel the RR tadpole, we introduce one stack of auxiliary D6-branes which wraps on the ΩR orientifold and has no intersection with three observable D6-branes. However, the auxiliary D6-brane breaks above 4-dimensional $N = 1$ supersymmetry. So, our model is quasi-supersymmetric,¹ and there may exist the uncancelled NSNS tadpoles. Concretely, Model I describes $U(4)^3$ gauge theory with odd-family chiral fermion spectrum, and Model II $U(4)^3$ gauge theory with even-family chiral fermion spectrum, Model III $U(8)^3$ gauge theory with even-family chiral fermion spectrum. In all these models, the Standard Model fermions and Higgs particles are embedded into the bi-fundamental representations, and the symmetric, anti-symmetric or any other unnecessary massless representations are absent. In particular, we just have three families of fermions and three pairs of Higgs particles for Model I. We show that in Model I the $U(4)^3$ gauge symmetry can indeed be broken down to the Standard Model gauge symmetry by introducing the light open string states, and similar mechanism works for the Models II and III. Furthermore, we discuss the supersymmetry breaking due to the auxiliary D6-brane, and find that the 1 TeV scale soft masses imply the intermediate string scale around 10^{11} – 10^{12} GeV, which is a reasonable unification scale for the Pati–Salam model [42] and can be realized in large extra dimension scenario [40,41]. However, the unification gauge coupling

¹ In this Letter, the quasi-supersymmetry means that the observable D6-branes preserve the same 4-dimensional $N = 1$ supersymmetry as the orbifold background, which is broken by the auxiliary D6-brane.

(α_{GUT}) is seriously suppressed to 10^{-8} , which implies the fine-tuning in the RGE runnings of the gauge-couplings.

2. Supersymmetric model buildings from $T^6/(Z_2 \times Z_2)$ orientifolds with intersecting D6-branes

In spite of non-supersymmetric essence of G^3 GUT models, the 4-dimensional $N = 1$ supersymmetry are required to be locally preserved in the observable sector in order to solve the gauge hierarchy problem. So, we first review the rules to construct the supersymmetric models from Type IIA orientifolds on $T^6/(Z_2 \times Z_2)$ with D6-branes at generic angles, and to obtain the spectrum of massless open string states [32]. Here, we follow the notation in Ref. [33].

The starting point is Type IIA string theory compactified on a $T^6/(Z_2 \times Z_2)$ orientifold. We consider T^6 to be a six-torus factorized as $T^6 = T^2 \times T^2 \times T^2$ whose complex coordinates are $z_i, i = 1, 2, 3$ for each of the 2-torus, respectively. The θ and ω generators for the orbifold group $Z_2 \times Z_2$, which are associated with their twist vectors $(1/2, -1/2, 0)$ and $(0, 1/2, -1/2)$, respectively, act on the complex coordinates of T^6 as

$$\begin{aligned} \theta : (z_1, z_2, z_3) &\rightarrow (-z_1, -z_2, z_3), \\ \omega : (z_1, z_2, z_3) &\rightarrow (z_1, -z_2, -z_3). \end{aligned} \quad (1)$$

The orientifold projection is implemented by gauging the symmetry ΩR , where Ω is worldsheet parity, and R acts as

$$R : (z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3). \quad (2)$$

So, there are four kinds of orientifold 6-planes (O6-planes) for the actions of $\Omega R, \Omega R\theta, \Omega R\omega,$ and $\Omega R\theta\omega$, respectively. To cancel the RR charges of O6-planes, we introduce some stacks of N_a D6-branes, which wrap on the factorized three-cycles. Meanwhile, we have two kinds of complex structures consistent with orientifold projection for a torus—rectangular and tilted [18,32,33]. If we denote the homology classes of the three cycles wrapped by the D6-brane stacks as $n_a^i[a_i] + m_a^i[b_i]$ and $n_a^i[a'_i] + m_a^i[b_i]$ with $[a'_i] = [a_i] + \frac{1}{2}[b_i]$ for the rectangular and tilted tori respectively, following the notation of Ref. [33], we can label a generic two cycle by (n_a^i, l_a^i)

in either case, where in terms of the wrapping numbers $l_a^i \equiv m_a^i$ for a rectangular torus and $l_a^i \equiv 2\tilde{m}_a^i = 2m_a^i + n_a^i$ for a tilted torus. Note that for a tilted torus, $l_a^i - n_a^i$ must be even. For a stack of N_a D6-branes along the cycle (n_a^i, l_a^i) , we also need to include their ΩR images $N_{a'}$ with wrapping numbers $(n_a^i, -l_a^i)$. For D6-branes on the top of O6-planes, we count the D6-branes and their images independently. So, the homology three-cycles for stack a of N_a D6-branes and its orientifold image a' take the form

$$\begin{aligned} [\Pi_a] &= \prod_{i=1}^3 (n_a^i[a_i] + 2^{-\beta_i} l_a^i[b_i]), \\ [\Pi_{a'}] &= \prod_{i=1}^3 (n_a^i[a_i] - 2^{-\beta_i} l_a^i[b_i]), \end{aligned} \quad (3)$$

where $\beta_i = 0$ if the i th torus is rectangular and $\beta_i = 1$ if it is tilted. And the homology 3-cycles wrapped by the four O6-planes are

$$\Omega R : [\Pi_{\Omega R}] = 2^3[a_1] \times [a_2] \times [a_3], \quad (4)$$

$$\Omega R\omega : [\Pi_{\Omega R\omega}] = -2^{3-\beta_2-\beta_3}[a_1] \times [b_2] \times [b_3], \quad (5)$$

$$\Omega R\theta\omega : [\Pi_{\Omega R\theta\omega}] = -2^{3-\beta_1-\beta_3}[b_1] \times [a_2] \times [b_3], \quad (6)$$

$$\Omega R\theta : [\Pi_{\Omega R\theta}] = -2^{3-\beta_1-\beta_2}[b_1] \times [b_2] \times [a_3]. \quad (7)$$

Then, the intersection numbers are

$$I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^3 (n_a^i l_b^i - n_b^i l_a^i), \quad (8)$$

$$I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k} \prod_{i=1}^3 (n_a^i l_b^i + n_b^i l_a^i), \quad (9)$$

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k} \prod_{i=1}^3 (n_a^i l_a^i), \quad (10)$$

$$\begin{aligned} I_{aO6} &= [\Pi_a][\Pi_{O6}] \\ &= 2^{3-k} (-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3), \end{aligned} \quad (11)$$

where

$$[\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}]$$

is the sum of O6-plane homology three-cycles wrapped by the four O6-planes, and $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted tori.

Table 1

General spectrum on intersecting D6-branes at generic angles which is valid for both rectangular and tilted tori. The representations in the table make sense to $U(N_a/2)$ due to $Z_2 \times Z_2$ orbifold projection [32]. In supersymmetric situations, scalars combine with the fermions to form the chiral supermultiplets

Sector	Representation
aa	$U(N_a/2)$ vector multiplet 3 Adj. chiral multiplets
$ab + ba$	I_{ab} ($\square_a, \bar{\square}_b$) fermions
$ab' + b'a$	$I_{ab'}$ (\square_a, \square_b) fermions
$aa' + a'a$	$-\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6})$ \square fermions $-\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6})$ $\bar{\square}$ fermions

The general spectrum on intersecting D6-branes at generic angles, which is valid for both rectangular and tilted tori, is given in Table 1. And the 4-dimensional chiral supersymmetric ($N = 1$) models from Type IIA orientifolds with intersecting D6-branes are mainly constrained in two aspects:

I. Tadpole cancellation conditions. As sources of RR fields, D6-branes and orientifold 6-planes are required to satisfy the Gauss law in a compact space, i.e., the total RR charges of D6-branes and O6-planes must vanish since the RR field flux lines cannot escape. The RR tadpole cancellation conditions are

$$\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi_{a'}] - 4[\Pi_{O6}] = 0, \tag{12}$$

where the last contributions come from the O6-planes which have -4 RR charges in the D6-brane charge unit by exchanging RR field while scattering.

Tadpole cancellation directly leads to the $SU(N)^3$ cubic non-abelian anomaly cancellation [20,21,32]. And the cancellation of $U(1)$ mixed gauge and gravitational anomaly or $[SU(N)]^2 U(1)$ gauge anomaly can be achieved by Green–Schwarz mechanism mediated by untwisted RR fields [20,21,32].

II. Conditions for 4-dimensional $N = 1$ supersymmetric D6-brane. The 4-dimensional $N = 1$ supersymmetric models require that 1/4 supercharges from 10-dimensional Type I T-dual be preserved, i.e., they should survive two supersymmetry breaking mechanisms: orientation projection of the intersecting D6-branes, and orbifold projection on the background manifold. Concrete analysis shows that the $N = 1$ su-

persymmetry can be preserved only if the rotation angle of any D6-brane with respect to the ΩR -plane is an element of $SU(3)$, or in other words, $\theta_1 + \theta_2 + \theta_3 = 0$, where θ_i is the angle between the D6-brane and the ΩR -plane in the i th torus. In Ref. [33], this condition is rewritten as

$$-x_A l_a^1 l_a^2 l_a^3 + x_B l_a^1 n_a^2 n_a^3 + x_C n_a^1 l_a^2 n_a^3 + x_D n_a^1 n_a^2 l_a^3 = 0, \tag{13}$$

$$-n_a^1 n_a^2 n_a^3 / x_A + n_a^1 l_a^2 l_a^3 / x_B + l_a^1 n_a^2 l_a^3 / x_C + l_a^1 l_a^2 n_a^3 / x_D < 0, \tag{14}$$

where $x_A = \lambda$, $x_B = \lambda 2^{\beta_2 + \beta_3} / \chi_2 \chi_3$, $x_C = \lambda 2^{\beta_1 + \beta_3} / \chi_1 \chi_3$, $x_D = \lambda 2^{\beta_1 + \beta_2} / \chi_1 \chi_2$, and $\chi_i = R_2^i / R_1^i$ are the complex structure moduli where R_1^i and R_2^i are radii for the i th torus due to $T^2 \equiv S^1 \times S^1$. λ is a positive parameter without physical significance.

3. Quasi-supersymmetric G^3 unification

Generally speaking, the RR-tadpole cancellation conditions and the 4-dimensional supersymmetry preservation conditions are too stringent to find the realistic GUT models, and the existing GUT models always tend to produce extra gauge interactions and extra fermions beyond the SM or MSSM. However, by relaxing the supersymmetry preserving condition for the auxiliary D6-brane which is introduced to cancel the RR tadpole, we can construct the natural GUT models with the four properties emphasized in introduction.

Let us look at the tadpole cancellation conditions first. If we consider $N^{(i)}$ auxiliary D6-branes wrapped along the i th orientifold plane whose wrapping numbers are given in Table 2, the tadpole cancellation conditions are modified to

$$-2^k N^{(1)} - \sum_{\sigma} N_{\sigma} n_{\sigma}^1 n_{\sigma}^2 n_{\sigma}^3 = -16, \tag{15}$$

$$-2^k N^{(2)} + \sum_{\sigma} N_{\sigma} n_{\sigma}^1 l_{\sigma}^2 l_{\sigma}^3 = -16, \tag{16}$$

$$-2^k N^{(3)} + \sum_{\sigma} N_{\sigma} l_{\sigma}^1 n_{\sigma}^2 l_{\sigma}^3 = -16, \tag{17}$$

$$-2^k N^{(4)} + \sum_{\sigma} N_{\sigma} l_{\sigma}^1 l_{\sigma}^2 n_{\sigma}^3 = -16. \tag{18}$$

Table 2
Wrapping numbers of the four O6-planes

Orientifold action	O6-plane	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$
ΩR	1	$(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$
$\Omega R\omega$	2	$(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$
$\Omega R\theta\omega$	3	$(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$
$\Omega R\theta$	4	$(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$

Suppose there are three stacks of observable D6-branes, a, b , and c . Adding S_3 symmetry onto D6-branes configuration and $T_2 \times T_2 \times T_2$ geometry, i.e., $N^{(2)} = N^{(3)} = N^{(4)}$, $N_a = N_b = N_c = 2N$ and $\chi_1 = \chi_2 = \chi_3$, we notice that among Eqs. (16)–(18), only one is independent. Similarly for the $N = 1$ supersymmetry preserving conditions. If one stack of the observable D6-brane preserves $N = 1$ supersymmetry, all three stacks of D6-branes will preserve the $N = 1$ supersymmetry automatically. The simplest case is that $N^{(2)} = N^{(3)} = N^{(4)} = 0$, and one stack of auxiliary D6-brane wrapped along the ΩR orientifold plane are needed for RR-tadpole cancellation in these models. Then the gauge group of our models is G^3 where $G = U(N)$.

For simplicity, we consider three stacks of observable D6-branes (a, b and c) with one zero wrapping number. Without loss of generality, we have two possibilities

$$\begin{aligned} n_a^1 = n_b^2 = n_c^3 = 0 \quad (i) \\ l_a^1 = l_b^2 = l_c^3 = 0 \quad (ii). \end{aligned} \tag{19}$$

For the first case (i), the models without symmetric and antisymmetric representations cannot be constructed. So, we focus on the second case (ii).

In addition, we only consider the models with bifundamental representations which the Standard Model fermions and Higgs particles can be embedded into. To avoid the symmetric and anti-symmetric representations, we require that

$$\begin{aligned} l_a^2 n_a^3 = -n_a^2 l_a^3, \\ l_b^3 n_b^1 = -n_b^3 l_b^1, \\ l_c^1 n_c^2 = -n_c^1 l_c^2, \end{aligned} \tag{20}$$

which are equivalent to the supersymmetry preserving conditions. Because of the S_3 symmetry among the three stacks of D6-branes or three 2-tori, Eq. (20)

implies

$$\begin{aligned} l_b^3 n_a^3 = -n_b^3 l_a^3, \\ l_c^1 n_b^1 = -n_c^1 l_b^1, \\ l_a^2 n_c^2 = -n_a^2 l_c^2, \end{aligned} \tag{21}$$

and vice versa. This means that at massless level, the representations $(N_a/2, N_b/2, 1)$, $(1, N_b/2, N_c/2)$, $(N_a/2, 1, N_c/2)$ (or their complex conjugations) will appear or disappear together with the symmetric and anti-symmetric representations in the models with G^3 unification. As for the determination of N in $U(N)^3$ gauge group, we only have two choices: 4 or 8, which can be figured out from the tadpole cancellation conditions in our setup:

$$\begin{aligned} N_a n_a^1 l_a^2 l_a^3 = -16, \\ N_b l_b^1 n_b^2 l_b^3 = -16, \\ N_c l_c^1 l_c^2 n_c^3 = -16, \end{aligned} \tag{22}$$

$$\begin{aligned} -(N_a n_a^1 n_a^2 n_a^3 + N_b n_b^1 n_b^2 n_b^3 + N_c n_c^1 n_c^2 n_c^3) \\ - N_g n_g^1 n_g^2 n_g^3 = -16, \end{aligned} \tag{23}$$

where $N_a = N_b = N_c = 2N$. Obviously, N cannot be larger than 8 since the four O6-planes in our setup can only provide -16 RR charges in the D6-brane charge unit, while $N = 2$ is ruled out from the phenomenological concern. We emphasize that for $U(4)^3$ model, the three tori can be tilted, but, for $U(8)^3$ model, the three tori cannot be tilted since $n_a^1 - l_a^1$ is odd.

There are three typical solutions corresponding to three G^3 models. The D6-brane configurations for Model I, Model II, and Model III are given in Tables 3, 4, and 5, respectively. We also present the chiral open string spectrum for those models in Table 6. In short, we have $2p + 1$, $8p$ and $2p$ generations

Table 3
Model I. D6-brane configuration in $(2p + 1)$ -generation quasi-supersymmetric $U(4)^3$ model. This model is built on three tilted 2-tori with $Z_2 \times Z_2$ orbifold symmetry and p is a non-negative integer

N_i	(n_i^1, l_i^1)	(n_i^2, l_i^2)	(n_i^3, l_i^3)
$N_a = 8$	$(2, 0)$	$(2p + 1, 1)$	$(2p + 1, -1)$
$N_b = 8$	$(2p + 1, -1)$	$(2, 0)$	$(2p + 1, 1)$
$N_c = 8$	$(2p + 1, 1)$	$(2p + 1, -1)$	$(2, 0)$
N_g	$N_g n_g^1 n_g^2 n_g^3 = -48(2p + 1)^2 + 16$		

Table 4

Model II. D6-brane configuration in $(8p)$ -generation quasi-supersymmetric $U(4)^3$ model. This model is built on three rectangular 2-tori with $Z_2 \times Z_2$ orbifold symmetry and p is a positive integer

N_i	(n_i^1, l_i^1)	(n_i^2, l_i^2)	(n_i^3, l_i^3)
$N_a = 8$	(2, 0)	(p , 1)	(p , -1)
$N_b = 8$	(p , -1)	(2, 0)	(p , 1)
$N_c = 8$	(p , 1)	(p , -1)	(2, 0)
N_g	$N_g n_g^1 n_g^2 n_g^3 = -48p^2 + 16$		

Table 5

Model III. D6-brane configuration in $(2p)$ -generation quasi-supersymmetric $U(8)^3$ model. This model is built on three rectangular 2-tori with $Z_2 \times Z_2$ orbifold symmetry and p is a positive integer

N_i	(n_i^1, l_i^1)	(n_i^2, l_i^2)	(n_i^3, l_i^3)
$N_a = 16$	(1, 0)	(p , 1)	(p , -1)
$N_b = 16$	(p , -1)	(1, 0)	(p , 1)
$N_c = 16$	(p , 1)	(p , -1)	(1, 0)
N_g	$N_g n_g^1 n_g^2 n_g^3 = -48p^2 + 16$		

Table 6

Chiral open string spectrum for the $U(N)^3$ GUT models. $N = 4$ for Model I and Model II, and $N = 8$ for Model III. $N_f = 2p + 1$, $8p, 2p$ for Model I, Model II, and Model III, respectively

Sector	$U(N) \times U(N) \times U(N)$	Q_a	Q_b	Q_c
$ab + ba$	$N_f \times (N, \bar{N}, 1)$	1	-1	0
$bc + cb$	$N_f \times (1, N, \bar{N})$	0	1	-1
$ca + ac$	$N_f \times (\bar{N}, 1, N)$	-1	0	1

of bifundamental representations under $U(N)^3$ gauge symmetry which include the Standard Model fermions and Higgs particles. In particular, in Model I, we can only have three families of fermions and three pairs of Higgs particles.

One may notice that in Tables 3, 4, and 5, the number of the auxiliary branes (N_g) is negative if we have at least three family fermions. This means that the auxiliary branes are anti-D6-branes. And then, the 4-dimensional $N = 1$ supersymmetry, which is preserved by the observable D6-branes and orbifold background, is broken by the auxiliary D6-branes. Therefore, the models are quasi-supersymmetric, and the NSNS tadpoles do not vanish.

4. Comments on phenomenology of G^3 models

4.1. Gauge coupling unification

The gauge couplings have been discussed in Refs. [35,37]. Since the gauge couplings are associated with different stacks of D6-branes, usually they do not have a conventional gauge coupling unification, although the value of each gauge coupling at the string scale is predicted in terms of the moduli χ_i and the ratio of the Planck scale to string scale. Let us calculate the 4-dimensional gauge coupling in detail, and show that in our models, we do have the gauge coupling unification.

Dp -branes provide us a world where the gauge sectors are localized on $(p + 1)$ -dimensional spacetime while gravity propagates in 10-dimensional spacetime. Before compactification, the gravitational and gauge interaction on Dp -brane can be generally described by an effective action [43]

$$S_{10} \supset \int d^{10}x \frac{M_s^8}{(2\pi)^7 g_s^2} R_{10d} + \int d^{p+1}x \frac{M_s^{p-3}}{(2\pi)^{p-2} g_s} F_{p+1}^2, \quad (24)$$

where $M_s = 1/\sqrt{\alpha'}$ is the string scale, and g_s is the string coupling. Upon the compactification, the 4-dimensional Planck scale M_{Pl} and the gauge coupling g_{YM}^σ on the D6-brane stack σ are

$$M_{Pl}^2 = \frac{M_s^8 V_6}{(2\pi)^7 g_s^2},$$

$$(g_{YM}^\sigma)^2 = \frac{(2\pi)^4 g_s}{M_s^3 V_3^\sigma}, \quad (25)$$

where

$$V_6 = \frac{(2\pi)^6}{4} \prod_{i=1}^3 R_1^i R_2^i, \quad (26)$$

is the physical volume of T^6 and

$$V_3^\sigma = \frac{1}{4} (2\pi)^3 \prod_{i=1}^3 \sqrt{(n_\sigma^i R_1^i)^2 + (2^{-\beta_i} l_\sigma^i R_2^i)^2}, \quad (27)$$

is the physical volume of three-cycle wrapped by the D6-brane stack σ . So, we obtain

$$(g_{\text{YM}}^\sigma)^2 = \frac{\sqrt{8\pi} M_s}{M_{\text{Pl}}} \frac{1}{\prod_{i=1}^3 \sqrt{(n_\sigma^i)^2 \chi_i^{-1} + (2^{-\beta_i} l_\sigma^i)^2 \chi_i}}. \tag{28}$$

Because in our models, $\chi_1 = \chi_2 = \chi_3 = \chi$, we do have the gauge coupling unification. In general, we can expect that n_σ^i , l_σ^i and χ_i are the order one integer or real number. Then the 4-dimensional gauge coupling $(g_{\text{YM}}^\sigma)^2$ is about M_s/M_{Pl} . Therefore, for the intersecting D6-brane models with low string scale on the spacetime $M^4 \times T^6$ or $M^4 \times T^6/(Z_2 \times Z_2)$, where the D6-branes wrap on the factorized three cycles of three 2-tori, the gauge couplings are generically very small and may lead to the fine-tuning in the RGE runnings of gauge couplings. However, in the general Calabi–Yau threefolds, one can make the physical volume of the 6-dimensional compact manifold large without affecting the physical volume of the compact three cycles wrapped by the D6-branes [44,45], so, the low string scale in D6-brane models does not imply the very small gauge couplings in general.

4.2. Gauge symmetry breaking

In our models, the $U(N)^3$ gauge symmetry can be broken down to the Standard Model gauge symmetry by introducing the light open string states. As an example, we only consider the Model I, and similarly, one can discuss the gauge symmetry breaking in Model II and Model III.

In Model I, we have 3 families by choosing $p = 1$. The gauge group is $U(4) \times U(4) \times U(4)$, which has subgroup $SU(4) \times SU(2) \times SU(2)$, i.e., the Pati–Salam model. The left-handed fermions come from

the $(4, \bar{4}, 1)$ representations, the right-handed fermions come from the $(\bar{4}, 1, 4)$ representations, and the pair of Higgs doublets come from the $(1, 4, \bar{4})$ representations. Then, we will have three pairs of Higgs doublets. However, in order to have the D-flat and F-flat directions, we find that there are no Higgs particles at massless state level which can break the $U(4) \times U(4) \times U(4)$ gauge symmetry down to the $SU(4) \times SU(2) \times SU(2)$ or Standard Model gauge symmetry. Thus, the GUT breaking Higgs fields must arise from the light open string spectrum.

Indeed, we do have such kind of Higgs fields. The “ a ” stack of D6-branes a is parallel to the orientifold (ΩR) image b' of the “ b ” stack of D6-branes along the third torus, i.e., the “ b ” stack of D6-branes b is parallel to the orientifold (ΩR) image a' of the “ a ” stack of D6-branes along the third torus. Then, there are open strings which stretch between the branes a and b' (or say a' and b). If the minimal distance squared $Z_{(ab')}^2$ (in α' units) between these two branes on the third torus is small, i.e., the minimal length squared of the stretched string is small, we have the light scalars with masses $Z_{(ab')}^2/(4\pi^2\alpha')$ from the NS sector, and the light fermions with the same masses from the R sector [20,21]. These scalars and fermions form the 4-dimensional $N = 2$ hypermultiplets. Similarly, the “ b ” stack of D6-branes b is parallel to the orientifold (ΩR) image c' of the “ c ” stack of D6-branes along the first torus, and the “ c ” stack of D6-branes c is parallel to the orientifold (ΩR) image a' of the “ a ” stack of D6-branes along the second torus. Thus, we can also have the light hypermultiplets from the open strings which stretch between the branes b and c' , and between the branes c and a' .

The light open string spectrum is given in Table 7. These light Higgs fields can break the $U(4)^3$ down

Table 7

Light open string spectrum in the Model I which can break the $U(4)^3$ gauge symmetry down to the Standard Model gauge symmetry

Sector	$U(N) \times U(N) \times U(N)$	Q_a	Q_b	Q_c	Mass square
$ab' + ba'$	$4 \times (4, 4, 1)$	1	1	0	$Z_{(ab')}^2/(4\pi^2\alpha')$
$ab' + ba'$	$4 \times (\bar{4}, \bar{4}, 1)$	-1	-1	0	$Z_{(ab')}^2/(4\pi^2\alpha')$
$bc' + cb'$	$4 \times (1, 4, 4)$	0	1	1	$Z_{(bc')}^2/((4\pi^2\alpha')$
$bc' + cb'$	$4 \times (1, \bar{4}, \bar{4})$	0	-1	-1	$Z_{(bc')}^2/(4\pi^2\alpha')$
$ca' + ac'$	$4 \times (4, 1, 4)$	1	0	1	$Z_{(ca')}^2/(4\pi^2\alpha')$
$ca' + ac'$	$4 \times (\bar{4}, 1, \bar{4})$	-1	0	-1	$Z_{(ca')}^2/(4\pi^2\alpha')$

to the Standard Model gauge symmetry. Roughly speaking, the Higgs fields in the $(1, 4, 4)$ and $(1, \bar{4}, \bar{4})$ representations can break the $U(4) \times U(4) \times U(4)$ gauge symmetry down to the $U(4) \times SU(2) \times SU(2)$ gauge symmetry, and the Higgs fields in the $(4, 1, 4)$ and $(\bar{4}, 1, \bar{4})$ representations can break the $U(4) \times SU(2) \times SU(2)$ gauge symmetry down to the Standard Model gauge symmetry. The detail symmetry breaking pattern and phenomenology are under investigation. By the way, we do not need the particles in the $(4, 4, 1)$ and $(\bar{4}, \bar{4}, 1)$ representations to be light because we do not need them to break the gauge symmetry.

4.3. Supersymmetry breaking and possible problems

In our models, the observable D6-branes preserve the same 4-dimensional $N = 1$ supersymmetry as the orbifold background does. But, this supersymmetry is broken by the auxiliary D6-brane, which has no intersections with the observable D6-branes. So, the supersymmetry breaking effects can be mediated by the heavy bifundamental messenger fields with string scale masses which are the open strings stretching between the observable D6-brane and auxiliary D6-brane, and by the gravity supermultiplets in the bulk. Of course, the dominant contributions to the scalar masses and gaugino masses are from the gauge mediated supersymmetry breaking.

Similar to the discussions in [26,27], the quadratic divergences for scalars (for example, Higgs fields) are absent up to one-loop. The supersymmetry breaking soft masses for scalars generated from two-loop diagrams are the same order as the gaugino masses generated from one-loop diagrams. The soft masses-squared for scalars ϕ_a typically are

$$\tilde{m}_a^2 \propto \left[\frac{\alpha_i}{4\pi} \right]^2 M_s^2. \quad (29)$$

In our models, $\chi_1 = \chi_2 = \chi_3 = \chi$. Using Eq. (28), we obtain

$$M_s^2 \sim \frac{4\pi^2}{\sqrt{8\pi}} \tilde{m}_a M_{\text{Pl}} \prod_{i=1}^3 \sqrt{(n_\sigma^i)^2 \chi^{-1} + (2^{-\beta_i} l_\sigma^i)^2 \chi}. \quad (30)$$

Considering the Model I with three families and $\chi = 1$, we obtain that the string scale M_s is about 5.6×10^{11} GeV if $\tilde{m}_a \sim 1$ TeV. This is a reasonable unification scale for the Pati–Salam model [42] and

can be generated by introducing relatively large extra dimension [40,41] or small string coupling g_s . However, the gauge coupling (α_{GUT}) at string scale is seriously suppressed to 10^{-8} , which implies the fine-tuning in the RGE runnings of gauge couplings. For the RGE runnings of gauge couplings, we should include the additional contributions from the extra adjoint fields and their KK modes, and the KK modes of gauge fields. Whether we can have such small gauge coupling at string scale is a question deserving further detail study. By the way, if χ , which is a positive real number, is larger or smaller than 1, we can increase the string scale. However, the unification gauge coupling at string scale is the same.

5. Discussions and conclusions

Adding S_3 symmetry onto the observable D6-brane configuration and complex structure moduli, we obtain three natural quasi-supersymmetric GUT models with four interesting properties. In Model I and Model II, the gauge group is $U(4)^3$, while in Model III the gauge group is $U(8)^3$. The three tori of T^6 are all tilted for Model I, and they are all rectangular for Model II and Model III. The D6-brane configurations and chiral open string spectrum at massless level are given in Tables 3–6. In all our three models, the Standard Model fermions and Higgs particles can be embedded into the bifundamental representations, and there is no any other unnecessary massless representations. In particular, we only have three families of fermions and three pairs of Higgs particles for Model I. Moreover, we show that there exists the gauge coupling unification in our models. We consider the gauge symmetry breaking, too. Explicitly, we show that in Model I, the $U(4) \times U(4) \times U(4)$ gauge symmetry can indeed be broken down to the Standard Model gauge symmetry by introducing the light open string states, and similar mechanism works for the Models II and III. Furthermore, we find that the 1 TeV scale soft masses imply the intermediate string scale (10^{11} – 10^{12} GeV), which is a reasonable unification scale for the Pati–Salam model. However, the unification gauge coupling at string scale is very small and may lead to the fine-tuning in the RGE runnings of gauge couplings.

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