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Optimal transmission schedules in TWDM optical passive star networks

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Abstract

This paper is concerned with optimal transmission schedules in the TWDM (time and wavelength division multiplexed) optical passive star network. Our model of the network consists of a set V of N nodes with N tunable transmitters and N fixed-tuned receivers (where each node is assigned a transmitter–receiver pair), k wavelengths (each wavelength is shared by N/k receivers where N/k is an integer), and tuning time $\delta > 0$ (in units of time slot, for transmitters to tune from one wavelength to another). We assume that at any given time slot, at most one transmission can be done per wavelength. An *optimal transmission schedule* is defined to be the one that schedules transmissions such that for each node v in V, the transmitter in v transmits once to every receiver in $V - \{v\}$ within a repeating cycle of minimum length. We present an optimal transmission schedule for each tuning time, and show that the cycle length of any optimal transmission schedule is $\max\{N(N-1)/k, k\delta + N - 1\}$.

1. Introduction

The interconnection network in a multiprocessing system is of major importance to the performance of the system. Massively parallel processing requires massively parallel interconnects. It is well understood that electronic interconnect faces its fundamental physical limits as the performance of processors and their affiliated memories grow. Optical interconnects (primarily because of their higher bandwidth advantages) are viable alternatives for the traditional electronic interconnects when designing massively parallel processing systems [5,9,12,13]. Recently, there has been a lot of interest in using the time and wavelength division multiplexed (TWDM) optical communication network as the underlying architecture to support many different communication patterns by embedding them directly into the system hardware [1,2,6,8,11,15,16].

In this paper, we consider the following TWDM optical passive star network [3,7,8]. The physical architecture of the network has N inputs and N outputs connected by an

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optical passive star. Fig. 1 shows N nodes connected via an optical passive star. Each input has a single optical transmitter and each output has a single optical receiver. Each node v_i $(0 \leqslant i \leqslant N-1)$ in the network consists of a pair of transmitter t_i and receiver r_i. Transmitters and receivers can be tuned to any wavelength. In order to communicate from a transmitter to a receiver, the corresponding transmitter and receiver must be tuned to the same wavelength. Transmitters and receivers are called *tunable* if they can tune from one wavelength to another, while ones that cannot are called *fixed-tuned.*

Our model of the network assumes that receivers are fixed-tuned and transmitters are tunable; it also assumes that there are k $(2 \leqslant k \leqslant N)$ wavelengths in the network and N/k receivers share a wavelength, where N/k is an integer for simplicity. Furthermore, we assume that at any given time slot, at the most one transmission can be done per wavelength. Hence, at the most k transmissions can be done simultaneously. Time is divided into time slots of equal duration with the slot length equal to the *packet duration* (i.e., the amount of time to transmit a fixed size packet). For transmitters to tune from one wavelength to another, tuning time $\delta > 0$ (expressed in units of time slot) is required, and idle transmitters can tune to wavelengths just-in-time to start their transmissions. (It is noted that if packet sizes are small such as ATM cells, δ is expected to be large.) The problem considered in this paper is formulated as follows:

Optimal transmission schedule problem: A TWDM optical passive star network consists of a set V of N nodes with N tunable transmitters and N fixed-tuned receivers (where each node is assigned a transmitter–receiver pair), k wavelengths such that each wavelength w_i for $0 \leqslant i \leqslant k-1$ is shared by N/k receivers where N/k is an integer (the set of receivers tuned to wavelength w_i is $R_i = \{r_j \mid j \bmod k = i\}$), and tuning time δ. Given a virtual interconnection topology G with N nodes which is embedded in the network, the problem is to schedule transmissions in such a way that the time slots are arranged into repeating cycles of minimum length and each node in G transmits once to each of its neighboring nodes within a cycle. In this paper, G is assumed to be a complete graph. Thus, an optimal transmission schedule for our model is the one

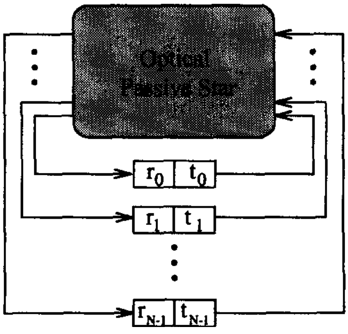

Fig. 1. An N-node optical passive star network

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	2	4				w_1	w_1	1	3	5				w_0	w_0
1	3	5				w_0	w_0	0	2	4				w_1	w_1
2	w_1	w_1	1	3	5				w_0	w_0	0	4			
3	w_0	w_0	0	2	4				w_1	w_1	1	5			
4				w_0	w_0	0	2				w_1	w_1	1	3	5
5				w_1	w_1	1	3				w_0	w_0	0	2	4

(a) T_1: $\delta = 2$, $N = 6$, $k = 2$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	2	4	w_1	w_1	w_1	w_1	w_1	1	3	5	w_0	w_0	w_0	w_0	w_0
1	3	5	w_0	w_0	w_0	w_0	w_0	0	2	4	w_1	w_1	w_1	w_1	w_1
2	w_1	w_1	1	3	5	w_0	w_0	w_0	w_0	w_0	0	4	w_1	w_1	w_1
3	w_0	w_0	0	2	4	w_1	w_1	w_1	w_1	w_1	1	5	w_0	w_0	w_0
4	w_0	w_0	w_0	w_0	w_0	0	2	w_1	w_1	w_1	w_1	w_1	1	3	5
5	w_1	w_1	w_1	w_1	w_1	1	3	w_0	w_0	w_0	w_0	w_0	0	2	4

(b) T_2: $\delta = 5$, $N = 6$, $k = 2$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0	2	4	w_1	w_1	w_1	w_1	w_1	w_1	w_1	w_1	1	3	5	w_0	w_0	w_0	w_0	w_0	w_0	w_0	w_0
1	3	5	w_0	w_0	w_0	w_0	w_0	w_0	w_0	w_0	0	2	4	w_1	w_1	w_1	w_1	w_1	w_1	w_1	w_1
2	w_1	w_1	1	3	5	w_0	w_0	w_0	w_0	w_0	w_0	w_0	w_0	0	4	w_1	w_1	w_1	w_1	w_1	w_1
3	w_0	w_0	0	2	4	w_1	w_1	w_1	w_1	w_1	w_1	w_1	w_1	1	5	w_0	w_0	w_0	w_0	w_0	w_0
4	w_0	w_0	w_0	w_0	w_0	0	2	w_1	w_1	w_1	w_1	w_1	w_1	w_1	w_1	1	3	5	w_0	w_0	w_0
5	w_1	w_1	w_1	w_1	w_1	1	3	w_0	w_0	w_0	w_0	w_0	w_0	w_0	w_0	0	2	4	w_1	w_1	w_1

(c) T_3: $\delta = 8$, $N = 6$, $k = 2$

Fig. 2. Allocation tables.

that schedules transmissions such that for each node v in V, v transmits once to every node in $V - \{v\}$ within a repeating cycle of minimum length.

A transmission schedule will be expressed as a table that represents a repeating cycle of time and wavelength allocations. As an example, consider a network with six nodes and two wavelengths, so $R_0 = \{r_0, r_2, r_4\}$ and $R_1 = \{r_1, r_3, r_5\}$. Let G be a complete graph with six nodes. Fig. 2 shows allocation tables: T_1 for $\delta = 2$, T_2 for $\delta = 5$, and T_3 for $\delta = 8$. Each entry $T(i, l)$ of the tables occurring in the ith row and the lth column is either j, w_d or *blank*: during the time slot l, $T(i,l) = j$ if t_i is transmitting to r_j, $T(i,l) = w_d$ if t_i is tuning to wavelength w_d, and $T(i,l) = blank$ if t_i is not involved in transmitting or tuning.

Pieris and Sasaki [14] proposed an all-to-all broadcasting model where each transmitter has N packets to be sent to N receivers; thus, N^2 transmissions need to be done. They then showed that (assuming the initial tuning time δ) a lower bound and an upper bound of the minimum schedule length are, respectively, $\max\{\delta + N^2/k, N\sqrt{\delta}\}$ and $\max\{\delta + N^2/k, k\delta + N - N/k + N^2/k^2\}$. Choi et al. [4] later proved that the above upper bound is the length of an optimal schedule. In the all-to-all broadcasting model studied in [4, 14], each transmitter has to send packets to k groups of receivers, where the size of each group is N/k. Our model of the network with N nodes requires each transmitter to transmit to $N - 1$ receivers since G is a complete graph. Thus, every transmitter has to transmit to k groups of receivers, where $k - 1$ groups are of the same size N/k but the size of one remaining group is $N/k - 1$, which makes our approach to optimal transmission schedules more analytically complex than that for the model in [14].

The rest of this paper is organized as follows. In Section 2, we evaluate a lower bound for the cycle length of any transmission schedule. An optimal transmission schedule for each tuning time δ is obtained in Section 3. Finally, we give our concluding remarks in Section 4.

2. Lower bound of schedule length

Lemma 1. *The cycle length of any transmission schedule is at least* $\max\{N(N-1)/k, k\delta + N - 1\}$.

Proof. Let $L_{\min}$ denote the minimum cycle length of any transmission schedule. Each node in G has $N - 1$ neighboring nodes, so $N(N - 1)$ transmissions must be done in a cycle. Since at the most k transmissions can be done simultaneously, we have $L_{\min} \geqslant N(N-1)/k$.

On the other hand, observe that any node v in G has $N - 1$ neighboring nodes whose receivers are tuned to k different wavelengths. Therefore, we need tuning time of at least, $k\delta$ for the transmitter assigned to v to tune to k different wavelengths. Since there have to be $N - 1$ transmissions from v, it implies that $L_{\min} \geqslant k\delta + N - 1$. We conclude that the cycle length $L_{\min}$ is at least $\max\{N(N-1)/k, k\delta + N - 1\}$. □

3. Optimal transmission schedules

We will construct an optimal transmission schedule (i.e., minimum length schedule) for each δ. First we will design an algorithm for an initial allocation schedule (satisfying the conditions (i)–(iii) below) for a cycle of $N(N-1)/k$ consecutive time slots. Next, in the proofs of Lemmas 2–4, we will construct an optimal transmission schedule for each δ from the initial allocation schedule obtained from *Algorithm Initial Allocation* (G).

In order to facilitate our discussion we need to introduce the following notation and a partition of the node set of G and a partition of $N(N-1)/k$ time slots, together with matrices M and Q corresponding to an initial allocation schedule. Let $P = \{V_i \mid 0 \leqslant i < N/k\}$ be a partition of the node set $V = \{v_i \mid 0 \leqslant i \leqslant N-1\}$ of G where $V_i = \{v_j \mid ik \leqslant j \leqslant (i+1)k-1\}$, and write $S_q = \{v_i \in V \mid r_i \in R_q\}$ $(0 \leqslant q < k)$. Note that $v_i \in S_q$ iff $q = i \bmod k$, and that $|V_i \cap S_q| = 1$ (i.e., V_i contains exactly one receiver in R_q) for each i and q $(0 \leqslant i < N/k,\ 0 \leqslant q < k)$.

Note that each node in G has $N-1$ neighboring nodes whose receivers are tuned to k different wavelengths, one of which is shared by $N/k-1$ receivers and each of the remaining $k-1$ wavelengths is shared by N/k receivers. In our optimal transmission schedule, each node transmits consecutively to all of its neighboring nodes sharing the same wavelength: the transmission from each node can be done using k blocks of time slots, one of which has length $N/k-1$ and each of the remaining $k-1$ blocks has length N/k. In the following, we first construct a matrix M which shows an arrangement of the contiguous time slots of length $N/k-1$ for each node. We next construct matrices M^* and Q to find an *initial allocation schedule*. This schedule will be modified later to satisfy the tuning duration.

Let $N/k = ak + b$ $(0 \leqslant b < k)$, so $a = \lfloor N/k^2 \rfloor$ and $b = (N/k) \bmod k$. First, define an $N/k \times k$ matrix $M = (m_{ij})$ $(0 \leqslant i < N/k,\ 0 \leqslant j < k)$ with $m_{ij} \in \{1, -1\}$ by: $m_{ij} = -1$ iff $i + j = N/k - 1 - tk$ for $0 \leqslant t \leqslant a$. Observe that for each row i of M, there is only one i' such that $m_{i\,i'} = -1$. Next, define an $N/k \times k$ matrix $M^* = (p_{ij})$ $(0 \leqslant i < N/k,\ 0 \leqslant j < k)$ by: each row i, $[p_{i0}\ \ p_{i1}\ \cdots\ \ p_{i\,k-1}]$, of M^* is obtained from $[0\ \ 1\ \ \cdots\ \ k-1]$ by cyclically shifting it to the right i' positions. Next, we define an $N \times k$ matrix $Q = (q_{sl})$ $(0 \leqslant s < N,\ 0 \leqslant l < k)$ by: each row $s = ik + j$ $(0 \leqslant j < k)$ of Q is obtained from row i of M^* by cyclically shifting it to the left j positions. Note that for each $s = ik + j$ $(0 \leqslant s < N,\ 0 \leqslant j < k)$, $q_{sl} = j$ iff $p_{il} = 0$, while $p_{il} = 0$ iff $m_{il} = -1$; hence $q_{sl} = j$ iff $m_{il} = -1$. This implies that for each $s = ik + j$ $(0 \leqslant s < N,\ 0 \leqslant j < k)$ and l $(0 \leqslant l < k)$, $v_s \in S_{q_{sl}}$ iff $m_{il} = -1$. Thus, the matrices M and Q can be used as indicators for an initial allocation schedule: $m_{il} = -1$ ($m_{il} = 1$, respectively) corresponds to transmissions of each $v_s \in V_i$ to the nodes in $S_{q_{sl}} - \{v_s\}$ (in $S_{q_{sl}}$, respectively). Fig. 3. shows a construction of M, M^* and Q. Now, let $B = \{B_l \mid 0 \leqslant l < k\}$ be a partition of $N(N-1)/k$ consecutive time slots with $|B_l| = (N/k)^2 - (a+1)$ for $0 \leqslant l < b$ and $|B_l| = (N/k)^2 - a$ for $b \leqslant l < k$ such that the last time slot in B_l is followed by the first time slot in $B_{(l+1) \bmod k}$.

$$\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

(a) M

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

(b) M*

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

(c) Q

Fig. 3. Matrices M, M^* and Q for $N = 24$ and $k = 3$

Now, we construct an initial allocation schedule for a cycle of $N(N-1)/k$ consecutive time slots which satisfies the following conditions:

(i) For each $v_s \in V_i$ $(0 \leqslant i < N/k)$ and each block B_l, if $m_{il} = -1$ then v_s transmits to only the nodes in $S_{q_{sl}} - \{v_s\}$ (since $v_s \in S_{q_{sl}}$) using $|S_{q_{sl}} - \{v_s\}|$ consecutive time slots, while if $m_{il} = 1$ then v_s transmits to only the nodes in $S_{q_{sl}}$ (since $v_s \notin S_{q_{sl}}$) using $|S_{q_{sl}}|$ consecutive time slots; t_s tunes to wavelength w_q (where $q = q_{s\ ((l+1) \bmod k)}$) using all the time slots between the block of transmissions for v_s in B_l and the block of transmissions for v_s in $B_{(l+1) \bmod k}$. (We denote by T_{sl} the tuning allocations for v_s between the block of transmissions for v_s in B_l and the block of transmissions for v_s in $B_{(l+1) \bmod k}$.)

(ii) For each time slot in B_l $(0 \leqslant l < k)$, exactly k nodes which belong to some member of P transmit simultaneously.

(iii) For each l $(0 \leqslant l < k)$, $|T_{il}| = |T_{jl}|$ for i, j $(0 \leqslant i, j < N)$.

The following algorithm gives an initial allocation schedule of each node v_s in V during each time block B_l. [Each tuning allocation T_{sl} $(0 \leqslant s < N,\ 0 \leqslant l < k)$ in the initial allocation schedule will be modified to satisfy the δ tuning time requirement when we construct an optimal transmission schedule (Lemmas 2–4) from the initial allocation schedule.]

Algorithm Initial Allocation (Complete-Graph)
Input: A complete graph G with N nodes.
Output: An initial allocation schedule $(B_0, B_1, B_2, \ldots, B_{k-1})$.
For $l = 0$ to $k-1$ do
/* schedule of each node in V during time block B_l */
  $\alpha \leftarrow 0$;
  For $i = 0$ to $N/k - 1$ do
  /* schedule of each node in V_i during time block B_l */
    For $j = 0$ to $k-1$ do
      $s \leftarrow ik + j$; $q \leftarrow q_{sl}$;
      t_s tunes to wavelength w_q using the first α time slots in B_l;
      If $(m_{il} = -1)$ /* $v_s \in S_q$ */
        then v_s transmits to all nodes in $S_q - \{v_s\}$ using
        the next $N/k - 1$ consecutive time slots in B_l
      else /* $v_s \notin S_q$ */
        v_s transmits to all nodes in S_q using the next N/k consecutive
        time slots in B_l
      Endif
      $q \leftarrow q_{s\,((l+1) \bmod k)}$;
      t_s tunes to wavelength w_q using the remaining
      time slots in B_l
    Endfor
    If $(m_{il} = -1)$ then $\alpha \leftarrow \alpha + N/k - 1$ else $\alpha \leftarrow \alpha + N/k$ Endif
  Endfor
Endfor
Return $(B = \{B_i \mid 0 \leqslant i < k\})$
End *Algorithm.*

Fig. 4 shows the initial allocation schedule (B_0, B_1, B_2) obtained from Algorithm Initial Allocation (G) for $N = 24$ and $k = 3$ (see the corresponding matrices M and Q in Fig. 3).

We show that the initial allocation schedule $(B_0, B_1, B_2, \ldots, B_{k-1})$ for a cycle of $N(N-1)/k$ time slots obtained from *Algorithm Initial Allocation (G)* satisfies the conditions (i–iii). Observe that for each $v_s \in V_i$ $(0 \leqslant i < N/k)$ and each block B_l, if $m_{il} = -1$ then v_s transmits to only the nodes in $S_{q_{sl}} - \{v_s\}$ using $|S_{q_{sl}} - \{v_s\}|$ consecutive time slots, while if $m_{il} = 1$ then v_s transmits to only the nodes in $S_{q_{sl}}$ using $|S_{q_{sl}}|$ consecutive time slots; furthermore, t_s tunes to wavelength w_q (where $q = q_{s\,((l+1) \bmod k)}$) using all the time slots between the block of transmissions for v_s in B_l and the block of transmissions for v_s in $B_{(l+1) \bmod k}$. Since the first α time slots in B_l are used for tuning all nodes in V_i, it follows that for each time slot in B_l, exactly k nodes which belong to some member of P transmit simultaneously. For any node $v \in V_i$ $(i > 0)$, the number of time slots for transmitting v in B_l is equal to the number of time slots for transmitting any node of V_{i-1} in $B_{(l+1) \bmod k}$, which implies that $|T_{il}| = |T_{jl}|$ for each i

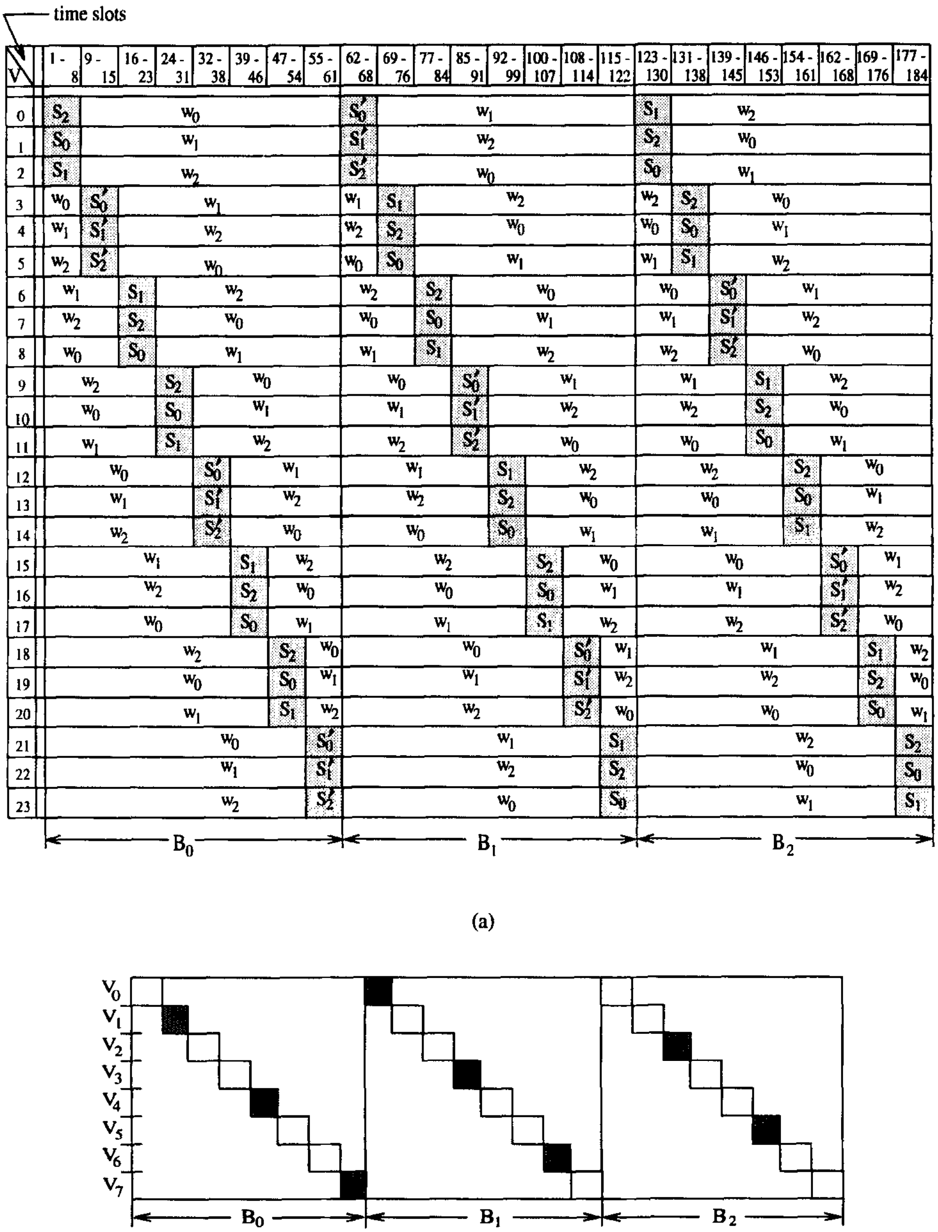

Fig. 4. (a) Initial allocation schedule (B_0, B_1, B_2) from Algorithm Initial Allocation(G) for $N=24$ and $k=3$. Row s ($0 \leqslant s \leqslant 23$) is the transmission schedule for $v_s \in V$: S_i (S_i', respectively) indicates that v_s transmits to all nodes in S_i ($S_i - \{v_s\}$, respectively); w_i indicates that t_s tunes to wavelength w_i. (b) Block diagram of (a), $V_i = \{v_j \mid 3i \leqslant j \leqslant 3i+2\}$ ($0 \leqslant i \leqslant 7$).

and j $(0 \leqslant i,\ j < N)$. Thus, the initial allocation schedule $(B_0, B_1, B_2, \ldots, B_{k-1})$ for a cycle of $N(N-1)/k$ time slots obtained from Algorithm Initial Allocation (G) satisfies the conditions (i–iii). Also, the initial allocation schedule satisfies the condition that for each $v \in V$, v transmits to all nodes in $V - \{v\}$.

Now, it remains to adjust each T_{ij} $(0 \leqslant i < N,\ 0 \leqslant j < k)$ of the initial allocation schedule to satisfy the δ tuning time requirement. In the proofs of Lemmas 2–4, we construct an optimal transmission schedule from the initial allocation schedule designed on $B = \{B_j \mid 0 \leqslant j < k\}$ with a cycle of $N(N-1)/k$ time slots, and then evaluate the cycle length of the optimal transmission schedule: For a construction of an optimal transmission schedule, the tuning allocations T_{ij} $(0 \leqslant i < N,\ 0 \leqslant j < k)$ of the initial allocation schedule are adjusted in order to satisfy the δ tuning time requirement by using additional time slots for a large δ, or by replacing redundant tunings with blanks for a small δ. Recall that $a = \lfloor N/k^2 \rfloor$ and $b = N/k \mod k$.

Lemma 2. *Suppose $\delta \leqslant (N/k)(N/k-1)-(a+1)$. Then the cycle length of any optimal transmission schedule is $N(N-1)/k$.*

Proof. By Lemma 1, we need only show that there exists a transmission schedule with cycle length $N(N-1)/k$. Observe from the initial allocation schedule that for each $v_i \in V$, $|T_{ij}| = (N/k)(N/k-1)-(a+1)$ for $0 \leqslant j < b-1$, and $|T_{ij}| = (N/k)(N/k-1)-a$ for $b-1 \leqslant j < k$; hence $\delta \leqslant |T_{ij}|$ for each i and j. We now replace the first $|T_{ij}| - \delta$ allocations of T_{ij} by blanks for each j $(0 \leqslant j < k)$ and each $v_i \in V$. The new allocation clearly constitutes a feasible transmission schedule with cycle length $N(N-1)/k$. This completes the proof. □

Lemma 3. *Suppose $\delta = (N/k)(N/k-1)-a$. Then the cycle length L of any optimal transmission schedule is given by*:

(1) $L = k\delta + N - 1$ *for* $a = 0$ *or* $b > 1$.
(2) $L = \frac{N(N-1)}{k}$ *for* $a > 0$ *and* $b \leqslant 1$.

Proof. Since $L \geqslant N(N-1)/k$ and $L \geqslant k\delta + N - 1$ by Lemma 1, we need only show that there exists a transmission schedule with cycle length $N(N-1)/k$ or $k\delta + N - 1$. For each of the following cases, we first evaluate $|T_{ij}|$ from the initial allocation schedule.

Case 1: $a = 0$ *or* $b > 1$. For each $v_i \in V$, we have $|T_{ij}| = (N/k)(N/k-1)-(a+1)$ for $0 \leqslant j < b-1$, and $|T_{ij}| = (N/k)(N/k-1)-a$ for $b-1 \leqslant j < k$. Thus, $|T_{ij}| = \delta - 1$ for $0 \leqslant j < b-1$, and $|T_{ij}| = \delta$ for $b-1 \leqslant j < k$. Now, for each j $(0 \leqslant j < b-1)$, insert one time slot s_j between B_j and $B_{j+1}$and assign to s_j the contiguous tuning of T_{ij} for each i $(0 \leqslant i < N)$. It follows that T_{ij} is extended to δ time slots for each j $(0 \leqslant j < b-1)$, and thus the new allocation has exactly δ tuning time between any two blocks of transmissions for each $v_i \in V$. We conclude that the new allocation schedule has length $k\delta + N - 1$.

Case 2: $a > 0$ *and* $b \leqslant 1$. First let $a > 0$ and $b=1$. Note that $|T_{ij}| = (N/k)(N/k-1)-a = \delta$ for each j $(0 \leqslant j < k)$ and each $v_i \in V$. Thus, the initial allocation schedule with length $N(N-1)/k$ satisfies the δ tuning time requirement. Next let $a > 0$ and $b = 0$. In this case we have for each $v_i \in V$, $|T_{ij}| = (N/k)(N/k-1) - a = \delta$ for each j $(0 \leqslant j < k-1)$, and $|T_{i\,k-1}| = (N/k)(N/k-1)-(a-1) = \delta+1$. Thus, for each $v_i \in V$, we replace the first tuning in $T_{i\,k-1}$ by a blank so that the new allocation satisfies the δ tuning time requirement. The new allocation schedule has length $N(N-1)/k$. This completes the proof. $\square$

Lemma 4. *Suppose $\delta > (N/k)(N/k-1) - a$. Then the cycle length L of any optimal transmission schedule is $L = k\delta + N - 1$.*

Proof. Since $L \geqslant k\delta + N - 1$, by Lemma 1, we need only show that there exists a transmission schedule with cycle length $k\delta + N - 1$. Note from the initial allocation schedule that for each $v_i \in V$, $|T_{ij}| \leqslant (N/k)(N/k-1) - a < \delta$ for each j $(0 \leqslant j < k-1)$, and $|T_{i\,k-1}| \leqslant (N/k)(N/k-1) - a + 1 \leqslant \delta$. Now, for each j $(0 \leqslant j < k)$, insert $\delta - |T_{ij}|$ time slots s_j between B_j and $B_{(j+1) \bmod k}$ and assign to each time slot in s_j the contiguous tuning of T_{ij} for each i $(0 \leqslant i < N)$. It follows that T_{ij} is extended to δ time slots for each j $(0 \leqslant j < k)$, and thus the new allocation has exactly δ tuning time between any two blocks of transmissions for each $v_i \in V$. We conclude that the new allocation schedule has length $k\delta + N - 1$. $\square$

From Lemmas 1–4, we establish the following theorem.

Theorem 1. *The cycle length L of any optimal transmission schedule is* $\max\{N(N-1)/k, k\delta + N - 1\}$. *Furthermore, L is given by*:
(1) $L = N(N-1)/k$ *if* $\delta \leqslant (N/k)(N/k-1) - (a+1)$.
(2) $L = N(N-1)/k$ *if* $\delta = (N/k)(N/k-1) - a$ *with* $a > 0$ *and* $b \leqslant 1$.
(3) $L = k\delta + N - 1$ *if* $\delta = (N/k)(N/k-1) - a$ *with* $a = 0$ *or* $b > 1$.
(4) $L = k\delta + N - 1$ *if* $\delta > (N/k)(N/k-1) - a$.

4. Concluding remarks

In this paper, we have constructed an optimal transmission schedule for each tuning time in the TWDM optical passive star network when the virtual interconnection topology G is a complete graph. Our model of the network with N nodes and k wavelengths requires each transmitter to transmit to k groups of receivers, where $k-1$ groups are of the same size N/k, but the size of one remaining group is $(N/k) - 1$. In a similar model considered in [14], each transmitter has to send packets to k groups of receivers, where the size of each group is N/k. It should be noted that optimal transmission schedules for our model is more diverse than those for the model in [14].

When the virtual interconnection topology G is a hypercube, an optimal transmission schedule is reported in [10]. Further extension includes construction of optimal transmission schedules for other virtual interconnection topologies including meshes, tori and trees.

References

[1] J.D. Attaway and J. Tan, 'HyperFast: Hypercube time slot allocation in a TWDM network,' preprint (1994).
[2] S. Bhattacharya, D.H. Du, and A. Pavan, 'A network architecture for distributed high performance heterogeneous computing,' in: Proceedings of the Heterogeneous Computing Workshop, 110–115.
[3] C.A. Brackett, 'Dense wavelength division multiplexing networks: principles and applications,' (Invited Paper) IEEE J. Selected Areas Comm. 8 (1990) 948–964.
[4] H.S. Choi, H.-A. Choi and M. Azizoglu, Efficient scheduling of transmissions in optical broadcast networks, to appear in IEEE/ACM Trans. on Networkings. (Preliminary version of this paper can be found in Proc. of ICC'95 Communications - Gateway to Globalization, 266-270.)
[5] P.W. Dowd, 'High performance interprocessor communication through optical wavelength division multiple access channels,' Proceedings of the Computer Architecture Conference, (1991) 96–105.
[6] P.W. Dowd, 'Wavelength division multiple access channel hypercube processor interconnection," IEEE Trans. Comput. 41 (1992) 1223–1241.
[7] A. Ganz and Y. Gao, 'Time-wavelength assignment algorithms for high performance WDM star based systems,' IEEE Trans. Comm. 42 (1994) 1827–1836.
[8] G.R. Green, Fiber Optic Networks, (Prentice Hall, Englewood Cliffs, NJ 1993).
[9] A. Guha, J. Bristow, C. Sullivan and A. Husain, 'Optical interconnections for massively parallel architectures,' Appl. Opt. 29 (1990).
[10] S.-K. Lee, A.D. Oh and H.-A. Choi, 'Hypercube interconnection in TWDM optical passive star networks," Technical report GWU-IIST 95-02, Department of Electrical Engineering and Computer Science, George Washington University, (1995). (Preliminary result of this can be found in Proceedings of the Massively Parallel Processing Using Optical Interconnections, San Antonio TX, 23 - 23 October (1995) 289–296.
[11] G. Liu, K.Y. Lee and H. Jordan, 'TDM Hypercube and TWDM mesh optical interconnections,' in: Proceedings of the IEEE GLOBECOM (1994) 1953–1957.
[12] A. Louri and H. Sung, 'A hypercube-based optical interconnection network: a solution to the scalability requirements for massively parallel computers,' Proceedings of the Ist International Workshop on Massively Parallel Processing using Optical Interconnections, Cancun, Mexico (1994) 81–93.
[13] A. Louri and H. Sung, 'An efficient 3-D optical implementation of binary de bruijn networks with applications to massively parallel processing,' Proceedings of the Massively Parallel Processing using Optical Interconnections, San Antonio, TX (1995) 152–159.
[14] G.R. Pieris and G.H. Sasaki, Scheduling transmissions in WDM broadcast-and-select networks, IEEE/ACM Trans. Networking 2 (2) (1944).
[15] K.A. Williams and D.H.C. Du, 'Efficient embedding of a hypercube in an irregular WDM network,' Technical report, Department of Computer Science, University of Minnesota (1991).
[16] K.A. Williams and D.H.C. Du, 'Time and wavelength division multiplexed architectures for optical passive star networks,' Technical report, Department of Computer Science, University of Minnesota (1991).