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NOTE

SUM-FREE SETS AND RAMSEY NUMBERS II

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In this note we obtain a new lower bound for the Ramsey number R(5, 6). The method is computational and the bound obtained is $R(5,6) \geq 57$.

In [1] we defined $f(k_1, k_2)$, $k_1, k_2 \geq 3$, to be the largest integer m such that the integers $1, 2, \ldots, m$ can be partitioned into two classes C_1 and C_2 in such a way that C_1 does not contain a solution to a system of equation (S_{k_1}) and C_2 does not contain a solution to a system of equations (S_{k_2}) where (S_k) is given by

$$x_{i,j} + x_{j,j+1} = x_{i,j+1}, \qquad 1 \leq i < j \leq k-1.$$

Let $R(k_1, k_2)$ be the smallest integer q such that if the edges of any complete graph on at least q nodes are coloured in any manner in colours c_1 and c_2, then the resulting graph contains either a complete graph of colour c_1 on k_1 nodes or a complete graph of colour c_2 on k_2 nodes.

In [1] we showed that $R(k_1, k_2) \geq f(k_1, k_2) + 2$, and based on some partial computer results conjectured that $f(5, 6) = 51$. With the aid of an I.B.M. 360/67 and approximately 4 hours c.p.u. time we have found that in fact $f(5, 6) = 55$ so that the Ramsey number $R(5, 6) \geq 57$. One such partitioning of the integers $1, 2, \ldots, 55$ is

$$C_1 = \{2, 3, 6, 9, 14, 16, 18, 19, 23, 24, 25, 27, 28, 29, 31, 32, 33, 37, 38, 40, 42, 47, 50, 53, 54\}$$

$$C_2 = \{1, 4, 5, 7, 8, 10, 11, 12, 13, 15, 17, 20, 21, 22, 26, 30, 34, 35, 36, 39, 41, 43, 44, 45, 46, 48, 49, 51, 52, 55\}.$$

All of the twelve such partitionings we found give rise to regular colourings in the sense of Kalbfleisch [2]. Based on this observation and our previous computations we would conjecture the following:

(I) For any $k_1, k_2 \geq 3$, there exists a partitioning of the integers $1, 2, \ldots, f(k_1, k_2)$ that gives rise to a regular colouring.

(II) For any $k_1, k_2 \geq 3$, there exists two partitionings of the integers $1, 2, \ldots, f(k_1, k_2)$ such that in one partitioning $1 \in C_1$ and in the other $1 \in C_2$.

If the first conjecture is true the computer time can very likely be shortened for such problems by using branch and bound techniques. The second conjecture, if always true, would effectively halve the computer search as we have approached it, since we are using backtrack methods.

In regards to the second conjecture, Kalbfleisch, [3], has observed that if in general

$$\{1,2,\ldots,n-1\} = C_1 \cup C_2 \cup \cdots \cup C_m$$

is a partitioning of the integers $1,2,\ldots,n-1$ into classes $C_1, C_2, \ldots, C_m$ and if k is an integer such that $(k,n)=1$, then multiplying all elements by k (mod n) gives a new partition

$$\{1,2,\ldots,n-1\} = C_1' \cup C_2' \cup \cdots \cup C_m'$$

and the new partition and the old define isomorphic colourings of the complete graph on n vertices. It thus follows that, if there exists a colouring in which C_j contains an integer prime to n, then there exists an isomorphic colouring in which $1 \in C_j$. For example, if C_1 and C_2 are the partitions given above for $f(5,6)$ then we may choose $k \in \{3, 5, 9, 11, 13, 15, 17, 19, 23, 25, 27\}$ which yields the twelve partitionings that we found.

Acknowledgements

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References

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[3] J.G. Kalbfleisch, Chromatic graphs and Ramsey's theorem, Ph.D. Thesis, University of Waterloo (1966).