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Revisiting the Dang Van criterion

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Abstract

The aim of this paper is to reanalyze the Dang Van High cycle fatigue criterion using recent results in crystal plasticity and micromechanics. The initial criterion is based on the estimation of the shakedown limit at the grain scale and assumes a plastic inclusion in an elastic matrix in order to localize the macroscopic stress at the grain scale. The work presented here starts from the classical assumptions of the criterion. It then refines the micromechanical modeling at the grain scale by considering on the one hand side a coupled plastic-damage polycrystalline material behavior and on the other hand side a numerical model based on crystal plasticity. Several aspects of metal fatigue will be discussed: notch and stress gradient effects, mean stress effects and finite lifetime estimation. The result permit to asses the criterion using refined modeling and enables to cover qualitatively and quantitatively new areas of application and extensions of the classical Dang Van proposal.

Keywords: high cycle fatigue, plasticity, damage, aggregates.

1. Introduction

In high cycle fatigue, Dang Van's approach [1] lead to the definition of a widely used multiscale criterion corresponding to the unlimited endurance condition. The basic framework is the following:

- the fatigue damage is controlled by mechanisms at the grain scale and therefore a description at this mesoscopic scale is necessary;
- at this scale, most of the metallic materials are aggregates of crystals with a random distributed crystallographic orientations, that can be considered isotropic and homogeneous at the macroscopic scale;
- among all grains and possible slip planes, only some well-oriented slip planes, maximizing the shear stress for a given loading path, will develop plasticity and create localized slip bands inducing crack initiation;
- below the fatigue limit, microscopic plastic strains homogenize to negligible macroscopic plastic strains, which matches the fact that macroscopic stresses are small with respect to the yield limit.

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The Dang Van criterion is then based on the Lin–Taylor homogenization assumption in order to relate meso- and macroscopic mechanical fields and, based on shakedown concepts, is defined in terms of a linear combination between the hydrostatic pressure, p_h, and the mesoscopic resolved shear stress amplitude τ_a. The objective of this paper is to justify some ingredients of the Dang Van criterion and, more particularly, the maximization of the shear stress and of the hydrostatic pressure, and to propose an extension of the criterion in the case where no elastic shakedown is obtained that lead to finite lifetimes.

2. Mean stress effects

Under low macroscopic loading as in the context of HCF, in the case of FCC structures, the plastic behavior is generally characterized by the activation of a predominant slip system and more precisely by the formation of strain localization bands that are the potential sites for microcracks nucleation. This general framework is detailed in [2] and corresponds to the classical hypothesis of the Dang Van approach. The originality here consists in the introduction of damage at the grain scale. In fact, in the particular case of persistent slip bands (PSB), the strain localization is also accompanied by a dislocation annihilation mechanism which leads to the formation of vacancies along such bands that has been modeled by Essmann et al. [3] by defining a porosity associated to this mechanism, η_a, depending on the cumulated plastic slip.

As the transition from vacancy production to the formation of microcracks is not yet well understood, it is assumed that it is the result of the agglomeration and the growth of vacancies formed by the previous process. Damage along slip bands is then the result of two mechanisms: vacancy production and voids growth that is the result of the combined effect of the slip-like plastic activity and pressure. The total porosity at the grain scale, η, is then decomposed into two terms corresponding respectively to nucleation, η_a, and the growth part, η_g. As voids nucleation and growth induce volume change, the plastic strain at the grain scale can be decomposed in the following form, $\underline{\underline{I}}$ being the second order identity tensor:

$$\underline{\underline{\varepsilon}}_p = \underline{\underline{\varepsilon}}_p^{\,d} + \varepsilon_p^{\,h}\,\underline{\underline{I}} \tag{1}$$

where the volumetric plastic strain $\varepsilon_p^{\,h}$ due to voids growth is related to η_g by using mass balance equation: $\eta_g = 1 - \exp(-3\varepsilon_p^{\,h})$. A local fatigue criterion corresponding to no crack initiation is then defined by considering a critical value of the porosity, η_c, ideally corresponding to a critical crack size at the slip band/matrix interface:

$$\eta_a + \eta_g < \eta_c \tag{2}$$

A first step of the modeling of void growth consists in the consideration of a single void growth in an infinite perfectly plastic medium using the well-known Rice and Tracey approach [4]. The volumetric plastic strain $\varepsilon_p^{\,h}$ is then the result of the combined action of pressure and the predominant slip plastic strain activity and is given by the following equation (see [2] for more details) :

$$\dot{\varepsilon}_p^{\,h} = \eta \frac{1}{2\sqrt{3}}.\sinh\left(\frac{3}{2}\frac{\sigma_h}{\tau_0}\right)\dot{\gamma}_{p\,cum} \tag{3}$$

where σ_h is the hydrostatic part of the microscopic stress tensor and $\dot{\gamma}_p$ is described by the Schmid law and the indice "cum" designates the cumulative plastic strain. Then, inspired by the HCF framework of Dang Van [1], in order to relate meso- and macroscopic mechanical fields, the more generalized self-consistent scheme of Kröner can be used which includes the Lin-Taylor model. Following Dang Van's reasoning, the elastic shakedown at the grain scale is a first necessary condition for the absence of crack initiation. The criterion (2) is here a sufficient condition [2]. The shakedown condition (that cumulated plastic strain is bounded) enables simplifications and a final expression of the macroscopic fatigue criterion, deduced from (2) and from the proposition of Essmann et al. [3], is the following [2]:

$$A_0\,(k_a\gamma_{p\,cum} - 1 + \exp(-k_a\gamma_{p\,cum})) + 3\varepsilon_p^{\,h} < \eta_c \tag{4}$$

Then, by first considering affine loadings, the Lin-Taylor scheme and a linear isotropic hardening model, this expression (4) leads exactly to the Dang Van criterion [2]. The criterion proposed by Monchiet et al. [2, 5] is therefore a micromechanical based generalization, which represents explicitly the role of the mean and alternate part of the hydrostatic stress postulated initially by Dang Van and which leads to the definition of an incremental evolution law for the damage, enabling the treatment of arbitrary loadings.

3. Stress gradient effects

In order to refine the understanding of the plastic phenomena at the grain scale several computational analyse were proposed in the last decade. The approach proposed here is based on an elastic-plastic analysis of an aggregate of grains embedded in different configurations: (a) homogeneous matrix, (b) at the hot spot near the tip of a notch and (c) on the surface of a bar. The following steps were performed:

(i) Determination of the shakedown state. Measured as the *relative shift of the cumulated plastic strain* of the last loading cycle when compared to the one before.

(ii) Computation of the Dang Van criterion for each slip system in each grain (slip system projection). The computation of the Dang Van criterion for each slip system.

(iii) Computation of the Dang Van fatigue criterion in each grain. The computation of the Dang Van criterion in each grain is done using the classical algorithms of the Dang Van criterion with the mean stress field computed over each grain as an input value.

(iv) Computation of the Dang Van fatigue criterion for the homogenous structure in a standard way.

A schematic view of the elastoplastic analysis at the grain scale (steps (i) to (iii)) is represented in Figure 1(a) and a typical outcome of the elastoplastic and fatigue analysis of an aggregate of grains is plotted in Figure 1(b). The "cloud" of critical instants of the stress path for all the grains compared to the critical instances obtained through a classical Dang Van fatigue analysis will permit to assess the assumptions of the Dang Van fatigue criterion and better explain certain phenomena [6, 7].

The computations under a macroscopic tension or shear when the grain aggregate is embedded in a homogenous matrix showed that the critical instant obtained by the classical analysis matched the instant of the most loaded grain confirming the assumptions of the Dang Van theory. Moreover, one could remark that the spread of the cloud along the hydrostatic stress axis increased with increasing applied stress showing on the one hand the appearance of a hydrostatic term in pure shear experiments due to the mesoscopic structure, and on the other hand the influence of the increasing residual stress. Several grain orientations have lead to similar results proving their independence of the grain structure.

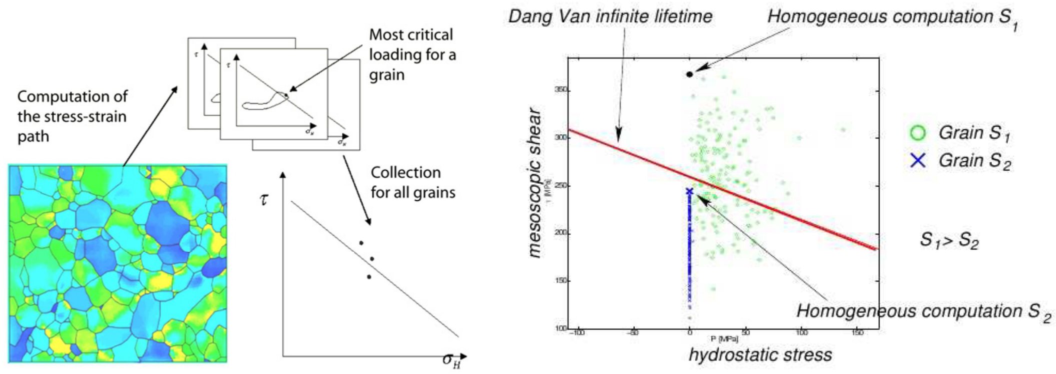

Fig. 1. Elastoplastic analysis at the grain scale: (a) schematic view (b) shear and mean stress distribution at the grain scale

The results in the case of a notched specimen or for the bar under bending, when a stress gradient was present were slightly different. In the case of a small gradient, i.e. when the macroscopic stress applied to all grains was of the same level compared to the yield limit, were similar to the ones reported before. However, when larger stress gradients were applied, one could consider that grains in one column were subject to the same macroscopic stress.

With increasing stress gradient we do not remark a peculiar evolution of the spread of cloud. However, there is remarkable difference (around 15%) between the most critical instant obtained from the grains and the classical computation of the Dang criterion performed at the hot spot (largest applied stress). The difference can be explained by the failure of the homogenisation assumptions of the Dang Van criterion and the difference can be estimated from a statistical analysis of the number of grains subject to a certain stress value.

4. Plasticity and extensions to finite lifetime

The initial Dang Van criterion has been proposed for determining finite lifetime under cyclic loading with a fixed amplitude. A first proposal to extend the ideas in the realm of finite lifetime has been presented in [8] more precisely to repeated blocks of variable amplitude loadings without cycle counting. The extension presented here starts from this initial ideas and is also based on a mesoscopic approach and the use of the accumulated inelastic mesoscopic strain as a damage variable. Failure is not related to a critical value of the accumulated plastic meso-strain, but is defined as a mesoscopic relation between the accumulated inelastic mesoscopic strain, the number of cycles to crack nucleation and some material and loading parameters.

The Schmid plasticity model is replaced for simplicity with von Mises model with a yield stress depending of the hydrostatic stress that is close to the classical Drucker-Prager model. The proposed model depending on mean stress has been justified in the second section and is assumed to represent the damage process (plasticity, void growth, micro-crack nucleation,...). The simplification of the polycrystalline plasticity with a von Mises model represents a mean value of the shear stress over all slip planes and leads to a decreased computational cost. Another step in this direction is the application of the simplified method [9, 10] for computing the plastic stress. The final fatigue life model has six parameters identified from a Woehler curve and two fatigue limits estimated from experimental results. The tests performed with data from literature for in-phase and out-of-phase constant amplitude loadings on 6082-T6 aluminium alloy and proportional multiaxial random loadings on 10HNAP are very promising [11].

5. Conclusion

The objective of this paper was to justify some ingredients of the Dang Van criterion and, in particular, the maximization of the shear stress and of the hydrostatic pressure. The role of hydrostatic pressure has been shown to be linked to the explicit incorporation of the damage process (plasticity, void growth, micro-crack nucleation,...) and the maximization of the shear stress has been justified by numerical simulation of polycrystalline aggregates. A general framework based on these elements enables to extend the initial Dang Van's approach to finite lifetimes.

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