
The generalized model for the description of prompt neutrons in the low-energy fission

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Abstract

The generalized model for the description of neutron emission from the spontaneous and neutron-induced fission in the energy interval up to 20 MeV is developed. For accurate calculations of nucleon composition and excitation energy of the fissioning nucleus at the scission point, the time-dependent statistical model including the pre-equilibrium neutron emission and nuclear friction effects is used. For each member of the compound nucleus ensemble at the scission point, the primary fission-fragment characteristics such as kinetic and excitation energies and yields are calculated using the scission-point fission model with nuclear shell and pairing effects, and based on the multimodal approach. The charge distribution for the primary fragment isobaric chains is considered as a result of the frozen quantal fluctuations of the isovector nuclear matter density at the finite scission neck radius. The post-scission neutron spectra are calculated as the result of the equilibrium emission from the fully accelerated fission fragments with calculated kinetic energies. The pre-scission neutron multiplicity and spectra for multi-chance fission are also calculated. The neutron and γ-ray emission during the saddle-to-scission time is also included in the consideration. This mechanism may partially explain the long standing problem of the so-called isotropic component of prompt fission neutrons.

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1. Introduction

Prompt fission neutrons are important probe of the dynamics at different stages of the fission process: (i) formation of the compound nucleus and of the stationary fission-probability flow over the fission barrier; (ii) particle evaporation during the pre-saddle stage (formation of fission chances); (iii) evaporation during the descent time from the saddle to the scission configuration; (iv) emission as a result of the fast dynamical processes near the scission point; and (v) evaporation from the moving heated fragments. The prompt neutron differential distributions may be presented as the sum of numerous different sources which dynamical and statistical parameters are still not properly known.

For spontaneous and low-energy neutron-induced fission (up to the second fission chance), there is no emission of neutrons at stages (i) and (ii). Several versions of the multiple-source post-scission neutron emission models were proposed for spontaneous and low-energy fission (Browne and Dietrich, 1974; Madland and Nix, 1982; Märten and Seeliger, 1984; Gerasimenko and Rubchenya, 1985; Hambsch et al., 2002; Lemaire et al., 2005; Tudora, 2009; Litaize and Serot, 2010; Talou et al., 2011). However none of them includes contributions from the emission mechanisms (iii) and (iv). It is known that, for the energy of the bombarding neutron going above the second fission thresholds and higher, the pre-scission neutron spectra and contributions from different compound nuclei to the post-scission particle emission should be included into consideration (Rubchenya, 2007). At the neutron bombarding energy above ~10 MeV, the fast pre-equilibrium emission mechanism should also be taken into account. In addition, the contribution of dynamical mechanisms (iii) and (iv)) to the prompt neutron spectra discovered many years ago (Bowman et al., 1962) is still not explained.

In this paper, we present the unified model approach for calculating of the prompt neutron characteristics in spontaneous and neutron-induced fission with energies up to 20 MeV.

2. Theoretical model

The prompt neutron double-differential spectrum integrated over the fragment mass and charge distributions can be presented as a sum of five main emission mechanisms

\[
\frac{d^2M}{dE d\Omega} = \frac{d^2M^{preeq}}{dE d\Omega} + \frac{d^2M^{presd}}{dE d\Omega} + \frac{d^2M^{sdsc}}{dE d\Omega} + \frac{d^2M^{sc}}{dE d\Omega} + \frac{d^2M^{post}}{dE d\Omega}.
\]  

(1)

Here \(E\) is the neutron energy in the laboratory frame and \(M\) is the prompt fission neutron multiplicity with the subscript related to emission at the pre-equilibrium, pre-saddle, saddle-to-scission, near scission, and post-scission stages of the fission process, respectively.

The pre-equilibrium particle emission process is described in the frame of the two-component exciton model using Monte Carlo simulation code, which allows one to incorporate the time duration criterion for the pre-compound stage of the reaction (Rubchenya, 2007). It is supposed that duration of the pre-equilibrium process is two orders of magnitude shorter than the average statistical decay time of the initial composite nucleus. Therefore formation and de-excitation stages of the compound nuclei are considered as decoupled.

The emission of neutrons and \(\gamma\) rays is assumed to start with full statistical decay width just after the end of the pre-equilibrium stage. The time-dependent fission width approaches an asymptotic value after some delay time and is influenced by the nuclear friction (Grangé et al., 1986). The pre-saddle prompt fission neutron
spectrum and multiplicity are defined by the competition between neutron, gamma, and fission channels.

The neutrons and $\gamma$-rays may be emitted during the saddle-to-scission $\tau_{sdsc}$ time, which is defined by the fission dynamics and increases with the nuclear dissipation strength (Hofman and Nix, 1983). The neutron spectrum and multiplicity at the descend stage is defined also by the available excitation energy and the saddle-to-scission descent time. Averaged dissipation energy at the descent from the saddle to the scission point may be approximated by the following expression (Davis et al., 1976):

$$E_{diss} = 0.03 \cdot A^{2/3} \left( \frac{Z^2}{A} - 26.12 \right).$$

(2)

The dissipated energy is high enough to allow emission of neutrons however the decisive role may be played by the available saddle-to-scission $\tau_{sdsc}$ time. In the numerical calculations, the excitation energy at the descend stage was assumed to be equal to the sum of the excitation energy just after passing through the fission barrier and of the half of the difference in the potential energy at saddle and scission points. The $\tau_{sdsc}$ value is considered as a parameter. The Monte Carlo approach is used to calculate neutron emission during the fixed time interval $\tau_{sdsc}$.

Neutron emission near the scission point (the $M^{sc}$ component) may be described as a result of the single-particle transition into unbound states during the diabatic transition, at the scission point, from the configuration with a thin neck to that with two separated fragments (Carjan et al., 2005). This process is of great fundamental interest but has not been included into numerical calculations, due to the lack of the quantitative description.

After the pre-scission particle emission, the compound nuclei arrive at the scission point with some distributions $W_{sc}(E_{sc}, A_{sc}, Z_{sc}, J_{sc})$ in excitation energies $E_{sc}^*$, masses and nuclear charges ($A_{sc}, Z_{sc}$), as well as spins $J_{sc}$. This ensemble is then transformed into the ensemble of the primary exited fission fragments. For each member of the compound nucleus ensemble at the scission point, the primary fragment isobaric chain yields $Y(A)$ are calculated using the multimodal fission approach. The five fission modes: symmetric (SY), standard-I (SI), standard-II (SII), superasymmetric-I (SAI) and superasymmetric-II (SAII) are taken into consideration to describe the primary mass distribution (Tsekhanovich et al., 2004; Rubchenya and Äystö, 2012).

The smoothed charge distribution of the primary-fragment isobaric chains is approximated by the Gaussian function. The averaged charge is calculated in the framework of the scission-point fission model. The dispersion of the charge distribution is considered as a result of frozen quantal fluctuations of the isovector nuclear matter density at the finite scission neck radius (Rubchenya and Äystö, 2012).

The post-scission neutron multiplicity and spectra are formed in the emission of neutrons from the primary fission fragments which have distributions over mass, kinetic energy, excitation energy, and spin. In the centre-of-mass frame of the compound nucleus (practically coinciding with the laboratory frame for the low-energy neutron-induced fission), the double-differential spectra can be presented in the simplified form

$$\frac{d^2M}{dE d\Omega} = \sum A_{sc} \sum J_{sc} \sum A \sum Z \int dE_{sc}^* W_{sc}(E_{sc}^*, A_{sc}, J_{sc}) \frac{M_F^f(A, Z)}{4\pi} \sqrt{\frac{E_{FF}}{E}} \cdot W_F(E^f, A, Z, E_{kin}^f) Y(A, Z)$$

(3)

Here, $E^f$ is neutron energy in the frame of the fission fragment moving with the average kinetic energy, $E_{kin}^f$. 
Neutrons are assumed to be evaporated isotropically from the fully accelerated fission fragments. The neutron multiplicities \( M_n \) and spectra \( W_f(E^F, A, E_{kin}^F) \) are calculated using statistical model taking into account the competition of the \( \gamma \)-ray channel. The excitation energies and total kinetic energies of the primary fragments are calculated in the framework of the fission scission-point model (Rubchenya, 2007). Contributions are summed from different fission fragments with the corresponding primary fragment yields \( Y(A,Z) \) for the compound nucleus ensemble \( W_{scc}^*(E_{sc}^c, A_{sc}, J_{sc}) \) at the scission point.

Fission fragment characteristics are calculated within the scission-point fission model approach, with inclusion of the shell and pairing effects. Total fragment excitation energy is defined by the expression:

\[
E^{*H}(L) = E^{*H}_{sc} + E_{def}^H + E_{rot}^H ,
\]

(4)

\( E^{*H}_{sc} \), \( E_{def}^H \), and \( E_{rot}^H \) are the internal excitation, deformation, and rotation energies of the heavy and light fragments at the scission point, where the energy balance equation is fulfilled:

\[
Q_f(A_{CN}^{sc}, Z_{CN}^{sc}, A, Z) + E^{*sc}_{CN} = V_{pot}^{sc} + E^*_{sc} + E_{kin}^{sc} + A_{rot}^{rel} L(L + 1).
\]

(5)

Here \( Q_f(A_{CN}^{sc}, Z_{CN}^{sc}, A, Z) \) is the fission energy release, \( E^{*sc}_{CN} \) is the compound nucleus excitation energy, and \( V_{pot}^{sc}, E_{kin}^{sc} \) are the potential energy and the fragment kinetic energy at scission point, \( A_{rot}^{rel} \) is the rotation constant of a two-fragment system, and \( L \) is angular momentum of the relative rotation. Thermal energy at the scission point is divided between fragments according to the thermal equilibrium condition:

\[
\frac{E^{*H}_{sc}}{E^{*L}_{sc}} = \frac{a^H}{a^L}, \quad E^*_L = E^*_{sc} + E^*_{sc},
\]

(6)

where \( a^H(L) \) are the level density parameters of deformed fragments at scission point. Total fragment kinetic energy is a sum of the Coulomb interaction energy, rotation energy and kinetic energy at the scission point:

\[
TKE = V_{coul} + E_{kin}^{sc} + A_{rot}^{rel} L(L + 1).
\]

(7)

At scission point the spin of the compound nucleus is divided between the fragments and the relative motion degree of freedom according to the relation

\[
\vec{J}_{CN} = \vec{J}_H + \vec{J}_L + \vec{L}.
\]

(8)

Fission fragment spins are mainly defined by their excitation energies at the scission point.
3. Results

The computing code FIPRODY (FIssion PRODuct Yyield) developed on the theoretical grounds shortly discussed in the previous section calculates the integral and differential characteristics from the fission induced by protons, neutrons, gamma rays and from spontaneous fission. The model parameters were determined from the comparison between calculated and experimental characteristics of the proton and neutron induced fission of the actinides from Th to Cf.

![Fig. 1. Calculated prompt-neutron pre-scission (green line), post-scission (blue line) and integral (red line) spectra for the spontaneous fission of $^{252}$Cf, compared to the Maxwell distribution with $T = 1.42$ MeV (black dotted line).](image)

In the spontaneous and neutron-induced fission at the energies below the second-chance threshold, the post-scission neutron emission from the fully accelerated fragments gives an overwhelming contribution. The contribution of the pre-scission emission during the saddle-to-scission time is small and increases with the fissility parameter $Z^2/A$. Results of the model calculation for the spontaneous fission of $^{252}$Cf is presented in Fig. 1. The pre-scission spectrum ($M^{sdsc} = 0.242$) was calculated with $\tau_{sdsc} = 10^{-20}$ S.

With increasing incident neutron energy the pre-saddle neutron emission gives contribution in accordance with the weight of different fission chances. Additionally, at the incident neutron energy above 10 MeV the pre-equilibrium emission becomes important. The comparison of the calculated spectrum (red line) with the experimental one (Boikov et al., 1991) (black points) in fission of $^{238}$U induced by 14.7 MeV neutrons is shown in Fig. 2. The calculated pre-equilibrium (purple line), pre-scission (green line), and post-scission (blue line) spectra are also presented in Fig. 2. The model describes satisfactorily the experimental data at $E \geq 1$ MeV including peculiarity at the high energy tail. However there is still a pronounced enhancement of the neutron yield in the low energy region $E < 1$ MeV over the theoretical predictions.

4. Conclusion

In conclusion, the advanced theoretical model for describing the fission prompt-neutron spectra and
multiplicities in the spontaneous and neutron-induced fission was developed. Decay of the excited compound nuclei, formed after the pre-equilibrium neutron emission, is described within the time-dependent statistical model with inclusion of nuclear friction. To calculate the primary fission fragment excitation and kinetic energies, the scission-point fission model with the shell, pairing, and temperature effects is used. The post-scission neutron characteristics are calculated for individual fragments and then averaged over calculated primary fission-fragment yields. For each member of the compound nucleus ensemble at scission, the primary fragment yields are calculated using the multimodal approach to the fission process. The model describes well the experimental prompt neutron spectra and multiplicities. The neutron emission during the descent from saddle to scission point may explain the isotropic component in the angular distribution relative to the fragment velocity direction in the spontaneous fission of heavy actinides (Bowman et al., 1962).

![Figure 2](image-url)

**Fig. 2.** Calculated prompt neutron spectrum (red line) and experimental spectrum (Boikov et al., 1991) (black points) in the 14.7 MeV neutron induced fission of $^{238}$U.

**References**