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Radiative correction to the Dirichlet Casimir energy for $\lambda\phi^4$ theory in two spatial dimensions

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ABSTRACT

In this Letter, we calculate the next to the leading order Casimir energy for real massive and massless scalar fields within $\lambda\phi^4$ theory, confined between two parallel plates with the Dirichlet boundary condition in two spatial dimensions. Our results are finite in both cases, in sharp contrast to the infinite result reported previously for the massless case. In this Letter we use a renormalization procedure introduced earlier, which naturally incorporates the boundary conditions. As a result our radiative correction term is different from the previously calculated value. We further use a regularization procedure which help us to obtain the finite results without resorting to any analytic continuation techniques.

1. Introduction

The Casimir effect can be observed in all systems with nontrivial boundary conditions (BCs) or background fields (*e.g.* solitons). During the last sixty years this effect has been an important topic of research with applications in many branches of physics [1–3]. The static Casimir effect, first calculated in 1948 [4], predicts an attraction between two perfectly conducting parallel plates, due to distortion of the electromagnetic vacuum state (for a general review on the Casimir effect, see Refs. [2,5,6]). Ten years later the first attempt to observe this phenomena was made by Sparnaay [7]. Since then, many experimental investigations have measured precisely the Casimir force in different cases, such as two parallel plates [8], or a sphere in front of a plane [9]. The majority of the theoretical investigations related to the zero order Casimir effects are for various fields, geometries, BCs [6,10–12], and various dimensions. Some of the major approaches used are: the mode summation method with a combination of the zeta function regularization technique [13,14], Green's function formalism [15], multiple-scattering expansions [16], heat-kernel series [17]. On the other hand, there exist many works on the first order and also second order radiative corrections to the Casimir energy for various cases [18–21]. Some of the major approaches used for the radiative corrections to the Casimir effect are the phase shift of the scattering states [22], or the replacement of the BCs by an appropriate potential term [23,24]. The Casimir effects have found many applications in physics. For example, the Casimir effect is the major contribution to the radiative correction to the mass of the solitons, and these corrections have been investigated in many papers [25–27].

The value of the Casimir energy has a complicated behavior as a function of the number of spatial dimensions, the type of fields, type of topology, and geometry. The case of even spatial dimensions is usually more complicated [12,21,28]. Since many interesting condensed matter systems are well-approximated by two-dimensional models, extracting finite results from the complicated divergencies, which usually plague such systems, is very important. The Casimir energies for scalar fields in even dimensions have been discussed for two parallel plates [21], spheres [12,28], and cylinders [14]. Some of those cases give divergent results, and some authors prescribe methods to extract a finite answer from those expressions [6,12,14]. However, those prescriptions are not universally applicable to all even dimensions. It seems that even for the simplest case of a scalar field in $2+1$ dimensions, it is not clear how the divergences can be removed [6].

In this Letter we calculate the first order radiative correction to the Dirichlet Casimir energy for two infinite parallel plates for massive and massless scalar fields in two spatial dimensions. The problems mentioned before, give us extra motivation to utilize an alternative renormalization program and regularization procedure for this problem. We have used these procedures to calculate this quantity in

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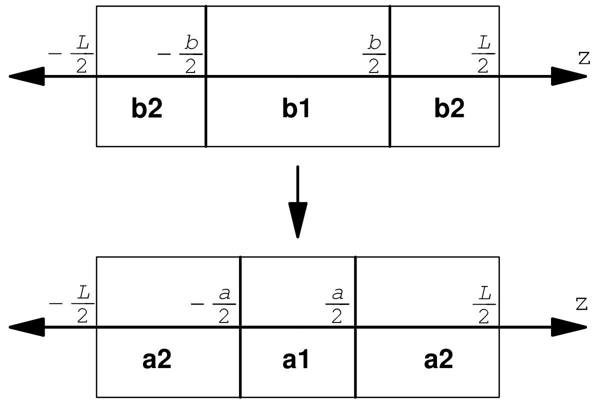

Fig. 1. The geometry of the two different configurations whose energies are to be compared. The labels $a1$, $b1$, etc., denote the appropriate sections in each configuration separated by plates. The left configuration is denoted by 'A' configuration, and the right one by 'B' configuration.

$1+1$ and $3+1$ dimensions [29,30]. As we shall see our procedure yields finite results for both massive and massless scalar fields which is different from the previously reported one [21]. As a matter of fact the previously reported result for the massless case is infinite [21]. It is worth mentioned that our finite result is obtained without any use of analytic continuation techniques due to our regularization procedure. The difference between our results and the previously reported one can be attributed to our alternative renormalization program.

In our Letter we combine two independent programs in order to calculate the radiative correction to the Casimir energy. First, we use an approach to the renormalization program which we believe to be systematic. The procedure to deduce the counterterms from the n-point functions in the renormalized perturbation theory is standard and has been available for over half a century [31]. We believe that all of the information about the nontrivial BCs or position dependent background fields should be carried by full set of the n-point functions. Therefore, all of the counterterms deduced from these n-point functions should also contain these information. Using this procedure we deduce the position dependent counterterms in our problem. We should mention that most of the authors use the free counterterms, by which we mean the ones that are relevant to the free cases with no nontrivial BCs, and are obviously position independent. However, the dependency of the counterterms on the distance between the plates has been noted in some references such as [32,33]. However, these authors use free counterterms in the space between the plates and place additional surface counterterms at the boundaries.

Another important part of our calculation is using a method to remove the divergences without resorting to any analytic continuation. In fact, we subtract two different configurations with similar nature. This subtraction scheme is based on the Boyer's subtraction scheme and it can regularize the infinities and help us to remove them without using any analytic continuation [34]. This method has been used in many previous works [3,35,36]. We place the two infinite parallel plates (with distance a) within two other plates (with distance $L>a$). We then construct a similar configuration of plates with distances $L>b$. We then subtract the Casimir energies of these two configurations. Finally in order to obtain the Casimir energy for the original configuration we let L and then b go to infinity. Therefore, the Casimir energy is now defined by

$$E_{\text{Cas.}} = \lim_{b/a\to\infty}\left[\lim_{L/b\to\infty}(E_A - E_B)\right], \tag{1}$$

where

$$E_A = E_{a1} + 2E_{a2}, \qquad E_B = E_{b1} + 2E_{b2}, \tag{2}$$

and E_{a1}, E_{a2}, E_{b1} and E_{b2} are the zero point energies of each region shown in Fig. 1.

We have already used this subtraction scheme to calculate the leading order part of the Casimir energy for a real massive scalar field and its massless limit with Dirichlet BC for two infinite parallel plates in arbitrary dimensions in Refs. [29,30]. Therefore, in this Letter we only report its final result for two spatial dimensions. We obtain

$$E^{(0)}_{\text{Cas.}} = -\frac{2L(ma)^3}{(4\pi)^{3/2}a^2}\sum_{j=1}^{\infty}\frac{K_{3/2}(2amj)}{(amj)^{3/2}}. \tag{3}$$

This expression for the leading order of the Casimir energy of a massive scalar field with Dirichlet BC in two spatial dimensions is the same as that reported in Refs. [3,10]. However, contrary to the methods used in Refs. [3,10], this expression is obtained without using any analytic continuation techniques. Two important limits should be considered at this stage. First is the small mass limit, $m\to 0$, and Eq. (3) becomes

$$E^{(0)}_{\text{Cas.}} = \frac{-L\zeta(3)}{16\pi a^2}, \tag{4}$$

where $\zeta(s)$ denotes the zeta function. This expression for the leading order Casimir energy of a massless scalar field is also the same as reported in Refs. [3,10]. Second is the large mass limit, $ma \gg 1$, and Eq. (3) becomes

$$E^{(0)}_{\text{Cas.}} = \frac{-L}{8\pi a^2}(am)e^{-2ma}. \tag{5}$$

In fact, in this limit the value of the Casimir energy decreases exponentially with increasing ma, and this is again the same as the previously reported result [3,10].

In Section 2, we first calculate the first order radiative correction to the Casimir energy for this problem. We then plot all of the result for the massive and massless cases. In Section 3, we summarize and discuss our results.

2. First order radiative correction

In this section we first calculate the leading order radiative correction to the Casimir energy for a massive scalar field within $\lambda\phi^4$ theory with Dirichlet BC in $2+1$ dimensions using the renormalized perturbation theory. As mentioned in the Introduction and also Refs. [29,30], the counterterms are computed from the appropriate n-point functions which, in the presence of the nontrivial BCs, are naturally position dependent. The renormalization procedure, the deduction of the counterterms, and the final general form of the first order correction to the Casimir energy for each region have been completely discussed in Refs. [29,30]. Therefore, in this Letter we use only the conclusions: the general expression for the first order radiative correction term to the Casimir energy is

$$E_{a1}^{(1)} = \frac{-\lambda}{8}\int_{a1} G_{a1}^2(x,x)\,d^2\mathbf{x}, \tag{6}$$

where $G_{a1}(x,x')$ is the propagator of a real scalar field in region $a1$ in two spatial dimensions. After the usual wick rotation, the expression for the Green's function or the propagator $G_{a1}(x,x')$ in three-dimensional Euclidean space is

$$G_{a1}(x,x') = \frac{2}{a}\int\frac{d^2k}{(2\pi)^2}\sum_{n=1}\frac{e^{-\omega(t-t')}e^{-i\mathbf{k}^\perp.(\mathbf{x}^\perp-\mathbf{x}'^\perp)}\sin[k_{a1,n}(z+\frac{a}{2})]\sin[k_{a1,n}(z'+\frac{a}{2})]}{k^2+k_{a1,n}^2+m^2+i\epsilon}, \tag{7}$$

where $k_{a1,n}=n\pi/a$, $x=(t,\mathbf{x})$, and $k=(\omega,\mathbf{k}^\perp)$. Using Eqs. (6), (7) and performing the spatial integration we obtain

$$\begin{aligned}E_{a1}^{(1)} &= \frac{-\lambda}{8}\left[\frac{4}{a^2}\sum_{n,n'=1}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2+k_{a1,n}^2+m^2+i\epsilon}\int\frac{d^2k'}{(2\pi)^2}\frac{1}{k'^2+k_{a1,n'}^2+m^2+i\epsilon}\left[\frac{aL}{4}\left(1+\frac{1}{2}\delta_{n,n'}\right)\right]\right]\\ &= \frac{-\lambda L}{32\pi^2 a}\left[\sum_{n,n'=1}\int_0^\infty dk\frac{k}{k^2+k_{a1,n}^2+m^2+i\epsilon}\int_0^\infty dk'\frac{k'}{k'^2+k_{a1,n'}^2+m^2+i\epsilon}+\frac{1}{2}\sum_{n=1}\left(\int_0^\infty dk\frac{k}{k^2+k_{a1,n}^2+m^2+i\epsilon}\right)^2\right].\end{aligned} \tag{8}$$

All of the integrals in Eq. (8) are logarithmically divergent, and we make them dimensionless by multiplying appropriate factors of a. Then, we use cutoff regularization for each integral, and expand the results in the limit in which the cutoffs go to infinity as follows,

$$\int_0^\Lambda\frac{dk\,k}{k^2+\omega^2} = \frac{1}{2}\ln\left(k^2+\omega^2\right)\Big|_0^\Lambda \overset{\Lambda\to\infty}{\longrightarrow} \ln\Lambda-\ln\omega. \tag{9}$$

Using Eq. (9) for each integral in Eq. (8) we obtain

$$E_{a1}^{(1)} = \frac{-\lambda L}{32\pi^2 a}\left[\sum_{n,n'=1}\left(\ln\Lambda_{a1}-\ln\omega'_{a1,n}\right)\left(\ln\Lambda_{a1}-\ln\omega'_{a1,n'}\right)+\frac{1}{2}\sum_{n=1}\left(\ln\Lambda_{a1}-\ln\omega'_{a1,n}\right)^2\right], \tag{10}$$

where $\omega'^2_{a1,n}=(n\pi)^2+m^2a^2$, and Λ_{a1} is a cutoff in the upper limit of the integrals in Eq. (8). The terms related to other regions in Fig. 1 are calculated analogously. Now, for the calculation of the Casimir energy in Eq. (1), we have four similar terms which should be subtracted from each other. By appropriately adjusting each cutoff Λ_{a1}, Λ_{a2}, Λ_{b1} and Λ_{b2}, all of the infinities cancel due to our box subtraction scheme. We obtain

$$\begin{aligned}E_A^{(1)}-E_B^{(1)} &= E_{a1}^{(1)}+2E_{a2}^{(1)}-E_{b1}^{(1)}-2E_{b2}^{(1)}\\ &= \frac{-\lambda L}{32\pi^2}\left[\sum_{n,n'=1}\left(\frac{\ln\omega'_{a1,n}\ln\omega'_{a1,n'}}{a}+4\frac{\ln\omega'_{a2,n}\ln\omega'_{a2,n'}}{L-a}-\frac{\ln\omega'_{b1,n}\ln\omega'_{b1,n'}}{b}-4\frac{\ln\omega'_{b2,n}\ln\omega'_{b2,n'}}{L-b}\right)\right.\\ &\quad\left.+\frac{1}{2}\sum_{n=1}\left(\frac{\ln^2\omega'_{a1,n}}{a}+4\frac{\ln^2\omega'_{a2,n}}{L-a}-\frac{\ln^2\omega'_{b1,n}}{b}-4\frac{\ln^2\omega'_{b2,n}}{L-b}\right)\right].\end{aligned} \tag{11}$$

Now we can use the Abel–Plana Summation Formula (APSF) which basically reduces the summations into integrations as follows,

$$\sum_{n=1}^\infty F(n) = \frac{-1}{2}F(0)+\int_0^\infty dt\,F(t)+i\int_0^\infty dt\frac{F(it)-F(-it)}{e^{2\pi t}-1}. \tag{12}$$

If the summation starts from $n=0$, the sign of the first term becomes positive. Now by applying the APSF to all of the summations in Eq. (11) we obtain

$$E_A^{(1)} - E_B^{(1)} = \frac{-\lambda L}{128\pi^2}\left[\mathcal{R}(a) + 2\mathcal{R}\left(\frac{L-a}{2}\right) - \{a \to b\}\right], \tag{13}$$

where

$$\mathcal{R}(x) = \frac{1}{x}\left(\frac{-1}{2}\ln m^2x^2 + \int_0^\infty dn\, \ln\bigl(n^2\pi^2 + m^2x^2\bigr) + B_1(x)\right)^2 + \frac{1}{2x}\left(\frac{-1}{2}\ln^2 m^2x^2 + \int_0^\infty dn\, \ln^2\bigl(n^2\pi^2 + m^2x^2\bigr) + B_2(x)\right), \tag{14}$$

and $B_1(x)$ and $B_2(x)$ are the branch-cut terms of the Abel–Plana summation formula and are calculated in Appendix A. Both of these two types of branch-cut terms are finite for $m \neq 0$. However, other integral terms which appear in Eq. (13) are divergent. At this stage our main purpose is to regularize these terms and show how they cancel each other, again due to our box subtraction scheme.

To regularize the integrals in Eq. (13), we set separate cutoffs, denoted again by Λs, for the upper limits of each integral. After the integrations, we expand the results in the limit $\Lambda \to \infty$. Now, by appropriate adjustment of the Λs, all of the divergent terms which depend on the cutoffs Λs, cancel in Eq. (13), due to our box subtraction scheme. Below, we present the details of these cancelations for both types of integrals. For the first type we have

$$\begin{aligned}
\int_0^\Lambda dn \ln\bigl(n^2\pi^2 + m^2a^2\bigr) &= \frac{ma}{\pi}\int_0^\Lambda dN\bigl(\ln\bigl(N^2+1\bigr) + \ln\bigl(m^2a^2\bigr)\bigr) \\
&= \frac{ma\Lambda}{\pi}\left[-2 + \ln\bigl(m^2a^2\bigr) + \ln\bigl(1+\Lambda^2\bigr) + \frac{2}{\Lambda}\arctan\Lambda\right] \\
&\xrightarrow{\Lambda\to\infty} \frac{ma}{\pi}\bigl(-2 + \ln\bigl(m^2a^2\Lambda^2\bigr)\bigr)\Lambda + ma - \frac{ma}{\pi\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda}\right)^3 \longrightarrow ma,
\end{aligned} \tag{15}$$

where in the first line we have used the following change of variable $N = n\pi/ma$. Therefore, only the finite terms $\{am, (L-a)m, bm, (L-b)m\}$ remain for the first type of integrals. For the second type of integrals we have

$$\begin{aligned}
\int_0^\Lambda dn \ln^2\bigl(n^2\pi^2 + m^2a^2\bigr) &= \frac{ma}{\pi}\int_0^\Lambda dN\,\bigl[\ln\bigl(a^2m^2\bigr) + \ln\bigl(N^2+1\bigr)\bigr]^2 \\
&= \frac{ma}{\pi}\int_0^\Lambda dN\,\bigl[\ln^2\bigl(a^2m^2\bigr) + 2\ln\bigl(a^2m^2\bigr)\ln\bigl(N^2+1\bigr) + \ln^2\bigl(N^2+1\bigr)\bigr] \\
&= \frac{ma}{\pi}\ln^2\bigl(a^2m^2\bigr)\Lambda + \frac{2ma}{\pi}\ln\bigl(a^2m^2\bigr)\underbrace{\bigl\{-2N + 2\arctan(N) + N\ln\bigl(N^2+1\bigr)\bigr\}\big|_0^\Lambda}_{\mathcal{K}(\Lambda)} \\
&\quad + \frac{ma}{\pi}\int_0^\Lambda dN \ln^2\bigl(N^2+1\bigr),
\end{aligned} \tag{16}$$

where $\mathcal{K}(\Lambda)$ in the limit $\Lambda \to \infty$ is

$$\mathcal{K}(\Lambda) \xrightarrow{\Lambda\to\infty} \bigl(-2 + \ln\Lambda^2\bigr)\Lambda + \pi - \frac{1}{\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda}\right)^2 \longrightarrow \pi. \tag{17}$$

The first term in the right-hand side of Eq. (16) in line three is divergent. The second integration is similar to the first type of integral terms which were considered above. The divergent terms in the sum of these two terms are removed by choosing appropriate adjustment of Λs and using the subtraction scheme indicated in Eq. (13), and only finite terms remain. The third term in the right-hand side of Eq. (16) and its counterparts in the other regions are also divergent and their calculations are very difficult. However, if we let first the cutoffs go to infinity, one can show that they exactly cancel each other in the box subtraction scheme. Therefore, the only contributions coming from this term is

$$\int_0^\infty dn \ln^2\bigl(n^2\pi^2 + m^2a^2\bigr) \longrightarrow 4ma\ln ma. \tag{18}$$

Using Eqs. (13), (15), (18), we have

$$\begin{aligned}
E_A^{(1)} - E_B^{(1)} = \frac{-\lambda L}{128\pi^2}\Biggl[&\frac{1}{a}\left(a^2m^2 + B_1^2(a) - ma\ln\bigl(m^2a^2\bigr) + 2maB_1(a) - B_1(a)\ln\bigl(m^2a^2\bigr) + \frac{1}{2}(4ma\ln ma) + \frac{1}{2}B_2(a)\right) \\
&+ \frac{2}{L-a}\left(\frac{(L-a)^2m^2}{4} + B_1^2\left(\frac{L-a}{2}\right) - \frac{m(L-a)}{2}\ln\left(\frac{m^2(L-a)^2}{4}\right) + m(L-a)B_1\left(\frac{L-a}{2}\right)\right. \\
&\left.- B_1\left(\frac{L-a}{2}\right)\ln\left(\frac{(L-a)^2m^2}{4}\right) + m(L-a)\ln\left(\frac{m(L-a)}{2}\right) + \frac{1}{2}B_2\left(\frac{L-a}{2}\right)\right) - \{a \to b\}\Biggr].
\end{aligned} \tag{19}$$

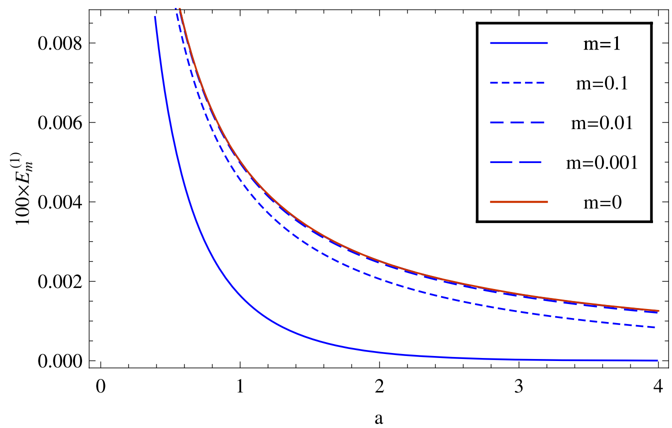

Fig. 2. The first order radiative corrections to the Casimir energy for massive and massless scalar fields in two spatial dimensions are plotted as a function of the distance between the lines (a), within the $\lambda\phi^4$ theory for $\lambda = 0.1$. The numerical values for the plots have been multiplied by a factor of 100, in order to make their absolute values comparable to the zero order terms shown in Fig. 3. In this figure we have shown the sequence of plots for $m = \{1, 0.1, 0.01, 0.001, 0\}$. It is apparent that the sequence of the massive cases converges rapidly to the massless case and there is an insignificant difference between the figures of the massive cases for $m < 0.01$, and the massless case.

There many internal cancelations in the above expressions. After these cancelations only the branch-cut terms remain. By using the values of the branch-cut terms obtained in Appendix A, we can write an explicit expression for the lowest order radiative correction to the Casimir energy in terms of parameters $m, a, \frac{L-a}{2}, b$ and $\frac{L-b}{2}$. As stated in Eq. (1), first the limit $L/b \to \infty$ should be calculated and then $b/a \to \infty$. In these limits all of the terms which depend on L and b disappear from our expression and only the terms which depend on the distance of the original plates (a) remain. Our final result is

$$E^{(1)}_{\text{Cas.}} = \frac{-\lambda L}{128\pi^2 a}\left[\left(am + \ln\left(1 - e^{-2am}\right)\right)^2 - m^2a^2 - \gamma \ln\left(1 - e^{-2am}\right) + \sum_{j=1}^{\infty} \frac{e^{-2amj}}{j}\left(\ln(maj) - e^{4maj}\Gamma(0, 4maj)\right)\right], \tag{20}$$

where γ is the Euler–Mascheroni constant, and $\Gamma(\alpha, x)$ is the incomplete Gamma function. Our result differs from the previously reported result [21], since they use the free counterterms, and we have used the ones dictated by the Green's function appropriate for this problem.

To calculate the massless limit, we go back to the original expression given in Eq. (19). The direct calculation of the massless case is extremely difficult. We use m as a regulator for this limit. However, multitude of difficulties appear. These difficulties are partly due to the fact that the branch-cut terms are also divergent in the limit $m \to 0$. Fortunately, there is no essential singularity and we obtain

$$\begin{aligned} E^{(1)}_{\text{Cas.}} &= \frac{-\lambda L}{128\pi^2 a}\left[\left(am + B_1(a)\right)^2 - m^2a^2 - 2B_1(a)\ln(ma) + \frac{1}{2}B_2(x)\right] \\ &\xrightarrow{m\to 0} \frac{-\lambda L}{128\pi^2 a}\left[\ln^2(2ma) - 2ma\int_1^{\infty} dN \frac{\ln(N^2-1)}{e^{2maN}-1}\right], \end{aligned} \tag{21}$$

where in the second line we have used the small mass limit of B_1,

$$B_1(a) = \ln\left(1 - e^{-2ma}\right) \xrightarrow{m\to 0} \ln(2ma) - ma + \mathcal{O}\left(m^2\right), \tag{22}$$

and used a suitable change of variables for B_2 which leads to some cancelations, and we have ignored terms of $\mathcal{O}(m^2)$. In the above expression all of the infinities cancel and we finally obtain the following finite result,

$$E^{(1)}_{\text{Cas.}} \xrightarrow{m\to 0} \frac{-\lambda L}{128\pi^2 a}(-0.6349208). \tag{23}$$

As shown in Fig. 2, the sequence of plots of the massive cases converges rapidly to the massless limit. It is obvious that the massless limit is finite, exactly as we have obtained, and as expected on physical grounds.

In Fig. 3 all of the values for the zero order and the first order radiative correction to the Casimir energy for a massive ($m = 1$) and massless scalar fields are plotted. We should mention that the correction terms are positive and their values are approximately 100 times smaller than their zero order counterparts.

3. Conclusion

In this Letter, the first order radiative correction to the Casimir energy with Dirichlet BC for two infinite parallel plates in two spatial dimensions has been calculated by a systematic approach to the renormalization program. This program automatically yields position dependent counterterms. Moreover, we used the Boyer's subtraction scheme which eliminates the need to use any analytic continuation techniques. The final results for the radiative correction of the Casimir energy for a massive and massless scalar fields are different from the reported results in the previous papers [21]. We believe that this difference is due to the use of different renormalization programs. It is important to note that our result for the massless case is finite, in sharp contrast to the previously reported result [21].

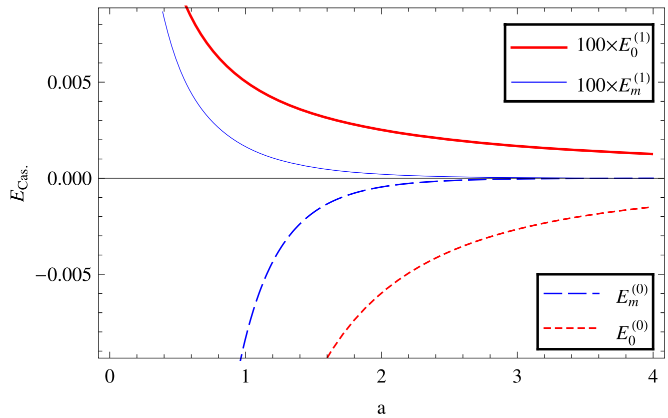

Fig. 3. The leading term for the Casimir energy and its first order radiative correction (multiplied by a factor of 100) in two spatial dimensions, are plotted as a function of the distance between the lines (a) for a massive ($m = 1$) and a massless scalar fields for $\lambda = 0.1$. The correction terms are always positive.

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Appendix A. Calculation of the branch-cut terms

In this appendix we present the calculation of two types of branch-cut terms which appear in the calculation of the first order radiative correction to the Casimir energy. We start with the first type of the branch-cut term which is denoted by B_1 in the main text. We have for $B_1(a)$,

$$B_1(a) = i\int_{\frac{ma}{\pi}}^{\infty} dn\,\frac{\ln((in)^2\pi^2 + m^2a^2) - \ln((-in)^2\pi^2 + m^2a^2)}{e^{2\pi n} - 1}$$
$$= i\int_{\frac{ma}{\pi}}^{\infty} dn\,\frac{\ln(e^{i\pi}n^2\pi^2 + m^2a^2) - \ln(e^{-i\pi}n^2\pi^2 + m^2a^2)}{e^{2\pi n} - 1} = -2\pi\int_{\frac{ma}{\pi}}^{\infty}\frac{dn}{e^{2\pi n} - 1} = \ln\left(1 - e^{-2am}\right). \tag{A.1}$$

The values of other branch-cut terms can be easily written only by the appropriate replacement in the argument of B_1. Analogous process is repeated for calculation of the second type of the branch-cut term. So, for $B_2(a)$ we have

$$B_2(a) = i\int_{\frac{ma}{\pi}}^{\infty} dn\,\frac{\ln^2((in)^2\pi^2 + m^2a^2) - \ln^2((-in)^2\pi^2 + m^2a^2)}{e^{2\pi n} - 1}$$
$$= i\int_{\frac{ma}{\pi}}^{\infty} dn\,\frac{(i\pi + \ln(n^2\pi^2 - m^2a^2))^2 - (-i\pi + \ln(n^2\pi^2 - m^2a^2))^2}{e^{2\pi n} - 1} = -4\pi\int_{\frac{ma}{\pi}}^{\infty} dn\,\frac{\ln(n^2\pi^2 - m^2a^2)}{e^{2\pi n} - 1}. \tag{A.2}$$

This integral cannot be performed in closed form. Expanding the denominator of the integrand we obtain

$$B_2(a) = -4\pi\sum_{j=1}^{\infty}\int_{\frac{ma}{\pi}}^{\infty} dn\, e^{-2\pi nj}\ln\left(n^2\pi^2 - m^2a^2\right)$$
$$= -4\pi\left\{\frac{\gamma}{2\pi}\ln\left(1 - e^{-2am}\right) + \sum_{j=1}^{\infty}\frac{e^{-2amj}}{2\pi j}\left(\ln(am/j) + e^{4amj}\Gamma(0, 4amj)\right)\right\}, \tag{A.3}$$

where $\Gamma(\alpha, x)$ is the incomplete Gamma function and in our case we have

$$\Gamma(0, x) = -e^{-x/2}\sqrt{\frac{x}{\pi}}\,\partial_\nu K_\nu(x/2)\big|_{\nu=-1/2}. \tag{A.4}$$

Both the first and second type of the branch-cut terms are finite for $m \neq 0$ and also their values go to zero when their arguments tend to infinity. So, when the limit $L/a \to \infty$ and $b/a \to \infty$ are taken, the contributions of these two branch-cut terms go to zero and therefore, only the branch-cut terms which depend on the original distance a remain.

References

[1] G. Plunien, B. Muller, W. Greiner, The Casimir effect, Phys. Rep. 134 (1986) 87.
[2] V.M. Mostepanenko, N.N. Trunov, The Casimir Effect and its Applications, Clarendon, Oxford, 1997.
[3] M. Bordag, U. Mohideen, V.M. Mostepanenko, New developments in the Casimir effect, Phys. Rep. 353 (2001) 1, arXiv:quant-ph/0106045.
[4] H.B.G. Casimir, D. Polder, The influence of retardation on the London–van der Waals forces, Phys. Rev. 73 (1948) 360;
H.B.G. Casimir, On the attraction between two perfectly conducting plates, Proc. Kon. Aa. Wet. 51 (1948) 793.
[5] K.A. Milton, The casimir effect: Physical manifestations of zero point energy, in: Invited Lectures at 17th Symposium on Theoretical Physics, Seoul National University, Korea, 29 June–1 July 1998, arXiv:hep-th/9901011.
[6] K.A. Milton, The Casimir Effect: Physical Manifestations of Zero-Point Energy, World Scientific Publishing Co. Pte. Ltd., 2001.
[7] M.J. Sparnaay, Measurements of attractive forces between flat plates, Physica 24 (1958) 751.
[8] G. Bressi, G. Carugno, R. Onfrio, G. Ruoso, Measurement of the Casimir force between parallel metallic surfaces, Phys. Rev. Lett. 88 (2002) 041804.
[9] S.K. Lamoreaux, Demonstration of the Casimir force in the 0.6 to 6 µm range, Phys. Rev. Lett. 78 (1997) 5;
R.S. Decca, D. López, H.B. Chan, E. Fischbach, D.E. Krause, C.R. Jamell, Constraining new forces in the Casimir regime using the isoelectronic technique, Phys. Rev. Lett. 94 (2005) 240401;
R.S. Decca, D. López, E. Fischbach, G.L. Klimchitskaya, D.E. Krause, V.M. Mostepanenko, Precise comparison of theory and new experiment for the Casimir force leads to stronger constraints on thermal quantum effects and long-range interactions, Ann. Phys. 318 (2005) 37.
[10] J. Ambjørn, S. Wolfram, Properties of the vacuum, 1. Mechanical and thermodynamic, Ann. Phys. (N.Y.) 147 (1983) 1.
[11] N.F. Svaiter, B.F. Svaiter, Casimir effect in a d-dimensional flat space–time and the cutoff method, J. Math. Phys. 32 (1991) 175.
[12] G. Cognola, E. Elizalde, K. Kiresten, Casimir energies for spherically symmetric cavities, J. Phys. A: Math. Gen. 34 (2001) 7311.
[13] A. Romeo, K.A. Milton, Casimir energy for a purely dielectric cylinder by the mode summation method, Phys. Lett. B 621 (2005) 309;
K.A. Milton, A.V. Nesterenko, V.V. Nesterenko, Mode-by-mode summation for the zero point electromagnetic energy of an infinite cylinder, Phys. Rev. D 59 (1999) 105009;
I.H. Brevik, V.V. Nesterenko, I.G. Pirozhenko, Direct mode summation for the Casimir energy of a solid ball, J. Phys. A 31 (1998) 8661.
[14] V.V. Nesterenko, I.G. Pirozhenko, Spectral zeta functions for a cylinder and a circle, J. Math. Phys. 41 (2000) 4521.
[15] K.A. Milton, L.L. Deraad, J. Schwinger, Casimir self-stress on a perfectly conducting spherical shell, Ann. Phys. (N.Y.) 115 (1978) 388.
[16] R. Balian, B. Duplantier, Electromagnetic waves near perfect conductors. II. Casimir effect, Ann. Phys. (N.Y.) 112 (1978) 165.
[17] T.P. Branson, P.B. Gilkey, The asymptotics of the Laplacian on a manifold with boundary, Commun. Partial Differential Eqs. 15 (1990) 245;
M. Bordag, K. Kiresten, Heat kernel coefficients and divergencies of the Casimir energy for the dispersive sphere, Int. J. Mod. Phys. A 17 (2002) 813.
[18] M. Bordag, D. Robaschik, E. Wieczorek, Quantum field theoric treatment of the Casimir effect, Ann. Phys. (N.Y.) 165 (1985) 192;
M. Bordag, J. Lindig, Radiative correction to the Casimir force on a sphere, Phys. Rev. D 58 (1998) 045003, arXiv:hep-th/9801129;
D. Robaschik, K. Scharnhorst, E. Wieczorek, Radiative corrections to the Casimir pressure under he influence of temperature and external fields, Ann. Phys. (N.Y.) 174 (1987) 401;
M. Bordag, K. Scharnhorst, $O(\alpha)$ radiative correction to the Casimir energy for penetrable mirrors, Phys. Rev. Lett. 81 (1998) 3815, arXiv:hep-th/9807121;
S.S. Xue, Casimir effect of scalar field on $S(n-1)$ manifold, Commun. Theor. Phys. (Wuhan) 11 (1989) 243.
[19] F. Ravndal, J.B. Thomassen, Radiative corrections to the Casimir energy and effective field theory, Phys. Rev. D 63 (2001) 113007.
[20] X. Kong, F. Ravndal, Radiative corrections to the Casimir energy, Phys. Rev. Lett. 79 (1997) 545;
K. Melnikov, Radiative corrections to the Casimir force and effective field theories, Phys. Rev. D 64 (2001) 045002.
[21] R.M. Cavalcanti, C. Farina, F.A. Barone, Radiative corrections to Casimir effect in the $\lambda\phi^4$ model, arXiv:hep-th/0604200, 2006;
F.A. Barone, R.M. Cavalcanti, C. Farina, Radiative corrections to the Casimir effect for the massive scalar field, arXiv:hep-th/0301238v1, 2003;
F.A. Barone, R.M. Cavalcanti, C. Farina, Radiative corrections to the Casimir effect for the massive scalar field, Nucl. Phys. B (Proc. Suppl.) 127 (2004) 118, arXiv:hep-th/0306011v2, 2003.
[22] N. Graham, R. Jaffe, H. Weigel, Casimir effects in renormalizable quantum field theories, Int. J. Mod. Phys. A 17 (2002) 846.
[23] N. Graham, R.L. Jaffe, V. Khemani, M. Quandt, M. Scandurra, H. Weigel, Calculating vacuum energies in renormalizable quantum field theories: A new approach to the Casimir problem, Nucl. Phys. B 645 (2002) 49;
N. Graham, R.L. Jaffe, V. Khemani, M. Quandt, O. Schröder, H. Weigel, The Dirichlet Casimir problem, Nucl. Phys. B 677 (2004) 379.
[24] K.A. Milton, The Casimir effect: Recent controversies and progress, J. Phys. A: Math. Gen. 37 (2004) 209.
[25] H.J. Vega, Two-loop quantum correction to the soliton mass in two-dimensional scalar field theories, Nucl. Phys. B 115 (1976) 411;
M.A. Lohe, D.M. O'Brien, Soliton mass corrections and explicit models in two dimensions, Phys. Rev. D 23 (1981) 1771;
N. Graham, R.L. Jaffe, Fermionic one-loop corrections to soliton energies in $1+1$ dimensions, Nucl. Phys. B 549 (1999) 516;
A.A. Izquierdo, W.G. Fuertes, M.A. González León, J.M. Guilarte, Generalized zeta functions and one-loop corrections to quantum kink masses, Nucl. Phys. B 635 (2002) 525;
A. Rebhan, P. van Nieuwenhuizen, R. Wimmer, The anomaly in the central charge of the supersymmetric kink from dimensional regularization and reduction, Nucl. Phys. B 648 (2003) 174;
A.A. Izquierdo, W.G. Fuertes, M.A. González León, J.M. Guilarte, One-loop corrections to classical masses of kink families, Nucl. Phys. B 681 (2004) 163.
[26] R.F. Dashen, B. Hasslacher, A. Neveu, Nonperturbative methods and extended-hadron models in field theory. II. Two-dimensional models and extended hadrons, Phys. Rev. D 10 (1974) 4130.
[27] H. Yamagishi, Soliton mass distributions in $(1+1)$-dimensional supersymmetric theories, Phys. Lett. B 147 (1984) 425.
[28] C.M. Bender, K.A. Milton, Scalar Casimir effect for a D-dimensional sphere, Phys. Rev. D 50 (1994) 6547.
[29] R. Moazzemi, M. Namdar, S.S. Gousheh, The Dirichlet Casimir effect for ϕ^4 theory in $(3+1)$ dimensions: A new renormalization approach, JHEP 0709 (2007) 029, arXiv:hep-th/0708.4127v1.
[30] R. Moazzemi, S.S. Gousheh, A new renormalization approach to the Dirichlet Casimir effect for ϕ^4 theory in $1+1$ dimensions, Phys. Lett. B 658 (2008) 255, arXiv:hep-th/0708.3428v2.
[31] M.E. Peskin, D.V. Schroeder, An Introduction to Quantum Field Theory, Addison–Wesley Pub. Co., 1995.
[32] L.C. de Albuquerque, Casimir pressure at two loops and soft boundaries at finite temperature, Phys. Rev. D 55 (1997) 7754.
[33] C.D. Fosco, N.F. Svaiter, Finite size effects in the anisotropic $\lambda(\phi_1^4+\phi_2^4)/4!$ model, J. Math. Phys. 42 (2001) 5185.
[34] T.H. Boyer, Quantum electromagnetic zero-point energy of a conducting spherical shell and the Casimir model for a charged particle, Phys. Rev. 174 (1968) 1764.
[35] W. Lukosz, Electromagnetic zero-point energy and radiation pressure for a rectangular cavity, Physica 56 (1971) 109.
[36] M.A. Valuyan, R. Moazzemi, S.S. Gousheh, A direct approach to the electromagnetic Casimir energy in a rectangular waveguide, J. Phys. B: At. Mol. Opt. Phys. 41 (2008) 145502.