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A bulk inflaton from large-volume extra dimensions

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ABSTRACT

The universe may have extra spatial dimensions with large volume that we cannot perceive because the energy required to excite modes in the extra directions is too high. Many examples are known of manifolds with a large volume and a large mass gap. These compactifications can help explain the weakness of four-dimensional gravity and, as we show here, they also have the capacity to produce reasonable potentials for an inflaton field. Modeling the inflaton as a bulk scalar field, it becomes very weakly coupled in four dimensions and this enables us to build phenomenologically acceptable inflationary models with tunings at the few per mil level. We speculate on dark matter candidates and the possibility of braneless models in this setting.

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1. Introduction

Modern theories suggest that although the universe appears to have three spatial dimensions, there may in fact be more. As is well known, if the extra dimensions are sufficiently small, they would escape observation. If the extra dimensional volume were large, however, a number of attractive features emerge, including an appealing explanation for the small value of Newton's constant. But familiar intuition suggests that as the internal volume grows, it becomes energetically easier to excite modes in the extra directions—the mass gap to the Kaluza–Klein states decreases. This raises the question: why does the universe appear to be three-dimensional? Or, put another way, why haven't we seen the Kaluza–Klein states?

A standard response to this question is to focus on fields that are localized on a 3-brane so they do not probe the Kaluza–Klein states. However, as an alternative response, we point to an infinite number of examples that circumvent the familiar intuition. We will discuss known examples of spaces that have a large mass gap and a large volume. Consequently, even fields that did live in the bulk would find the lowest Kaluza–Klein state energetically difficult to excite.

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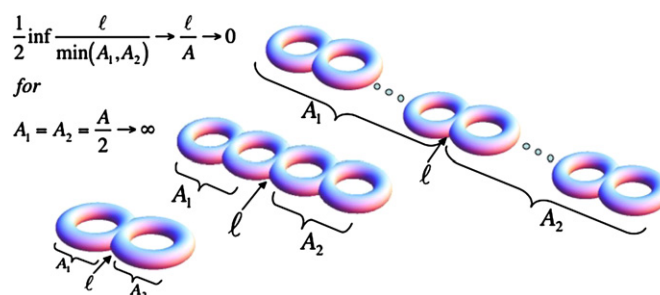


Fig. 1. The lowest mode on the surface made by linking doughnuts together wobbles between the two halves divided by the curve ℓ . As links are added, the tone gets lower as the two symmetric areas grow.

The essential reason why some surfaces have large minimum eigenvalue is related to the question famously posed by Mark Kac in the 1966 paper, “Can you hear the shape of a drum” [1]. While two drums can sound the same, as was shown nearly 30 years later [2], some features of the drum can be heard—you can ring out the eigenmodes of the Laplacian by banging the manifold [3,4]. A reasonable guess is that the bigger the drum the lower the tone. For instance, imagine the lowest frequencies on a surface made from stringing together doughnuts as drawn in Fig. 1. The lowest tone will result when roughly half of the surface wobbles out of phase with the other half. This conforms with Cheeger's bound on the minimum eigenvalue [5], which for a two-dimensional surface has the form,

$$k_1 \geq \frac{1}{2} \inf \frac{\ell}{\min(A_1, A_2)} \quad (1)$$

where ℓ is the length of a path that divides the surface into two areas, A_1 and A_2 , and the infimum is taken over all area dividing paths. In the case of the string of doughnuts, the minimum (non-zero) eigenvalue does indeed go down with area. The larger the area, the lower the tone.

However, there are counter-examples. For instance there are hyperbolic spaces, as we'll elaborate, that correspond to large mass gap and large volume. Compactification on these spaces, and the associated cosmology, has been studied in [6–9]. While in two dimensions these spaces are topologically equivalent to a string of doughnuts, they are not metrically equivalent. There are no thin bottlenecks that divide the space into roughly equal parts, so there is no mode that wobbles a large area of the surface at once. The lowest tone amounts to wobbling a small area. In another analogy, like waves in a pond full of barriers, the eigenmodes can only excite small areas at a time due to the intricate arrangement of holes. No matter how big you make the drum by adding more handles and holes, the lowest tone does not get any lower.

Large-volume extra dimensions can be put to good use in diluting the strength of gravity, thereby accounting for the small value of Newton's constant. Besides this phenomenological advantage, they are a curious intellectual possibility: at every point in space there might be some large transverse volume that we simply cannot perceive, not because we're confined to a brane, and not because the internal dimensions are small, but because it is simply too costly to do so at the low energies of our everyday experience. We discuss the mathematical constructions in Section 2.

As additional motivation for considering these spaces, they provide an attractive inflaton in the form of a bulk scalar field. We discuss this in general in Section 3 and study a concrete model in Section 4. Inflation in a large volume, large gap compactification has the following attractive features: (1) a suppression of the 4d coupling constant so the inflaton potential is flattened, (2) a 4d description which remains valid, even during inflation, thanks to the large gap, (3) a 4d vacuum expectation value (vev) for the inflaton driven up to the 4d Planck scale M_4 , (4) an inflaton mass at the fundamental scale of the bulk M , (5) inflation which takes place at an intermediate energy density $\sim M^2 M_4^2$, and (6) a standard cosmological evolution protected from copious and disruptive KK mode production by energetics.

These models have some more speculative advantages. The inflaton is very weakly coupled, which means it can double as a dark matter candidate. It is also tempting to revive the Kaluza–Klein idea in this context and construct a braneless model in which we are prohibited from detecting the extra dimensions by the large mass gap. We return to these possibilities in Section 5.

2. Large volume, large mass gap

First we review the familiar arguments about the energetic expense of exciting modes in the internal space. Consider the action for a scalar field in higher dimensions

$$\int d^{N+1}x \sqrt{-G} M^n \left(-\frac{1}{2} G^{IJ} \partial_I \phi \partial_J \phi - \frac{1}{2} m^2 \phi^2 \right) \quad (2)$$

Here M is the fundamental scale of the higher-dimensional description; we have included an overall factor of M^n so that all fields, masses, and coupling constants will have the same units as in $(3+1)$ -dimensions. Take a product geometry for the $N=3+n$ spatial dimensions $\mathbb{R}^3 \times \mathcal{M}^{(n)}$, with metric

$$ds^2 = G_{IJ} dx^I dx^J = \eta_{\mu\nu} dx^\mu dx^\nu + b^2 h_{ij} dy^i dy^j \quad (3)$$

Here $\mu, \nu = 0 \dots 3$ and $i, j = 4 \dots N$, and we have pulled out of the internal metric a dimensionful scale factor b . As usual this leads to a Kaluza–Klein tower of massive states, $m_k^2 = m^2 + k^2/b^2$, where k^2 is a dimensionless eigenvalue of the Laplacian on $\mathcal{M}^{(n)}$. For instance, in the case of a circle S^1 of size b , the masses are $m \sim n/b$ for $n \in \mathbb{Z}$, which illustrates the well-known fact that for larger b the modes are easier to excite. In the absence of a brane, the circle would have to be smaller than $b \sim \text{TeV}^{-1} \sim 10^{-16}$ mm to hide excitations of standard model fields from experiments.

There is, however, an alternative mechanism for hiding the Kaluza–Klein modes [6–9]. The intuition that the minimum energy mode will necessarily decrease into an observable domain as the volume of the internal space increases cannot be applied to all manifolds. Indeed there are an infinite number of compact manifolds whose minimum eigenvalue is *large*, implying a large mass gap, despite a having large volume. We consider these now.

We begin with hyperbolic surfaces, that is, two-dimensional surfaces with constant negative curvature as summarized in the Ricci scalar $\mathcal{R} = -2/b^2$. The Gauss–Bonnet theorem connects the area of these spaces with their topology, $A = 4\pi(g-1)b^2$ where g is the genus and b , again, is a dimensionful scale factor. The larger the genus, the larger the area of the surface for the same value of b . In most familiar examples, such as the string of doughnuts, the minimum eigenvalue goes down with the area for fixed b . But there is an extensive literature on the construction of hyperbolic surfaces of arbitrary genus that possess a large first eigenvalue: large in the sense that the lowest non-zero eigenvalue is bounded below by the curvature scale b^{-2} , and is independent of the area even as the area goes to infinity for fixed b [10–15].

In studying these surfaces it was originally conjectured by Buser in 1978 that the minimum eigenvalue k_1 would go to zero for large genus [12]. However, he later disproved his own conjecture by exhibiting surfaces of arbitrarily large genus with minimum eigenvalue squared $k_1^2 \geq 3/16$ [13]. The surfaces in Buser's proof come from number theoretic constructions. This therefore gives us hyperbolic surfaces with arbitrarily large genus g , and correspondingly large area, that maintain a large mass gap, to use the physics lexicon. Since the work of Buser, the number-theoretic lower bound has been improved slightly to $k_1^2 \geq 171/784$ (for the same surfaces) [16], while the construction was improved by Brooks and Makover to allow surfaces of arbitrary genus with first eigenvalue obeying nearly the same bound [10]. If Selberg's conjecture that the square of the minimum can be replaced by $1/4$ [14] is ever proven, then the theorem of Refs. [10,11] would deliver the bound $k_1^2 \geq 1/4$ for these same surfaces.

In a separate construction, Brooks and Makover show that in fact a *random* surface has large first eigenvalue. More precisely, take a large number N of equilateral triangles and glue them together in a random way by pairing up the edges to obtain a triangulated surface. The resulting surface has a canonical conformal structure, and by the Uniformization Theorem there is a unique hyperbolic metric in the conformal class. Then there is a constant C so that this hyperbolic metric will satisfy $k_1^2 \geq C$ with a probability that goes to 1 as N goes to infinity. (However, they do not give an explicit value for C , and their proof would probably give a very bad bound.) This shows that for surfaces that are “random” in a certain sense the first eigenvalue behaves moderately well.

For our purposes, it is more important that we have a good bound on k_1^2 than that the surfaces be generic. We therefore continue with the number-theoretic surfaces, and will use Selberg's conjectured bound $k_1^2 \geq 1/4$, although the difference between $171/784$ and $1/4$ is negligible for our purposes.

In practice then, there are surfaces of arbitrarily large genus, with area $A \sim 4\pi g b^2$ and a minimum eigenvalue bounded from

below. For $b = \text{TeV}^{-1}$ the mass gap, $kb^{-1} \sim \text{TeV}$ is too large to overcome except in the highest energy settings and yet the area is large if g is large. For $g \sim 10^{30}$, $A \sim (\text{mm})^2$. Despite such a large area, we would be unable to excite modes in the higher dimensions and would experience a 4d universe. Only at the energy scales of the Large Hadron Collider (LHC) could we expect to witness excitation of modes in the bulk.

These 2-surfaces are illustrative but there are presumably similar constructions in higher dimensions. Three-dimensional hyperbolic internal spaces of arbitrarily large volume are known [17] and have the particularly nice feature of being rigid—all metrical quantities are fixed by the topology and the requirement of constant curvature [18]. In other words, if the volume is stabilized, all moduli would be stabilized as a result of the rigidity.

So far, we have considered only the Laplacian (scalar) spectrum. Spinors also need to see a large mass gap in a realistic theory. The Dirac eigenspectrum is less well studied and it is not yet known if the large genus hyperbolic surfaces discussed above have a suitable spectrum. Ammann, Humbert, and Jammes have constructed surfaces (of any genus, with bounded volume) with a zero mode followed by an arbitrarily large gap in the Dirac spectrum [19], although these surfaces (dubbed “Pinocchio surfaces”, formed by stretching out a long nose from the surface) do not have a suitable Laplacian spectrum.

Although we have focused on hyperbolic spaces, there are other constructions. For instance one can obtain a large gap on a flat 2-torus, simply by allowing the complex structure to degenerate [20]. Another example, which gives the desired Kaluza–Klein tower for both scalars and fermions, is a rectangular n -torus of volume $\sim b^n$ with $n \gg 1$. The mass gap stays fixed even as the volume can be sent to infinity by sending the number of dimensions to infinity. This is less remarkable than the hyperbolic construction: each individual direction is small and the large volume is simply a result of a large number of dimensions. Also there is a huge spinor degeneracy since the number of spinors grows exponentially with the number of dimensions. Still, the n -dimensional torus demonstrates the existence of a space that has the required large mass gap for both scalars and fermions.

3. Bulk inflation

One phenomenological advantage to having a large volume is that it weakens the observed force of gravity in four dimensions. But any other bulk interactions will be suppressed as well. In this section we use this to help construct inflationary potentials [21].

We begin with a ϕ^4 theory in the bulk, with action

$$\int d^{4+n}x \sqrt{-G} M^n \left[-\frac{1}{2} G^{IJ} \partial_I \phi_B \partial_J \phi_B - \frac{1}{4} \lambda_B (\phi_B^2 - v_B^2)^2 \right] \quad (4)$$

where bulk quantities carry a B . Integrating over the internal dimensions the action becomes

$$\int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{4} \lambda (\phi^2 - v^2)^2 \right], \quad (5)$$

where we canonically normalize the kinetic term by redefining $\phi = \mathcal{V}^{1/2} \phi_B$. Here

$$\mathcal{V} = b^n M^n \int d^n y \sqrt{h} \quad (6)$$

is a dimensionless measure of the volume of the internal space, and the 4d coupling and vev are related to the bulk values through

$$\lambda = \lambda_B \mathcal{V}^{-1}, \quad (7)$$

$$v^2 = v_B^2 \mathcal{V}. \quad (8)$$

It follows that the mass is the same in the bulk and 4d descriptions $m^2 = m_B^2 = \lambda_B v_B^2$. (9)

These simple equations highlight the main features of large-volume compactification: we naturally get models with tiny couplings and huge vevs.

To get a sense of scale we compare to the gravitational action under dimensional reduction

$$\int d^{4+n}x \sqrt{-G} \frac{1}{2} M^{2+n} \mathcal{R} \rightarrow \int d^4x \sqrt{-g} \left[\frac{1}{2} M_4^2 \mathcal{R}^{(4)} + \dots \right]. \quad (10)$$

Here M is the underlying higher-dimensional scale and the effective reduced four-dimensional Planck mass is

$$M_4^2 = M^2 \mathcal{V} \sim (10^{18} \text{ GeV})^2. \quad (11)$$

Leaving M the unknown, this requires the volume adjust by $\mathcal{V} = M_4^2/M^2$. If the bulk coupling constant $\lambda_B \sim \mathcal{O}(1)$ and the bulk vev $v_B = \mathcal{O}(M)$ then

$$\lambda \sim (M/M_4)^2, \quad v \sim M_4, \quad m \sim M. \quad (12)$$

Taking $M \sim \text{TeV}$, for example, the coupling in the 4d theory is minute. The vev is at the 4d Planck scale, while the mass is much below Planck scale. Intriguingly, this implies that if there exist fundamental scalar fields in the bulk their interactions should be brutally suppressed. We would not easily observe such scalar fields, as indeed we do not. Furthermore, any scalar field potential would be exceedingly flat as slow-roll inflation requires: a very small coupling and a very large vacuum expectation value. And, neatly enough, any remnant scalar particles from the early universe would be dark matter candidates, with a mass set by the underlying higher-dimensional Planck scale M .

We note that although ϕ has mass set by the bulk scale M , inflation occurs at a much higher energy scale. Near the maximum of the potential, where $\phi \ll v$, the effective 4d energy density is

$$V = \lambda v^4 = \mathcal{V} \lambda_B v_B^4 \sim M^2 M_4^2 \sim (10^{10} \text{ GeV})^4 \quad (13)$$

where we're assuming the bulk energy density $\lambda_B v_B^4 \sim M^4 \sim \text{TeV}^4$. So an intriguing observation about the inflaton potential is that the energy scale of inflation would be 10^{10} GeV despite being driven by a field with an electroweak scale mass.

Although the choice $M \sim \text{TeV}$ is natural from the point of view of electroweak physics, the resulting inflationary scale $\sim 10^{10}$ GeV does not generate density perturbations of the required magnitude. Instead, as we'll see in the next section, the observed density perturbations favor the existence of an intermediate fundamental scale, with $M \sim 10^{11}$ GeV and $\mathcal{V} \sim 10^{14}$.

4. A slow-roll model

In this section we study a concrete model of bulk inflation and show that we can get a reasonable power spectrum, density perturbations of the right magnitude, and the requisite number of e -folds, all with tunings of the inflaton potential at the few per mil level.

We emphasize that any reasonable potential could be chosen for the inflaton. For simplicity we take a potential of the form

$$V = \frac{1}{4} \lambda e^{2\alpha\phi^2/v^2} (\phi^2 - v^2)^2 \quad (14)$$

where λ, v are set as in (7), (8). Setting $\alpha = 0$ recovers the usual ϕ^4 potential, while setting $\alpha = 1$ makes the second derivative of the potential vanish at the origin (see Fig. 2). We could equally well have used a potential of the Coleman–Weinberg type [22] or any other variant of inflaton potential. There are various phenomenological constraints that must be satisfied.

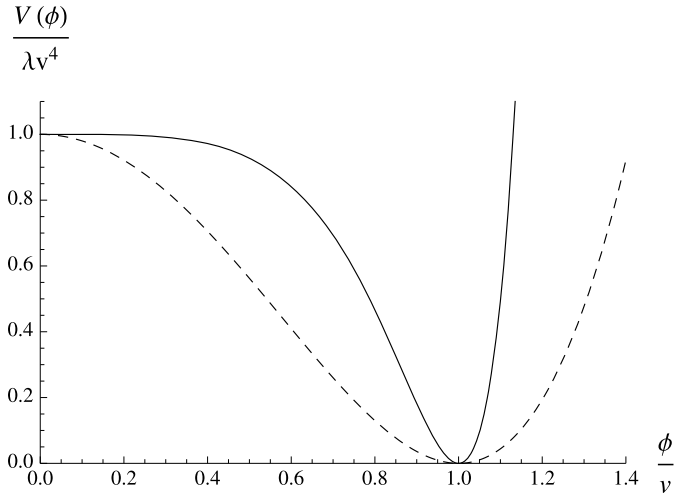


Fig. 2. Top dotted line has $\alpha = 0$ and so is a simple ϕ^4 style potential. The solid line has $\alpha = 1$.

4.1. Power spectrum

First we quantify the naturalness of V as a slow-roll inflaton potential using the parameters described in [23]. Slow-roll inflation is a consistent assumption if the slope and the curvature of the potential are small as quantified by the slow-roll parameters ϵ and η . Denoting $v = \beta M_4$, for small-field inflation $\phi \ll v$ and we have

$$\begin{aligned} \epsilon &= \frac{M_4^2}{2} \left(\frac{V'}{V} \right)^2 \approx \frac{8}{\beta^2} \frac{\phi^2}{v^2} \left[\alpha - 1 - \frac{\phi^2}{v^2} \right]^2, \\ \eta - \epsilon &= M_4^2 \left(\frac{V'}{V} \right)' \approx -\frac{4}{\beta^2} \left[1 - \alpha + 3 \frac{\phi^2}{v^2} \right]. \end{aligned} \quad (15)$$

We now study two special cases in turn.

- $\alpha < 1$:

When $\phi \ll v$ we have

$$\epsilon \ll |\eta|, \quad \eta \sim -\frac{4}{\beta^2} (1 - \alpha) \quad (16)$$

leading to a power spectrum $P_S \propto k^{n_S - 1}$ with scalar spectral index

$$n_S = 1 - 4\epsilon + 2\eta \approx 1 - \frac{8}{\beta^2} (1 - \alpha). \quad (17)$$

So for $\alpha < 1$ we have a red spectrum. In fact for generic values of $\alpha < 1$ the spectrum of scalar fluctuations is too red unless $v \gg M_4$ —the usual issue for quartic potentials for massive inflatons. Requiring that $n_S > 0.95$ for $\alpha = 0$, for instance, would demand the uncomfortable value $\beta = v/M_4 > 12$. As an alternative to a trans-Planckian vev one can tune α close to 1. For instance taking $\beta = 1$ and requiring $n_S > 0.95$ implies $1 - \alpha < 6 \times 10^{-3}$.

- $\alpha = 1$:

Setting $\alpha = 1$ and taking $\phi \ll v$ we have

$$\epsilon \ll |\eta|, \quad \eta \sim -\frac{12}{\beta^2} \frac{\phi^2}{v^2} \quad (18)$$

leading to

$$n_S = 1 - \frac{24}{\beta^2} \frac{\phi^2}{v^2}. \quad (19)$$

Provided inflation occurs at sufficiently small ϕ this is an acceptable, slightly red spectrum. For example, as we'll see below, taking inflation to begin at $\phi = 0.04M$ leads to a reasonable number of e -folds. For $\beta = 1$ this leads to $n_S = 0.96$. But achieving this does require some fine-tuning of the potential. For the approximation $\alpha = 1$ to be valid we need

$$1 - \alpha < 3\phi^2/v^2 \quad (20)$$

which for the values mentioned above leads to

$$1 - \alpha < 5 \times 10^{-3}. \quad (21)$$

4.2. Number of e -folds

The number of e -folds is given by

$$N = \frac{1}{M_4} \int_{\phi}^{\phi_e} \frac{d\phi}{\sqrt{2\epsilon(\phi)}}. \quad (22)$$

As a concrete example, consider taking $\alpha = 1$, so that

$$N = \frac{\beta^2}{8} \left(\frac{v^2}{\phi^2} - \frac{v^2}{\phi_e^2} \right). \quad (23)$$

Slow roll inflation ends when $\epsilon \sim 1$, or roughly when the field settles into its minimum, so that $\phi_e \sim v$. Sufficient e -folds then requires $\phi_i < \beta v / \sqrt{8N}$. For $N \sim 60$ and $\beta \sim 1$ the condition amounts to $\phi_i < v/20$ which is reasonable.

4.3. Density perturbations

Density perturbations are crucial in determining the energy density during inflation. In our case they will set the value of M or equivalently \mathcal{V} . The size of scalar perturbations is given by

$$\frac{\delta\rho}{\rho} = \frac{H}{\pi M_4} \frac{1}{\sqrt{8\epsilon}}. \quad (24)$$

During slow-roll $H^2 \approx V/3M_4^2$, and assuming inflation begins at $\phi_i \ll v$, the energy density during inflation $V \approx \frac{1}{4}\lambda v^4$. Taking $\alpha = 1$, the slow-roll parameter $\epsilon \approx 8M_4^2\phi^6/v^8$ so

$$\frac{\delta\rho}{\rho} = \frac{\lambda^{1/2}\beta^6}{16\pi\sqrt{3}} \left(\frac{M_4}{\phi_i} \right)^3. \quad (25)$$

With the number of e -folds $N \approx \beta^4 M_4^2 / 8\phi_i^2$ we have

$$\frac{\delta\rho}{\rho} = \frac{1}{\pi} \sqrt{\frac{2}{3}} \lambda^{1/2} N^{3/2}. \quad (26)$$

Sufficient inflation requires $N \sim \mathcal{O}(60)$, and observation requires $\delta\rho/\rho \sim 10^{-5}$. This leads to $\lambda \simeq 10^{-14}$. From the four-dimensional point of view this would be viewed as a very fine-tuned coupling. But in the context of large-volume extra dimensions it's quite easy to achieve. Let's take the bulk coupling $\lambda_B = \mathcal{O}(1)$. Then the required four-dimensional coupling translates into

$$\mathcal{V} = \lambda_B / \lambda \simeq 10^{14} \quad (27)$$

implying a bulk scale intermediate between the electroweak and 4d Planck scales:

$$M = M_4 / \sqrt{\mathcal{V}} \simeq 10^{11} \text{ GeV}. \quad (28)$$

This implies an energy density during inflation

$$V = \frac{1}{4}\lambda v^4 = \frac{1}{4}\lambda_B\beta^4 M^2 M_4^2 \sim (10^{14} \text{ GeV})^4 \quad (29)$$

where we've taken λ_B and β to be $\mathcal{O}(1)$.

Although this seems like the most natural way to realize bulk inflation, there are other possibilities. For instance we could demand that $M \sim 1$ TeV is of order the electroweak scale. This leads to $\mathcal{V} = M_4^2/M^2 \sim 10^{30}$ and, taking $\lambda_B = \mathcal{O}(1)$, $\lambda = \lambda_B/\mathcal{V} \sim 10^{-30}$. Then acceptable density perturbations require an extended period of inflation, $N \sim 10^7$, which requires that inflation begin at a very small value of ϕ : $\phi_i \sim 10^{-4}\beta v$. This can be arranged but may not be an appealing condition. Regardless of the particular potential or exit method, the gist is that density perturbations in this approach set the bulk scale, and this scale will fall somewhere between the Planck scale and the electroweak scale depending on the details.

5. Discussion and speculation

In summary, we have discussed internal manifolds with both a large volume and a large mass gap. From a mathematical point of view such manifolds seem generic in the space of all compactifications. From a physical point of view they are interesting because the large volume accounts for the weakness of four-dimensional gravity, while the large mass gap makes the extra dimensions invisible in current experiments.

A bulk scalar field, if present in such a compactification, has some curious features. From the 4d point of view its vev is large, of order the 4d Planck scale: $v \sim M_4$. But its coupling is tiny, $\lambda \sim 1/\mathcal{V}$, which suppresses its mass and energy density. These features are attractive for building inflationary potentials. Here we comment on some of the fine-tuning issues which are involved. As we saw in Section 4, it is not easy to satisfy the slow-roll conditions and obtain an acceptable perturbation spectrum—the so-called η problem of inflationary cosmology. We finessed this by tuning the potential at the few parts per mil level. Given this tuning, it is fairly easy to get enough e -folds of inflation. But the big payoff of a large-volume compactification is in generating density perturbations of the right magnitude. Normally this requires a tiny fine-tuned coupling from the 4d point of view. But in the extra dimensional scenario such a coupling is quite natural, and leads us to identify a fundamental bulk scale of perhaps 10^{11} GeV.

Most of these features rely on having a large internal volume. But the large gap plays an important role as well, because we need to ask: is the use of 4-dimensional effective field theory valid during inflation? In this regard it's reassuring that the energy density during inflation $V \sim \lambda M_4^4$ is well below the 4d Planck scale, so 4d quantum gravity effects should be negligible. But what about the Kaluza–Klein tower? To address this note that for the potential (14) the Hubble parameter during inflation $H^2 \approx V/3M_4^2 \approx \lambda_B\beta^4 M^2/12$. This corresponds to a de Sitter temperature

$$T = \frac{H}{2\pi} \approx \frac{\sqrt{\lambda_B}\beta^2 M}{4\pi\sqrt{3}}.$$

Given the bounds discussed in Section 2 and taking $b \approx 1/M$, the Kaluza–Klein tower begins at the scale $M/2$. So a naive estimate is that Kaluza–Klein excitations are suppressed by a Boltzmann factor $\exp(-2\pi\sqrt{3}/\sqrt{\lambda_B}\beta^2)$. Even for $\lambda_B \approx \beta \approx 1$ this is a suppression by almost 10^{-5} . By tuning λ_B and β to be slightly less than one—something which is desirable in any case, to avoid strong coupling in the bulk and a trans-Planckian vev—the contribution of the Kaluza–Klein tower can be made negligible. Similar remarks apply

to the effects of possible higher-derivative terms in the bulk gravitational action (10), which are suppressed by powers of

$$\mathcal{R}/M^2 \approx 12H^2/M^2 \approx \lambda_B\beta^4.$$

By tuning λ_B and β slightly these terms can be brought under control.

Putting this differently, if the mass gap were small there would be good astrophysical reasons to be concerned that a standard cosmology would not be possible. Although weakly coupled, Kaluza–Klein modes of the scalar field could still be copiously produced if the volume were large and the modes correspondingly easy to produce. Our large volume, large mass gap manifolds provide a protective energetic barrier and allow for a standard cosmological evolution (which resonates with the perspective of [6]). Thus inflation driven by a bulk scalar field seems like an attractive possibility.

We have implicitly assumed that the radion, and all other moduli, are stabilized during inflation.¹ Incorporating a mechanism for radion stabilization would be an important next step in developing this model. A mechanism for stabilizing moduli is required for all higher-dimensional cosmologies, and many scenarios have been developed. Stabilization might be achieved via twisted scalar fields [24], string windings [25], Casimir energy [26], fluxes [27], or some other motivated set of potentials [28]. Also, some evolution of the moduli could be phenomenologically interesting if both a bulk scalar and a radion are at play in double-field inflation. To keep our focus clear, we have not addressed moduli stabilization, but rather defer to the long list of possible mechanisms discussed in the literature.

We conclude with two more speculative possibilities which may be realized within the large-volume, large-gap scenario.

First, any remnant scalar particles from the early universe would be dark matter candidates. As a result of the suppression of the coupling constant the particles are effectively non-interacting, that is to say, dark. At the end of inflation, the flow of ϕ particles into standard model particles through parametric resonance could potentially overcome the very weak coupling and produce appropriate abundances of dark and baryonic matter.

Second, and more speculatively, one could imagine constructing fully braneless models along these lines. That is, one could allow all fields—including standard model fields—to propagate in the bulk. The large volume would account for the weakness of gravity by diluting its strength along the lines of [29], while the large gap would keep the extra dimensions from being directly detected. The challenges in realizing this scenario are (i) obtaining realistic interactions since the strength of all forces would be diluted over the large internal volume, and (ii) obtaining a realistic spectrum of chiral fermions. Regarding point (i), excited Kaluza–Klein modes are localized at around the curvature scale and so are not diluted over the entire internal volume. Consequently their interactions can be of reasonable strength. One might therefore hope to model massive gauge fields along these lines. Regarding point (ii), we note that the spectrum of the Dirac operator on these spaces is not well understood. Clearly the details of the phenomenology will depend crucially on the specific internal geometry, the eigenspectra of the various operators, and the overlap integrals of eigenmodes. Quantum effects on the finite internal volume will naturally have something to say about these issues. In a realistic approach, chiral fermions must be generated, the Einstein equations must be

¹ The radion must have a large enough mass that its Boltzmann factor $\exp(-4\pi\sqrt{3}M_{\text{radion}}/\sqrt{\lambda_B}\beta^2 M)$ can be neglected during inflation.

satisfied,² coupling constants must be resuscitated, and the extra dimensions must be stabilized.

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² The curvature term from a hyperbolic internal space would be large, $1/b^2 \approx M^2$, and would dominate the energy density of the universe, contrary to observations.