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Behavioural contracts with request–response operations[☆]

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ABSTRACT

In the context of service-oriented computing, behavioural contracts are abstract descriptions of the message-passing behaviour of services. They can be used to check properties of service compositions such as, for instance, client-service compliance. To the best of our knowledge, previous formal models for contracts consider unidirectional *send* and *receive* operations. In this paper, we present two models for contracts with bidirectional *request–response* operations, in the presence of unboundedly many instances of both clients and servers. The first model takes inspiration from the abstract service interface language WSCL, the second one is inspired by Abstract WS-BPEL. We prove that two different notions of client-service compliance (one based on client satisfaction and another one requiring mutual completion) are decidable in the former while they are undecidable in the latter, thus showing an interesting expressiveness gap between the modelling of *request–response* operations in WSCL and in Abstract WS-BPEL.

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1. Introduction

One interesting aspect of Service Oriented Computing (SOC) and Web Services technology is the need to describe in rigorous terms not only the format of the messages exchanged among interacting parties (as done, e.g., with the standard language WSDL [36]), but also the order in which such messages should be received and transmitted (as done, e.g., with the languages WSCL [35], WSCI [34], and Abstract WS-BPEL [29]). This specific aspect is clearly described in the Introduction of the Web Service Conversation Language (WSCL) specification [35], one of the proposals of the World Wide Web Consortium (W3C) for the description of the so-called Web Services *abstract interfaces*.

Defining which XML documents are expected by a Web service or are sent back as a response is not enough. It is also necessary to define the order in which these documents need to be exchanged; in other words, a business level conversation needs to be specified. By specifying the conversations supported by a Web service – by defining the documents to be exchanged and the order in which they may be exchanged – the external visible behaviour of a Web service, its abstract interface, is defined.

The abstract interface of services, sometimes called *behavioural contracts* (simply *contracts* in the following) can be used in several ways. For instance, one could check the *compliance* between a client and a service, that is, a guarantee for the client that the interaction with the service will in any case be completed successfully. One could also check, during the service discovery phase, the *conformance* of a concrete service to a given abstract interface by verifying whether the service implements at least the expected functionalities and does not require more.

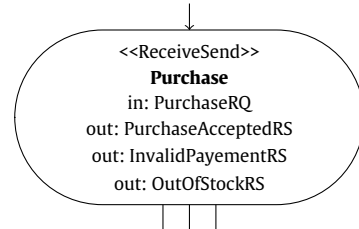
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Formal models are called for to devise rigorous forms of reasoning and verification techniques for services and abstract interfaces. To this aim, theories of *behavioural contracts* based on CCS-like process calculi [25] have been thoroughly investigated (see e.g. [5,8,10,13,14]). However, these models lack expressiveness in at least one respect: they cannot be employed to describe bidirectional *request–response* interactions, in contexts where several instances of the client and of the service may be running at the same time. This situation, on the other hand, is commonly found in practice-oriented contract languages, like the abstract service interface language WSCL [35], WSCI [34], and Abstract WS-BPEL [29] where also bi-directional interactions are supported. The typical trick for implementing bi-directional on top of uni-directional interactions, that exploits call backs, requires the generation of fresh sessions or operation names in the presence of multiple dynamically generated instances. But this is not possible if the underlying calculus is CCS, as it does not provide mechanisms for generation and communication of new names.

The WSCL language is a graphical notation similar to traditional flowcharts. There are four classes of basic actions: the one-way actions *Send* and *Receive*, and the two-way actions *SendReceive* and *ReceiveSend*. An example of *ReceiveSend* action is on the right.



In the example, an operation is represented that receives a purchase order and replies with three possible messages for acceptance, invalid payment, or out of stock, respectively.

WSCI is a richer XML-based language used to describe the behaviour of the actors involved in a multiparty service composition. Different from WSCL, in WSCI it is possible to indicate an activity to be executed between the receive and the send actions of a two-way input operation, as the process `tns:BookSeats` executed within the request–response operation `tns:TAToTraveler/bookTickets` in the example below:

```

<action name = "ReceiveConfirmation"
  role = "tns:TravelAgent"
  operation = "tns:TAToTraveler/bookTickets">
  <call process = "tns:BookSeats" />
</action>
  
```

Abstract WS-BPEL is an XML-based language, but different from WSCI, the request–response operations are modelled with two distinct *receive* and *reply* actions, that are correlated because they are executed with the same partner (see the example on the right). Between the *receive* and the *reply* actions any other action could be specified.

```

<receive partnerLink="purchase"
  portType="lns:OrderPT"
  operation="sendOrder"
  variable="PO">
  ...
<reply partnerLink="purchase"
  portType="lns:OrderPT"
  operation="sendOrder"
  variable="Invoice">
  
```

In this paper, we present a formal investigation of contract languages of the type described above, that is allowing bidirectional request–response interactions, taking place between instances of services and clients. We present two formal contract languages that, for simplicity, include only the request–response pattern¹: the first language is inspired by WSCL while the second one by Abstract WS-BPEL. We consider these two approaches as they represent the two ends of the spectrum of the different forms of request–response operations described above: in WSCL there is only one *ReceiveSend* event, while in Abstract WS-BPEL an arbitrary amount of other actions can be performed between two correlated *receive* and *reply* activities.

In both the two models that we present, the request–response interaction pattern is decomposed into sequences of more fundamental *send–receive–reply* steps: the client first *sends* its invocation, then the service *receives* such an invocation, and finally the service sends its *reply* message back to the client. The binding between the requesting and the responding sides (instances) of the original operation is maintained by employing naming mechanisms similar to those found in the π -calculus [26]. In both models, we do not put any restriction on the number of client or service instances that can be generated at runtime, so that the resulting systems are in general of infinite-state. The difference between the two models is that in the former it is not possible to describe intermediate activities of the service taking place between the *receive* and the *reply* steps, while this is possible in the latter.

We define client–service compliance on the basis of the *must testing* relation of [16]: a client and a service are compliant if all completed interactions between them leads the client to a successful state. Our main results show that client–service

¹ As discussed in Section 4, the languages that we propose are sufficiently expressive to model also the one-way communication pattern.

compliance is decidable in the WSCL-inspired model, while it is undecidable in the Abstract WS-BPEL model: this points to an interesting expressiveness gap between the two approaches for the modelling of the request–response interaction pattern. In the former case, the decidability proof is based on a translation of contracts into finite Petri nets. This translation does not reproduce faithfully all steps of computation (namely, intermediate steps of the request–response interaction are not represented in the Petri net), however the translation is complete, in the sense that it preserves and reflects the existence of unsuccessful computations, which is enough to reduce the original compliance problem to a decidable problem in finite Petri nets. This yields a practical compliance-checking procedure, obtained by adaptation of the classical Karp–Miller coverability tree construction [23].

We check the robustness of our approach for verifying client-service compliance by taking into consideration also a different notion of compliance. In multiparty service compositions, for instance, there is no clear distinction between clients and services as one partner could play both the roles. In those cases, a symmetric notion of compliance in which all the involved partners should reach successful completion is more appropriate. We first define a more restrictive notion of compliance, that we call *mutual compliance*, that guarantees completion of both the client and the service in any possible computation. We show then that our decision procedure can be slightly modified to cope also with this notion of compliance.

1.1. Structure of the paper

The rest of the paper is organized as follows. In Section 2, we present the two formal models and the definition of client-service compliance. Section 3 contains the Petri nets semantics and the proof of decidability of client-service compliance for the WSCL model. Section 4 reports on undecidability for the Abstract WS-BPEL model. In Section 5, we consider mutual compliance and we show how to modify the decision procedure defined in Section 3 to cope with this symmetric version of compliance. Finally, in Section 6 we draw some conclusions and discuss related and further work.

This paper is the full and extended version of [4]: the proofs of the technical results have been included, some additional examples have been introduced, a new part on mutual compliance has been added, and the related work section has been enriched.

2. Behavioural contracts with request–response

We presuppose a denumerable set of contract variables Var ranged over by X, Y, \dots , a denumerable set of names $Names$ ranged over by a, b, r, s, \dots . We use I, J, \dots to denote sets of indices.

Definition 2.1 (*WSCL Contracts*). The syntax of WSCL contracts, C, D, \dots , is defined by the following grammar (the auxiliary syntactic category G is used to denote guarded contracts)

$$\begin{aligned} G &::= \text{invoke} \left(a, \sum_{i \in I} b_i.C_i \right) \mid \text{recreply} \left(a, \sum_{i \in I} b_i.C_i \right) \mid \surd \\ C, D &::= \sum_{i \in I} G_i \mid C \mid C \mid X \mid \text{rec}X.C \end{aligned}$$

where $\text{rec}X.C$ is a binder for the contract variable X . We assume *guarded recursion*, that is, given a contract $\text{rec}X.C$ all the free occurrences of X in C are inside a guarded contract G . In the following, we will consider only closed contracts, i.e. contracts in which any occurrence of a variable X is bound by a corresponding $\text{rec}X.C$ primitive.

A *client contract* is a contract containing at least one occurrence of the guarded *success* contract \surd , while a *service contract* is a contract not containing \surd . In the following, we will denote the set of all WSCL contracts as \mathcal{C} .

G is used to denote guarded contracts, ready to perform either an invoke or a receive on a request–response operation a : the selection of the continuation C_i depends on the actual reply message b_i . A set of guarded contracts G_i can be combined into a choice $\sum_{i \in I} G_i$; if the index set I is empty, we denote this term by $\mathbf{0}$. Contracts can be composed in parallel. Note that infinite-state contract systems can be defined using recursion (see example later in the section). In the following, we use $Names(C)$ to denote the set of names occurring in C , and C and S to denote, respectively, client and service contracts. Before presenting the semantics of WSCL contracts, we introduce BPEL contracts as well.

Definition 2.2 (*BPEL Contracts*). BPEL contracts are defined like WSCL contracts in Definition 2.1, with the only difference that guarded contracts are as follows

$$G ::= \text{invoke} \left(a, \sum_{i \in I} b_i.C_i \right) \mid \text{receive}(a).C \mid \text{reply}(a,b).C \mid \surd.$$

Note that $\text{receive}(a)$ acts as a binder for a . Consequently, notions of free and bound names and alpha equivalence arise as expected. In the following, we will identify terms up to alpha-equivalence.

We now define the operational semantics of both models. We will interpret the WSCL contract $\text{recreply} \left(a, \sum_{i \in I} b_i.C_i \right)$ as the BPEL contract $\text{receive}(a). \sum_{i \in I} (\text{reply}(a,b_i).C_i)$, which receives an invocation on the operation a and then replies with one

Table 1
Operational semantics of contracts.

$$\begin{array}{c}
\text{(i) } \frac{G_1 \xrightarrow{\alpha} G'_1 \quad I \in I}{\sum_{i \in I} G_i \xrightarrow{\alpha} G'_i} \quad \text{(ii) } \frac{r \notin \text{Names} \left(\sum_{i \in I} b_i.C_i \right)}{\text{invoke} \left(a, \sum_{i \in I} b_i.C_i \right) \xrightarrow{(r)} \sum_{i \in I} (r\langle b_i \rangle.C_i) \mid \bar{a}(r)} \quad \text{(iii) } \frac{r \notin \text{Names}(C)}{\text{receive}(a).C \xrightarrow{a(r)} C\{r/a\}} \\
\text{(iv) } \text{reply}(r,b).C \xrightarrow{\tau} C \mid \bar{r}(b) \quad \text{(v) } \frac{C_1 \xrightarrow{\bar{a}(b)} C'_1 \quad C_2 \xrightarrow{a(b)} C'_2}{C_1|C_2 \xrightarrow{\tau} C'_1|C'_2} \quad \text{(vi) } \bar{r}(b) \xrightarrow{\tau} \mathbf{0} \quad \text{(vii) } r\langle b \rangle.C \xrightarrow{r(b)} C \\
\text{(viii) } \checkmark \xrightarrow{\tau} \mathbf{0} \quad \text{(ix) } \frac{C_1 \xrightarrow{\alpha} C'_1 \quad \alpha \neq (r)}{C_1|C_2 \xrightarrow{\alpha} C'_1|C_2} \quad \text{(x) } \frac{C_1 \xrightarrow{(r)} C'_1 \quad r \notin \text{Names}(C_2)}{C_1|C_2 \xrightarrow{(r)} C'_1|C_2} \quad \text{(xi) } \frac{C\{\text{rec}X.C/X\} \xrightarrow{\alpha} C'}{\text{rec}X.C \xrightarrow{\alpha} C'} \\
(a, b, r \in \text{Names}, \text{symmetric version of the rules for parallel composition omitted})
\end{array}$$

of the messages b_i . We shall rely on a run-time syntax of contracts, which is obtained from the original one by extending the clause for guarded contract, thus

$$G ::= \dots \mid \bar{a}(r) \mid r\langle b \rangle.C.$$

The terms $\bar{a}(r)$ and $r\langle b \rangle.C$ are used to represent an emitted but pending invocation of a request–response operation a : the name r represents a (fresh) channel r that will be used by the invoked operation to send the reply message back to the invoker. From now onwards we will call (WSCL) contract any term that can be obtained from this run-time syntax. In the following, we let $\text{Labels} \triangleq \{\tau, \checkmark\} \cup \{a\langle b \rangle, \bar{a}(b), (a) \mid a, b \in \text{Names}\}$. Moreover, by $C\{r/a\}$ we denote the term obtained from C by replacing with r every free occurrence of a , while $C\{\text{rec}X.C/X\}$ denotes the usual substitution of free contract variables with the corresponding definition.

Definition 2.3 (*Operational Semantics*). The operational semantics of contracts is given by the minimal labelled transition system, with labels taken from the set Labels, satisfying the axiom and rules in Table 1.

In the following, we use $C \xrightarrow{\alpha}$ to say that there is some C' such that $C \xrightarrow{\alpha} C'$. Moreover, we use $C \longrightarrow C'$ to denote reductions: $C \longrightarrow C'$ if $C \xrightarrow{\tau} C'$ or $C \xrightarrow{(r)} C'$ for some r . A *computation* is a finite or infinite sequence of reduction steps $D_1 \longrightarrow D_2 \longrightarrow \dots \longrightarrow D_n \longrightarrow \dots$. It is a *maximal computation* if it is infinite or it ends in a state D_n such that D_n has no outgoing reductions.

We now formalize the notion of client–service compliance resorting to *must-testing* [16]. Intuitively, a client C is *compliant* with a service contract S if all the computations of the system $C|S$ lead to the client’s success. Other notions of compliance have been put forward in the literature [8,9]; we have chosen this one, based on client satisfaction, because of its technical and conceptual simplicity (it has been adopted e.g. in [13], one of the pioneering papers on client–service compliance).

Definition 2.4 (*Client–Service Compliance*). A client contract C is *compliant* with a service contract S if for every maximal computation $C|S \longrightarrow D_1 \longrightarrow \dots \longrightarrow D_l \longrightarrow \dots$ there exists k such that $D_k \xrightarrow{\checkmark}$.

Example 2.5 (*Internal and External Nondeterminism*). One aspect that traditionally influences client–service compliance is the interplay between internal and external nondeterminism. In any conversation, the set of alternatives that the client expects as a reply should cover all the possible choices of the service. Consider the following WSCL contracts C_1 and S_1 representing a client that invokes a service on an operation “ op ”: the service can reply “ yes ” or “ no ”, while the client is ready to accept the replies “ yes ”, “ no ”, and “ $maybe$ ”.

$$\begin{array}{l}
C_1 \triangleq \text{invoke}(op, \text{yes}.\checkmark + \text{no}.\checkmark + \text{maybe}.\checkmark) \\
S_1 \triangleq \text{recreply}(op, \text{yes} + \text{no}).
\end{array}$$

It is easy to see that $C_1|S_1$ has only two possible maximal computations, both of them ending in a state in which the success action \checkmark is offered. Hence, we can conclude that C_1 is compliant with the service S_1 . If we swap the possible choices between the client and the service, we obtain the following contracts.

$$\begin{array}{l}
C_2 \triangleq \text{invoke}(op, \text{yes}.\checkmark + \text{no}.\checkmark) \\
S_2 \triangleq \text{recreply}(op, \text{yes} + \text{no} + \text{maybe}).
\end{array}$$

In this case, the service can *internally* decide to reply with “ $maybe$ ” (by applying rule (iv)), and such a reply cannot be received by the client. Formally, $C_2|S_2$ has a computation leading to a deadlock state in which the client is waiting to receive the reply.

This maximal computation does not offer the client's success \checkmark , hence we can conclude that C_2 is not compliant with the service S_2 .

Example 2.6 (*An Impatient Client and a Latecomer Service*). This example shows that even very simple WSCL scenarios could result in infinite-state systems. Consider a client C that either asks the box office service S for some tickets or decides to wait for them by listening to an *offerTicket*. Our client is impatient: at any time, it can decide to stop waiting and issue a new request. This behaviour can be described in WSCL as follows

$$C \triangleq \text{rec}X.(\text{invoke}(\text{requireTicket}, \text{ok}.X) + \text{recreply}(\text{offerTicket}, \text{ok}.\checkmark)).$$

Consider the box office service S , defined below, that is always ready to receive a *requireTicket* invocation and then responds by notifying (performing a call-back) on *offerTicket*.

$$S \triangleq \text{rec}X.\text{recreply}(\text{requireTicket}, \text{ok}.\text{invoke}(\text{offerTicket}, \text{ok})|X).$$

It could happen that $\text{invoke}(\text{offerTicket}, \text{ok})$ on the service side and $\text{recreply}(\text{offerTicket}, \text{ok}.\checkmark)$ on the client side never synchronize, as below

$$\begin{aligned} C | S &\rightarrow \overline{\text{requireTicket}}\langle r \rangle | r(\text{ok}).C | S && \text{(by (ii))} \\ &\rightarrow r(\text{ok}).C | \bar{r}(\text{ok}) | S | \text{invoke}(\text{offerTicket}, \text{ok}) && \text{(by (v))} \\ &\rightarrow C | S | \text{invoke}(\text{offerTicket}, \text{ok}) && \text{(by (v))} \\ &\rightarrow \rightarrow \rightarrow C | S | \text{invoke}(\text{offerTicket}, \text{ok}) | \text{invoke}(\text{offerTicket}, \text{ok}) && \text{(by (ii), (v), (v))} \\ &\vdots && \vdots \\ &\rightarrow \rightarrow \rightarrow C | S | \underbrace{\text{invoke}(\text{offerTicket}, \text{ok}) | \dots | \text{invoke}(\text{offerTicket}, \text{ok})}_n. \end{aligned}$$

Hence, the term $C|S$ could generate an infinite-state system where each state is characterized by an arbitrary number n of $\text{invoke}(\text{offerTicket}, \text{ok})$ parallel components. This infinite computation, moreover, does not traverse states in which the client can perform its \checkmark action, thus C is not compliant with S according to [Definition 2.4](#).²

Example 2.7 (*Unboundedly Many Instances of Client and Service*). A BPEL client C can recursively spawn instances that invoke the operation “service” provided by a service S . This repetitive behaviour is controlled by a “continue” operation of the client (representing, for instance, the interaction with a human agent) that nondeterministically replies either with *yes* to continue the spawning of instances, or with *no* to stop the client.

$$C \triangleq \text{rec}X. \left(\text{invoke} \left(\text{continue}, \begin{array}{l} \text{yes}.\text{invoke}(\text{service}, \text{ok}) | X \\ + \text{no}.\checkmark \end{array} \right) \right) | \text{rec}Y.\text{receive}(\text{continue}).(\text{reply}(\text{continue}, \text{yes}).Y + \text{reply}(\text{continue}, \text{no})).$$

Also the service S is able to spawn unboundedly many instances, one for each of the replies to the invocations coming from the client instances.

$$S \triangleq \text{rec}X.\text{receive}(\text{service}).(\text{reply}(\text{service}, \text{ok}) | X).$$

We complete this section by presenting the notion of stable WSCL contract that will be used in the proof of [Theorem 3.11](#). Informally, a contract is stable if there are no pending replies.

Definition 2.8 (*Stable Contracts*). A WSCL contract C (in the run-time syntax) is said to be *stable* if it contains neither unguarded $\text{reply}(r, b)$ actions nor pairs of matching terms of the form $\bar{r}(b)$ and $r(b)$.

Notice that every WSCL contract (according to the syntax of [Definition 2.1](#)) is stable. The following lemma shows that if one stable contract performs one computation and becomes unstable, then it can always return stable.

Lemma 2.9. *Suppose C is stable and that $C \rightarrow C'$. Then, there exists C'' stable such that $C' \rightarrow C_1 \rightarrow \dots \rightarrow C_l \rightarrow C''$ ($l \geq 0$) and for each $i = 1, \dots, l$ it holds that $C_i \not\rightarrow \checkmark$.*

Proof. If C' is not stable then it may contain both unguarded reply actions and pairs of the form $\bar{r}(b)$ and $r(b)$. According to the operational semantics of contracts, all unguarded reply actions and $\bar{r}(b)$ and $r(b)$ can be consumed performing a sequence of reductions. Therefore, a stable contract C'' can be reached from C' without traversing any state capable of \checkmark . \square

² Other definitions of compliance, see e.g. [8], resort to should-testing [31] instead of must-testing: according to these alternative definitions C and S turn out to be compliant due to the fairness assumption characterizing the should-testing approach.

3. Decidability of client-service compliance for WSCL contracts

We translate WSCL contract systems into finite place/transitions Petri nets [30], an infinite-state model in which several reachability problems are decidable (see, e.g., [18] for a review of decidable problems for finite Petri nets). The translation into Petri nets does not faithfully reproduce all the steps of computation of contracts. In particular, the operational semantics of contracts includes a mechanism for the generation of unboundedly many distinct names used to bind a service instance to the corresponding client instance in a bi-directional interaction. A faithful Petri net encoding should distinguish two distinct pairs of bound client-service instances, thus distinct places should be used. Obviously, this is not possible in a finite Petri net. The Petri net semantics that we present models bi-directional request–response interactions as a unique event, thus merging together events that are kept distinct in the operational semantics of contracts: the reception of the invocation and the emission and the reception of the reply. We will prove that this alternative modelling preserves client-service compliance because in WSCL the invoker and the invoked contracts do not interact with other contracts during the request–response.

Another difference with the operational semantics of contracts is that in the Petri net semantics when the client contract enters in a successful state, i.e. a state with an outgoing transition \surd , the corresponding Petri net enters a particular *successful* state and blocks its execution. This way, a client contract is compliant with a service contract if and only if in the corresponding Petri net all computations are finite and finish in a *successful* state. In fact, if a client is compliant with a service, we have that all the completed contract computations traverse a state in which the client is successful. As we will show, this last property is verifiable for finite Petri nets using a finite symbolic representation of all possible Petri net computations inspired by the so-called *coverability* tree [23].

3.1. A Petri net semantics for WSCL contracts

We first recall the definition of Petri nets. For any set S , we let $\mathcal{M}_{fin}(S)$ be the set of the finite multisets (*markings*) over S .

Definition 3.1 (*Petri Net*). A Petri net is a pair $N = (S, T)$, where S is the set of *places* and $T \subseteq \mathcal{M}_{fin}(S) \times \mathcal{M}_{fin}(S)$ is the set of *transitions*. A transition (c, p) is written $c \Rightarrow p$. A transition $c \Rightarrow p$ is *enabled* at a marking m if $c \subseteq m$. The *execution of the transition* produces the marking $m' = (m \setminus c) \oplus p$ (where \setminus and \oplus are the multiset difference and union operators). This is written as $m[]m'$. A *dead marking* is a marking in which no transition is enabled. A *marked* Petri net is a triple $M = (S, T, m_0)$, where (S, T) is a Petri net and m_0 is the *initial marking*. A computation in M leading to the marking m is a sequence $m_0[]m_1[]m_2 \cdots m_n[]m$.

Note that in $c \Rightarrow p$, the marking c represents the tokens to be “consumed”, while the marking p represents the tokens to be “produced”. The Petri net semantics that we present for WSCL contracts decomposes contract terms into multisets of terms, that represent sequential contracts at different stages of invocation. We introduce the decomposition function in Definition 3.3. Instrumental to this definition is the set $\text{Pl}(C)$, for C a WSCL contract, introduced in Definition 3.2. Let us first introduce the auxiliary functions $|\cdot|$ and $\text{unf}(\cdot)$. The function $|C|$ yields the syntactic size of a given contract C :

$$\left| \sum_{i \in I} G_i \right| = |X| = 1 \quad |C_1|C_2| = |C_1| + |C_2| \quad |\text{rec}X.C| = 1 + |C|.$$

The function $\text{unf}(C)$ performs the unfolding of any $\text{rec}X_$ in C not under the scope of one of the following prefixes: *invoke* (\cdot, \cdot) , *receive* (\cdot) , *reply* (\cdot, \cdot) , *a* (b) . $\text{unf}(C)$ is defined by induction on the pairs (n_1, n_2) , ordered lexicographically, where n_1 is the number of unguarded (i.e. not under an *invoke* (\cdot, \cdot) , *receive* (\cdot) , *reply* (\cdot, \cdot) , *a* (b)) sub-terms of the form $\text{rec}X.D'$ in C and $n_2 = |C|$.

$$\begin{aligned} \text{unf}(\text{rec}X.D) &= \text{unf}(D\{\text{rec}X.D/X\}) \\ \text{unf}(D_1|D_2) &= \text{unf}(D_1)|\text{unf}(D_2) \quad \text{unf}(C) = C, \text{ otherwise.} \end{aligned}$$

Definition 3.2 ($\text{Pl}(C)$). The set $\text{Pl}(C)$ is defined as follows:

$$\text{Pl}(C) \triangleq \left\{ \sum_{i \in I} G_i, a \uparrow \sum_{i \in I} b_i.C_i, c \downarrow \sum_{i \in I} b_i.C_i : \sum_{i \in I} G_i, \sum_{i \in I} b_i.C_i \text{ occur in } \text{unf}(C), a, c \in \text{Names}(\text{unf}(C)) \right\}.$$

The three kinds of places have the following meaning: $\sum_{i \in I} G_i$ represents a sequential process, $a \uparrow \sum_{i \in I} b_i.C_i$ represents a process that after emission of an invocation on a is waiting for the reply, while $c \downarrow \sum_{i \in I} b_i.C_i$ represents a blocked process because the reply c is different from the admitted replies b_i .

The function $\text{dec}(\cdot)$ transforms every WSCL contract C , as given in Definition 2.1, into a multiset $m \in \text{Pl}(C)$.

Definition 3.3 (*Decomposition*). The *decomposition* $\text{dec}(C)$ of a WSCL contract C is $\text{dec}_C(\text{unf}(C))$. The auxiliary function $\text{dec}_C(D)$ is defined in Table 2.

Note that by definition, contracts of the form $\sum_{i \in I} G_i$, as in Definition 2.1, are mapped by $\text{dec}(\cdot)$ into themselves: $\text{dec}(\sum_{i \in I} G_i) = \text{dec}_C(\sum_{i \in I} G_i) = \sum_{i \in I} G_i$, for any C .

Table 2The auxiliary function $dec_C(D)$.

$$\begin{aligned}
dec_C\left(\sum_{i \in I} r(b_i).D_i\right) &= a\uparrow \sum_{i \in I} b_i.D_i \quad \text{if } \bar{a}(r) \text{ occurs in } C \\
dec_C\left(\sum_{i \in I} r(b_i).D_i\right) &= c\downarrow \sum_{i \in I} b_i.D_i \quad \text{if } \bar{r}(c) \text{ occurs in } C \text{ and } c \neq b_i \text{ for every } i \in I \\
dec_C(\bar{a}(b)) &= \emptyset \quad dec_C(D_1|D_2) = dec_C(D_1) \oplus dec_C(D_2) \quad dec_C(D) = D, \text{ otherwise}
\end{aligned}$$

Table 3

Transitions schemata for the Petri net semantics of WSCL contracts.

$$\begin{aligned}
\left\{ \sum_{i \in I} G_i \right\} &\Rightarrow \left\{ a\uparrow \sum_{j \in J} b_j.C_j \right\} \quad \text{if } G_k = \text{invoke}\left(a, \sum_{j \in J} b_j.C_j\right) \text{ for some } k \in I \\
\left\{ a\uparrow \sum_{j \in J} b_j.C_j, \sum_{i \in I} G_i \right\} &\Rightarrow dec(C_y) \oplus dec(D_z) \quad \text{if } \begin{cases} G_k = \text{recreply}\left(a, \sum_{i \in L} c_i.D_i\right) \text{ for some } k \in I \text{ and} \\ b_y = c_z \text{ for some } y \in J, z \in L \end{cases} \\
\left\{ a\uparrow \sum_{j \in J} b_j.C_j, \sum_{i \in I} G_i \right\} &\Rightarrow \left\{ c_z\downarrow \sum_{j \in J} b_j.C_j \right\} \oplus dec(D_z) \quad \text{if } \begin{cases} G_k = \text{recreply}\left(a, \sum_{i \in L} c_i.D_i\right) \text{ for some } k \in I \text{ and} \\ z \in L \text{ s.t. } c_z \neq b_j \text{ for every } j \in J \end{cases}
\end{aligned}$$

Example 3.4. As an example of decomposition we consider the contract $C|S$ as defined in the [Example 2.6](#):

$$\begin{aligned}
dec(C|S) &= \{ \text{invoke}(\text{requireTicket}, \text{ok.recX}. (\text{invoke}(\text{requireTicket}, \text{ok.X}) \\
&\quad + \text{recreply}(\text{offerTicket}, \text{ok.}\sqrt{1})) \\
&\quad + \text{recreply}(\text{offerTicket}, \text{ok.}\sqrt{1}) \}, \\
&\quad \text{recreply}(\text{requireTicket}, \text{ok}.(\text{invoke}(\text{offerTicket}, \text{ok}) | \\
&\quad \text{recX.recreply}(\text{requireTicket}, \text{ok}.(\text{invoke}(\text{offerTicket}, \text{ok})|X)))) \}.
\end{aligned}$$

Note that the decomposition generates two tokens, one corresponding to the client and one corresponding to the service, both obtained by unfolding the corresponding recursive definitions.

There are three kinds of transitions in the Petri net we are going to define:

- transitions representing the emission of an invocation;
- transitions representing (atomically) the reception of the invocation and the emission and reception of the reply;
- transitions representing (atomically) the reception of the invocation and the emission of a reply that will never be received by the invoker because it is outside the set of admitted replies.

These three cases are taken into account in the definition below.

Definition 3.5 (*Petri Net Semantics*). Let C be a WSCL contract system as in [Definition 2.1](#). We define $Net(C)$ as the Petri net (S, T) where:

- $S = Pl(C)$;
- $T \subseteq \mathcal{M}_{fin}(S) \times \mathcal{M}_{fin}(S)$ includes all the transitions that are instances of the transitions schemata in [Table 3](#).

We define the *marked net* $Net^m(C)$ as the marked net (S, T, m_0) , where the initial marking is $m_0 = dec(C)$.

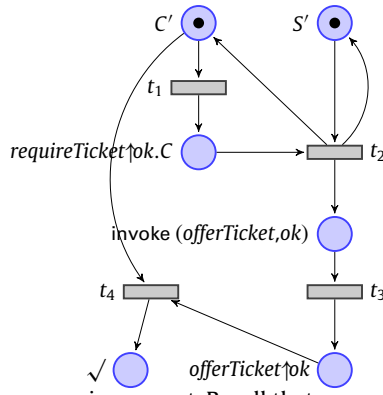
Example 3.6. We continue the [Example 3.4](#) by showing the Petri net associated to $C|S$ as defined in [Example 2.6](#). The places of the initial marking are fully defined in the [Example 3.4](#); here they are denoted more compactly as follows

$$C' \triangleq \text{invoke}(\text{requireTicket}, \text{ok}.C) + \text{recreply}(\text{offerTicket}, \text{ok}.\checkmark)$$

$$S' \triangleq \text{recreply}(\text{requireTicket}, \text{ok}.\text{invoke}(\text{offerTicket}, \text{ok})|S).$$

The marked net $\text{Net}^m(S|C)$ is depicted on the right. A bunch of unreachable places (like $\text{ok}\downarrow\text{ok}.\checkmark$, $\text{ok}\uparrow\text{ok}.\checkmark$, ...) has been omitted for the sake of clarity.

This example shows how the Petri net



semantics models bi-directional request–response interactions as a unique event. Recall that

$$\text{recreply}(\text{requireTicket}, \text{ok}.\text{invoke}(\text{offerTicket}, \text{ok})|S) = \text{receive}(\text{requireTicket}).\text{reply}(\text{requireTicket}, \text{ok}).\text{invoke}(\text{offerTicket}, \text{ok}) | S.$$

The first rule in Table 3 simulates the application of rule (ii). Indeed, in the Petri net above transition t_1 simulates the following:

$$C' | S' \xrightarrow{(r)} \overline{\text{requireTicket}}(r) | r\langle \text{ok} \rangle.C | S'.$$

The second rule in Table 3, which gives rise to transition t_2 , simulates the sequence of application of rules (v), (iv) and (v). The first application of (v) allows for the synchronization between the invocation $\text{requireTicket}(r)$ and $\text{receive}(\text{requireTicket})$:

$$\overline{\text{requireTicket}}(r) | r\langle \text{ok} \rangle.C | S' \xrightarrow{\tau} r\langle \text{ok} \rangle.C | \text{reply}(r, \text{ok}).\text{invoke}(\text{offerTicket}, \text{ok}) | S.$$

Thanks to (iv) the invoked part selects (the only) possible reply ok :

$$r\langle \text{ok} \rangle.C | \text{reply}(r, \text{ok}).\text{invoke}(\text{offerTicket}, \text{ok}) | S \xrightarrow{\tau} r\langle \text{ok} \rangle.C | \bar{r}\langle \text{ok} \rangle | \text{invoke}(\text{offerTicket}, \text{ok}) | S.$$

Finally, thanks to (v), the selected reply is received by the invoker

$$r\langle \text{ok} \rangle.C | \bar{r}\langle \text{ok} \rangle | \text{invoke}(\text{offerTicket}, \text{ok}) | S \xrightarrow{\tau} C | \text{invoke}(\text{offerTicket}, \text{ok}) | S.$$

We divide the proof of the correspondence between the operational and the Petri net semantics of WSCL contracts into two parts: we first prove a *soundness* result showing that all Petri net computations reflect computations of contracts, and then a *completeness* result showing that contract computations leading to a state in which there are no uncompleted request–response interactions are reproduced in the Petri net.

In the proof of the soundness result we use the following structural congruence rule to remove empty contracts and in order to rearrange the order of contracts in parallel compositions. Let \equiv be the minimal congruence for contract systems such as

$$C|0 \equiv C \quad C|D \equiv D|C \quad C|(D|E) \equiv (C|D)|E \quad \text{rec}X.C \equiv C\{\text{rec}X.C/X\}.$$

As usual, we have that the structural congruence respects the operational semantics.

Proposition 3.7. *Let C and D be two contract systems such that $C \equiv D$. If $C \xrightarrow{\alpha} C'$, then there exists D' such that $D \xrightarrow{\alpha} D'$ and $C' \equiv D'$.*

The following result establishes a precise relationship between the form of m and the form of C when $\text{dec}(C) = m$.

Lemma 3.8. *Let C be a WSCL contract system and suppose $\text{dec}(C) = m$. The following holds:*

- (1) $m = \{\sum_{i \in I} G_i\} \oplus m'$ if and only if $C \equiv \sum_{i \in I} G_i | D$, for some D such that $\text{dec}(D) = m'$;
- (2) $m = \{a \uparrow \sum_{j \in J} b_j.C_j\} \oplus m'$ if and only if $C \equiv \sum_{j \in J} r(b_j).C_j | \bar{a}(r) | D$, for some D and r such that $r \notin \text{Names}(D)$ and $\text{dec}(D) = m'$;
- (3) $m = \{c \downarrow \sum_{j \in J} b_j.C_j\} \oplus m'$ if and only if $c \neq b_j$ for each $j \in J$ and $C \equiv \sum_{j \in J} r(b_j).C_j | \bar{r}(c) | D$, for some D and r such that $r \notin \text{Names}(D)$ and $\text{dec}(D) = m'$.

Proof. Recall that $\text{dec}(C) = \text{dec}_C(\text{unf}(C))$ and note that $\text{unf}(C) \equiv C$.

(\Leftarrow) Follows from the definition of $\text{dec}(C)$.

(\Rightarrow) (1) If $m = \{\sum_{i \in I} G_i\} \oplus m'$ then rule $\text{dec}_C(C_1|C_2) = \text{dec}_C(C_1) \oplus \text{dec}_C(C_2)$ has been applied at least once. Therefore, there are C_1 and C_2 such that $C \equiv C_1|C_2$ and $\text{dec}_C(C_1) = \{\sum_{i \in I} G_i\}$ and $\text{dec}_C(C_2) = \text{dec}(C_2) = m'$. Moreover, the only rule that could have been used in order to infer that $\text{dec}_C(C_1) = \{\sum_{i \in I} G_i\}$ is $\text{dec}_C(D) = D$. Therefore, we have that $C_1 = \sum_{i \in I} G_i$ and $C = \sum_{i \in I} G_i|C_2$.

- (2) As before, if $m = \{a\uparrow\sum_{j \in J} b_j.C_j\} \oplus m'$ then there are C_1 and C_2 such that $C \equiv C_1|C_2$ and $dec_C(C_1) = \{a\uparrow\sum_{j \in J} b_j.C_j\}$ and $dec_C(C_2) = m'$. Moreover, the only rule that could have been used in order to infer that $dec_C(C_1) = \{a\uparrow\sum_{j \in J} b_j.C_j\}$ is $dec_C(\sum_{j \in J} r\langle b_j \rangle.D_j) = a\uparrow\sum_{j \in J} b_j.D_j$ and this guarantees that $\bar{a}\langle r \rangle \in C$. Recall that $dec_C(\bar{a}\langle r \rangle) = \emptyset$. Therefore, we have that $C_1 = \sum_{j \in J} r\langle b_j \rangle.C_j$ and $C = \sum_{j \in J} r\langle b_j \rangle.C_j \mid \bar{a}\langle r \rangle \mid C_2$.
Moreover, Given that C_2 is a run-time term, r is guaranteed to be fresh by rule (x), hence $r \notin \text{Names}(C_2)$ and $dec_C(C_2) = dec(C_2) = m'$.
- (3) Again, if $m = \{c\downarrow\sum_{j \in J} b_j.C_j\} \oplus m'$ then there are C_1 and C_2 such that $C \equiv C_1|C_2$ and $dec_C(C_1) = \{c\downarrow\sum_{j \in J} b_j.C_j\}$ and $dec_C(C_2) = m'$. Moreover, the only rule that could have been used in order to infer that $dec_C(C_1) = \{c\downarrow\sum_{j \in J} b_j.C_j\}$ is $dec_C(\sum_{j \in J} r\langle b_j \rangle.D_j) = c\downarrow\sum_{j \in J} b_j.D_j$ and this guarantees that $\bar{r}\langle c \rangle$ occurs in C and that $c \neq b_j$, for each $j \in J$. Recall that $dec_C(\bar{a}\langle r \rangle) = \emptyset$. Therefore, we have that $C_1 = \sum_{j \in J} r\langle b_j \rangle.C_j$ and $C = \sum_{j \in J} r\langle b_j \rangle.C_j \mid \bar{r}\langle c \rangle \mid C_2$.
Given that C_2 is a run-time term, r is guaranteed to be fresh by rule (x), hence $r \notin \text{Names}(C_2)$ and $dec_C(C_2) = dec(C_2) = m'$. \square

Proposition 3.9. *Let C be a WSCL contract. Consider the Petri net $\text{Net}(C) = (S, T)$ and a marking m of $\text{Net}(C)$ such that $dec(D) = m$, for some contract D . We have that m is dead if and only if D has no outgoing reductions.*

Proof. (\Rightarrow) Suppose m is dead, we have to prove that any D , with $dec(D) = m$, has no outgoing reductions. The proof is by induction on the structure of m . The case $m = \emptyset$ is trivial.

Suppose $m = \{\sum_{i \in I} G_i\} \oplus m'$, hence $D \equiv \sum_{i \in I} G_i|C'$, with $dec(C') = m'$ (Lemma 3.8). By definition, m' is dead, therefore, by induction, C' has no outgoing reductions. Moreover:

- $G_k \neq \text{invoke}(a, \sum_{j \in J} b_j.C_j)$, for each $k \in I$ (otherwise the first kind of transition would apply to m);
- $G_k = \text{recreply}(a_k, \sum_{l \in L_k} c_l.D_l)$ but m' does not contain $a_k \uparrow \sum_{j \in J} b_j.C_j$, for each $k \in I$ (otherwise either the second or the third kind of transition would apply to m). Therefore, by Lemma 3.8, there is no $\bar{a}_k\langle r \rangle \mid \sum_{j \in J} r\langle b_j \rangle.C_j$ in C' (by $dec(C') = m'$).

Therefore, by Proposition 3.7, $D \equiv \sum_{i \in I} G_i|C'$ has not outgoing reductions.

Suppose $m = \{a\uparrow\sum_{j \in J} b_j.C_j\} \oplus m'$, hence $D \equiv \sum_{j \in J} r\langle b_j \rangle.C_j \mid \bar{a}\langle r \rangle \mid C'$, with $r \notin \text{Names}(C')$ and $dec(C') = m'$ (Lemma 3.8). By definition, m' is dead; therefore, by induction, C' has no outgoing reductions. Moreover $m' \neq \{\sum_{i \in I} G_i\} \oplus m''$ with $G_k = \text{recreply}(a, \sum_{l \in L} c_l.D_l)$, for some $k \in I$. Otherwise either the second or the third kind of transition would apply to m . Hence, by definition of $dec(\cdot)$, C' cannot contain an unguarded subterm of the form $\text{recreply}(a, \sum_{l \in L} c_l.D_l)$ and D has no outgoing reductions.

Suppose $m = \{c\downarrow\sum_{j \in J} b_j.C_j\} \oplus m'$. Then $c \neq b_j$, for each j and $D \equiv \sum_{j \in J} r\langle b_j \rangle.C_j \mid \bar{r}\langle c \rangle \mid C'$, with $dec(C') = m'$ (Lemma 3.8). By definition, m' is dead, therefore, by induction, C' has no outgoing reductions and by inspection of the semantics of contracts it is easy to see that D has no outgoing reductions too.

(\Leftarrow) By induction on the structure of m and by Lemma 3.8 it can be easily seen that if D has an outgoing reduction then we have a contradiction and m is not dead. \square

In order to prove that the Petri net semantics preserves client-service compliance, we need to introduce the notion of *success marking*. A *success marking* m contains at least one token in a place corresponding to a successful client state, formally, $m(\sum_{i \in I} G_i) > 0$ for some contract $\sum_{i \in I} G_i$ such that $G_k = \surd$, for some $k \in I$.

We are now ready to prove the *soundness* result.

Theorem 3.10 (Soundness). *Let C be a WSCL contract. Consider the Petri net $\text{Net}(C) = (S, T)$ and let m be a marking of $\text{Net}(C)$. If $m \mid m'$ then for each D such that $dec(D) = m$ there exists a computation $D \stackrel{\Delta}{=} D_0 \longrightarrow D_1 \longrightarrow \dots \longrightarrow D_l$, with $dec(D_l) = m'$. Moreover, if m is not a success marking then there exists no $j \in \{0, \dots, l-1\}$ such that $D_j \xrightarrow{\surd}$.*

Proof. The proof proceeds by case analysis on the three possible kinds of transition.

- (1) If $m \mid m'$ by applying the first kind of transition then $m = \{\sum_{j \in J} G_j\} \oplus m''$, with $G_k = \text{invoke}(a, \sum_{i \in I} b_i.C_i)$ for a $k \in J$. Moreover, $m' = \{a\uparrow\sum_{i \in I} b_i.C_i\} \oplus m''$.

By Lemma 3.8 and by rule (ii), $D \equiv \sum_{j \in J} G_j \mid C' \longrightarrow \bar{a}\langle r \rangle \mid \sum_{i \in I} r\langle b_i \rangle.C_i \mid C' \stackrel{\Delta}{=} D'$ and $dec(D') = m'$.

- (2) If $m \mid m'$ by applying the second kind of transition then $m = \{a \uparrow \sum_{j \in J} b_j.C_j, \sum_{i \in I} G_i\} \oplus m''$, with $G_k = \text{recreply}(a, \sum_{l \in L} c_l.D_l)$, for some $k \in J$, and $b_y = c_z$ for some $y \in J$ and $z \in L$. By Lemma 3.8, if $dec(D) = m$ then $D \equiv \bar{a}\langle r \rangle \mid \sum_{j \in J} r\langle b_j \rangle.C_j \mid \sum_{i \in I} G_i \mid C'$, for any C' such that $dec(C') = m''$. Therefore, by $G_k = \text{recreply}(a, \sum_{l \in L} c_l.D_l)$:

$$\begin{aligned}
D &\longrightarrow \sum_{j \in J} r\langle b_j \rangle.C_j \mid \sum_{l \in L} \text{reply}(r, c_l).D_l \mid C' && \text{(by rule (v))} \\
&\longrightarrow \sum_{j \in J} r\langle b_j \rangle.C_j \mid \bar{r}\langle c_z \rangle \mid D_z \mid C' && \text{(by rule (iv))} \\
&\longrightarrow C_y \mid D_z \mid C' \stackrel{\Delta}{=} D' && \text{(by rule (v))}
\end{aligned}$$

with $\text{dec}(D') = \text{dec}(C_y) \oplus \text{dec}(D_z) \oplus m'' = m'$. Notice that each intermediate state in the reduction sequence from D to D' cannot perform a successful transition.

- (3) If $m[\]m'$ by applying the third kind of transition then $m = \{a\uparrow \sum_{j \in J} b_j.C_j, \sum_{i \in I} G_i\} \oplus m''$, with $G_k = \text{recreply}(a, \sum_{l \in L} c_l.D_l)$, for some $k \in J$, and there is $z \in L$ such that $b_y \neq c_z$ for each $y \in J$. By Lemma 3.8, if $\text{dec}(D) = m$ then $D \equiv \bar{a}(r) \mid \sum_{j \in J} r(b_j).C_j \mid \sum_{i \in I} G_i \mid C'$, for any C' such that $\text{dec}(C') = m''$. Therefore, by $G_k = \text{recreply}(a, \sum_{l \in L} c_l.D_l)$, we get

$$\begin{aligned} D &\longrightarrow \sum_{j \in J} r(b_j).C_j \mid \sum_{l \in L} \text{reply}(r, c_l).D_l \mid C' \quad (\text{by rule (v)}) \\ &\longrightarrow \sum_{j \in J} r(b_j).C_j \mid \bar{r}(c_z) \mid D_z \mid C' \stackrel{\Delta}{=} D' \quad (\text{by rule (iv)}) \end{aligned}$$

with $\text{dec}(D') = \{c_z \downarrow \sum_{j \in J} b_j.C_j\} \oplus \text{dec}(D_z) \oplus m'' = m'$. As in the previous case, each intermediate state in the reduction sequence from D to D' cannot perform a successful transition. \square

We now move to the completeness part.

Theorem 3.11 (Completeness). *Let C be a WSCL contract and let D be a contract reachable from C through the computation $C = C_0 \longrightarrow C_1 \longrightarrow \dots \longrightarrow C_n = D$. If D is stable then there exists a computation $m_0[\]m_1[\]m_2 \dots m_{i-1}[\]m_i$ of the marked Petri net $\text{Net}^m(C)$ such that $\text{dec}(D) = m_i$. Moreover, if there exists no $k \in \{0, \dots, n\}$ such that $C_k \xrightarrow{\checkmark}$ then for every $j \in \{0, \dots, l\}$ we have that m_j is not a success marking.*

Proof. The proof is by induction on the length n of the derivation $C = C_0 \longrightarrow C_1 \longrightarrow \dots \longrightarrow C_n = D$. The base case ($n = 0$) is trivial. In the inductive case there are two possible cases: C_{n-1} is stable or it is not stable. In the first case the proof is straightforward. In the second case, there are two possible scenarios to be considered: either C_{n-1} contains an unguarded action $\text{reply}(r, b)$ term, or it contains a pair of matching terms $\bar{r}(b)$ and $r(b)$ (but not both). We consider the first of these two cases, the second one can be treated similarly.

Let C_{n-1} be a non stable contract containing an unguarded action $\text{reply}(r, b)$. This action cannot appear unguarded in the initial contract C : let C_j , with $j > 0$, be the first contract traversed during the computation of C in which the action $\text{reply}(r, b)$ appears unguarded. Hence, we have that $C_{j-1} \longrightarrow C_j$ consists of the execution of a receive action. We now consider a different computation from C to D obtained by rearranging the order of the steps in the considered computation $C = C_0 \longrightarrow C_1 \longrightarrow \dots \longrightarrow C_n = D$. Namely, let $C = C_0 \longrightarrow C_1 \longrightarrow \dots \longrightarrow C_{j-1} \longrightarrow C'_j \longrightarrow \dots \longrightarrow C'_{n-2} \longrightarrow C_{n-1} \longrightarrow C_n = D$ be the computation obtained by delaying as much as possible the execution of the receive action generating the unguarded action $\text{reply}(r, b)$. In the new computation, this action appears for the first time in the contract C_{n-1} . Moreover, C'_{n-2} must be a stable contract otherwise C_n is not stable. Hence, we can straightforwardly prove the thesis by applying the inductive hypothesis to the shorter computation $C = C_0 \longrightarrow C_1 \longrightarrow \dots \longrightarrow C_{j-1} \longrightarrow C'_j \longrightarrow \dots \longrightarrow C'_{n-2}$ leading to the stable contract C'_{n-2} . \square

As a simple corollary of the last two theorems, we have that client-service compliance is preserved by the Petri net semantics.

Corollary 3.12 (Compliance Preservation). *Let C and S be respectively a WSCL client and service contract, as in Definition 2.1. We have that C is compliant with S if and only if in the marked Petri net $\text{Net}^m(C|S)$ all the maximal computations traverse at least one success marking.*

Proof. (\Rightarrow) Trivial by Theorem 3.10.

(\Leftarrow) Suppose that in $\text{Net}^m(C|S)$ all the maximal computations traverse at least one success marking and suppose by contradiction that C is not compliant with S . This means that there is a maximal computation from $C|S$ that does not traverse a state D such that $D \xrightarrow{\checkmark}$. This computation can either end in a state D' with no outgoing reductions or can be infinite.

In the first case we get a contradiction by Theorem 3.11. Indeed there would be a maximal computation from $\text{Net}^m(C|S)$ traversing only non-success markings.

Consider the second case. From the infinite sequence of reductions, we can build an infinite set of maximal computations of arbitrary length, starting from C and ending in a stable state (Lemma 2.9) without traversing a success state. By Theorem 3.11, for each of these maximal computations there exists a corresponding maximal computation in the net $\text{Net}^m(S|C)$ that does not traverse a success marking. We can arrange these computations so as to form a tree where m' is a child of m iff $m[\]m'$: this is an infinite, but finitely-branching, tree. By König's lemma, in $\text{Net}^m(S|C)$ there exists then an infinite computation that does not traverse a success marking and we get a contradiction. \square

3.2. Verifying client-service compliance using the Petri net semantics

In the light of Corollary 3.12, checking whether C is compliant with S reduces to verifying if all the maximal computations in $\text{Net}^m(C|S)$ traverse at least one success marking. In order to verify this property, we proceed as follows:

Table 4

An algorithm for checking the guaranteed reachability of success markings.

-
- (1) If the initial marking m_0 is not a success marking then consider a root node, label it with m_0 , and tag it “new”.
 - (2) While “new” nodes exist do the following:
 - (a) Select a “new” node labelled with the marking m (remove the tag “new”).
 - (b) If no transitions are enabled at m , return FALSE.
 - (c) While there exist enabled transitions at m , do the following for each of them:
 - (i) Obtain a marking m' that results from firing the transition.
 - (ii) If on the path from the root to the selected node there exists a marking m'' such that $m' \supseteq m''$ then return FALSE.
 - (iii) If m' is not a success marking introduce a new node labelled with m' , tag it “new”, and draw an arc from the selected node to the new one.
 - (d) Remove the tag “new” from the marking m .
 - (3) Return TRUE.
-

- we first modify the net semantics in such a way that the net computations block if they reach a success marking;
- we define a (terminating) algorithm for checking whether in the modified Petri net all the maximal computations are finite and end in a success marking.

The modified Petri net semantics simply adds one place that initially contains one token. All transitions consume such a token, and reproduce it only if they do not introduce tokens in success places, i.e., places $\sum_{i \in I} G_i$ such that $G_k = \surd$ for some $k \in I$.

Definition 3.13 (*Modified Petri Net Semantics*). Let C be a WSCL contract and $Net(C) = (S, T)$ the corresponding Petri net as defined in Definition 3.5. We define $ModNet(C)$ as the Petri net (S', T') where:

- $S' = S \cup \{run\}$, where run is an additional place;
- for each transition $c \Rightarrow p \in T$, then T' contains a transition that consumes the multiset $c \uplus \{run\}$ and produces either p , if p contains a place $\sum_{i \in I} G_i$ such that $G_k = \surd$ for some $k \in I$, or $p \uplus \{run\}$, otherwise.

The *marked modified net* $ModNet^m(C)$ is defined as the net $ModNet(C)$ with initial marking m_0 where

$$m_0 = \begin{cases} dec(C) \uplus \{run\} & \text{if } dec(C) \text{ is not a success marking} \\ dec(C) & \text{otherwise.} \end{cases}$$

We now state an important relationship between $Net^m(C)$ and $ModNet^m(C)$. It can be proved by relying on the definition of modified net.

Proposition 3.14. *Let C be a WSCL contract, $Net^m(C)$ (resp. $ModNet^m(C)$) the corresponding Petri net (resp. modified Petri net). We have that all the maximal computations of $Net^m(C)$ traverse at least one success marking if and only if in $ModNet^m(C)$ all the maximal computations are finite and end in a success marking.*

We now present the algorithm for checking whether in a Petri net all the maximal computations are finite and end in a success marking. In the algorithm and in the proof, we utilize the usual *covering* preorder over multisets on $Places(C)$: namely, $m \subseteq m'$ iff for each p , $m(p) \leq m'(p)$. It is well known by Highman’s Lemma [19] that this preorder, corresponding to the extension to multisets of the equality relation on a finite set (the set of places), is a *well-quasi-order*, that is, in any infinite sequence of multisets m_0, m_1, \dots there is a pair of multisets m_i and m_j , with $i < j$, such that $m_i \subseteq m_j$.

Theorem 3.15. *Let C be a WSCL contract as in Definition 2.1 and let $ModNet^m(C) = (S, T, m_0)$ be the corresponding modified Petri net. The algorithm described in Table 4 always terminates. Moreover, it returns TRUE iff all the maximal computations in $ModNet^m(C)$ are finite and end in a success marking.*

Proof. Suppose by contradiction that the algorithm does not terminate. This means that there exists an infinite computation from m_0 of the form $m_0 \llbracket m_1 \llbracket \dots \llbracket m_n \llbracket \dots$ such that, for each m_i : (i) m_i is not a success marking and (ii) for no m_j , with $0 \leq j < i$, it holds that $m_j \subseteq m_i$.

The last assertion implies that there exists an infinite sequence of elements in S that are not related by the preorder \subseteq , and this would violate the fact \subseteq is a well-quasi-order.

Assume now that the algorithm returns FALSE. This may happen at (b) or at (c)-ii. In the first case, we have found a maximal computation ending at m and not traversing a success state. In the second case, it is easy to see that we can build computations of arbitrary length that do not traverse success, again implying the existence of an infinite unsuccessful computation (via König’s lemma). The case when the algorithm returns TRUE is obvious. \square

4. Undecidability of client-service compliance for BPEL contracts

We now move to the proof that client-service compliance is undecidable for BPEL contracts. The proof is by reduction from the termination problem in Random Access Machines (RAMs) [27], a well known Turing powerful formalism based on

registers containing nonnegative natural numbers. The registers are used by a program, that is a set of indexed instructions I_i which are of two possible kinds:

- $i : Inc(r_j)$ that increments the register r_j and then moves to the execution of the instruction with index $i + 1$ and
- $i : DecJump(r_j, s)$ that attempts to decrement the register r_j ; if the register does not hold 0 then the register is actually decremented and the next instruction is the one with index $i + 1$, otherwise the next instruction is the one with index s .

Without loss of generality we assume that given a program I_1, \dots, I_n , it starts by executing I_1 with all the registers empty (i.e., all registers contain 0) and terminates when trying to perform the first undefined instruction I_{n+1} .

In order to simplify the notation, in this section we introduce a notation corresponding to standard input and output prefixes³ of CCS [25]. Namely, we model simple synchronization as a request–response interaction in which there is only one possible reply message. Assuming that this unique reply message is *ok* (with $ok \in Names$ not necessarily fresh), we introduce the following notation:

$$\bar{a}.P = \text{invoke}(a, ok.P) \quad a.P = \text{receive}(a).\text{reply}(a, ok).P.$$

In order to reduce RAM termination to client–service compliance, we define a client contract that simulates the execution of a RAM program, and a service contract that represent the registers, such that the client contract reaches the success \surd and only if the RAM program terminates.

Given a RAM program I_1, \dots, I_n , we consider the client contract C as follows

$$C \triangleq \prod_{i \in \{1, \dots, n\}} \llbracket I_i \rrbracket \mid \text{inst}_{n+1} \cdot \surd$$

$$\llbracket I_i \rrbracket \triangleq \begin{cases} \text{rec}X.(\text{inst}_i.\overline{\text{inc}}_j.\text{ack}.\overline{(\text{inst}_{i+1} \mid X)}) & \text{if } I_i = (i : \text{Inc}(r_j)) \\ \text{rec}X.(\text{inst}_i.\overline{\text{dec}}_j.(\text{ack}.\overline{(\text{inst}_{i+1} \mid X)} + \text{zero}.\overline{(\text{inst}_s \mid X)})) & \text{if } I_i = (i : \text{DecJump}(r_j, s)). \end{cases}$$

An increment instruction $Inc(r_j)$ is modelled by a recursive contract that invokes the operation inc_j , waits for an acknowledgement on ack , and then invokes the service corresponding to the subsequent instruction. On the contrary, a decrement instruction $DecJump(r_j, s)$ invokes the operation dec_j and then waits on two possible operations: ack or zero . In the first case the service corresponding to the subsequent instruction with index $i + 1$ is invoked, while in the second case the service corresponding to the target of the jump is invoked instead.

We now move to the modelling of the registers. Each register r_j is represented by a contract representing the initially empty register in parallel with a service modelling every unit subsequently added to the register

$$\llbracket r_j \rrbracket \triangleq \text{rec}X.(\text{dec}_j.\overline{\text{zero}}.X + \text{inc}_j.\overline{\text{unit}}_j.\text{invoke}(u_j, ok.\overline{\text{ack}}.X)) \mid \text{UNIT}_j$$

$$\text{UNIT}_j \triangleq \text{rec}X.\text{unit}_j.(X \mid \text{receive}(u_j).\overline{\text{ack}}.\text{rec}Y.(\text{dec}_j.\text{reply}(u_j, ok) + \text{inc}_j.\overline{\text{unit}}_j.\text{invoke}(u_j, ok.\overline{\text{ack}}.Y)).$$

The idea of the encoding is to model numbers with chains of nested request–response interactions. When a register is incremented, a new instance of a contract is spawn invoking the operation unit_j , and a request–response interaction is opened between the previous instance and the new one. In this way, the previous instance blocks waiting for the reply. When an active instance receives a request for decrement, it terminates by closing the request–response interaction with its previous instance, which is then re-activated. The contract that is initially active represents the empty register because it replies to decrement requests by performing an invocation on the *zero* operation.

We extend structural congruence \equiv , introduced in Section 3, to \equiv_{ren} to admit the injective renaming of the operation name

$$C \equiv_{ren} D \text{ if there exists an injective renaming } \sigma \text{ such that } C\sigma \equiv D.$$

Clearly, injective renaming is an equivalence and preserves the operational semantics.

Proposition 4.1. *Let C and D be two contract systems such that $C \equiv_{ren} D$. If $C \xrightarrow{\alpha} C'$, then there exists D' and a label α' obtained by renaming the operation names in α such that $D \xrightarrow{\alpha'} D'$ and $C' \equiv_{ren} D'$.*

Now, we introduce $\llbracket r_j, c \rrbracket$ that we use to denote the modelling of the register r_j when it holds the value c . Namely, $\llbracket r_j, 0 \rrbracket = \llbracket r_j \rrbracket$, while if $c > 0$ then

$$\llbracket r_j, c \rrbracket = \begin{cases} b_0(ok).\overline{\text{ack}}.\text{rec}X.(\text{dec}_j.\overline{\text{zero}}.X + \text{inc}_j.\overline{\text{unit}}_j.\text{invoke}(u_j, ok.\overline{\text{ack}}.X)) \mid \\ b_1(ok).\overline{\text{ack}}.\text{rec}Y.(\text{dec}_j.\text{reply}(b_0, ok) + \text{inc}_j.\overline{\text{unit}}_j.\text{invoke}(u_j, ok.\overline{\text{ack}}.Y)) \mid \\ \dots \mid \\ b_{c-1}(ok).\overline{\text{ack}}.\text{rec}Y.(\text{dec}_j.\text{reply}(b_{c-2}, ok) + \text{inc}_j.\overline{\text{unit}}_j.\text{invoke}(u_j, ok.\overline{\text{ack}}.Y)) \mid \\ \text{rec}Y.(\text{dec}_j.\text{reply}(b_{c-1}, ok) + \text{inc}_j.\overline{\text{unit}}_j.\text{invoke}(u_j, ok.\overline{\text{ack}}.Y)) \mid \\ \text{UNIT}_j. \end{cases}$$

³ The input and output prefixes correspond also to the representation of the one-way interaction pattern in contract languages such as those in [13, 14].

In the following theorem, stating the correctness of our encoding, we use the following notation: (i, c_1, \dots, c_m) to denote the state of a RAM in which the next instruction to be executed is I_i and the registers r_1, \dots, r_m respectively contain the values c_1, \dots, c_m , and $(i, c_1, \dots, c_m) \rightarrow_R (i', c'_1, \dots, c'_m)$ to denote the change of the state of the RAM R due to the execution of the instruction I_i .

Theorem 4.2. Consider a RAM R with instructions I_1, \dots, I_n and registers r_1, \dots, r_m . Consider also a state (i, c_1, \dots, c_m) of the RAM R and a corresponding contract C such that $C \equiv_{ren} \overline{inst_i} | \llbracket I_i \rrbracket | \dots | \llbracket I_n \rrbracket | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}}$. We have that

- either the RAM computation has terminated, thus $i = n + 1$
- or $(i, c_1, \dots, c_m) \rightarrow_R (i', c'_1, \dots, c'_m)$ and there exists $l > 0$ such that $C \longrightarrow C_1 \longrightarrow \dots \longrightarrow C_l$ and
 - $C_l \equiv_{ren} \overline{inst_{i'}} | \llbracket I_1 \rrbracket | \dots | \llbracket I_n \rrbracket | inst_{n+1} \cdot \sqrt{\{\{r_1, c'_1\}\} \dots \{\{r_m, c'_m\}\}}$
 - for each k ($1 \leq k < l$): $C_k \not\rightarrow$.

Proof. Suppose $i \neq n + 1$. The proof proceeds by distinguishing two cases depending on the instruction i :

$i : Inc(r_j)$: for the sake of simplicity, suppose $r_j = 0$. Then $(i, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m) \rightarrow_R (i + 1, c_1, \dots, c_{j-1}, 1, c_{j+1}, \dots, c_m)$.

The contract C corresponding to $(i, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m)$ is:

$$C \equiv_{ren} D \triangleq \overline{inst_i} | \dots | recX.(inst_i \cdot \overline{inc_j} \cdot ack.(\overline{inst_{i+1}} | X)) | \dots | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}} \\ | recX.(dec_j \cdot \overline{zero} \cdot X + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot X)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\}$$

and

$$D \rightarrow^* \dots | ack.(\overline{inst_{i+1}} | \llbracket I_i \rrbracket) | \dots | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}} \\ | \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot recX.(dec_j \cdot \overline{zero} \cdot X + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot X))) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\} \\ \rightarrow^* \dots | ack.(\overline{inst_{i+1}} | \llbracket I_i \rrbracket) | \dots | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}} \\ | r(ok) \cdot \overline{ack} \cdot recX.(dec_j \cdot \overline{zero} \cdot X + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot X)) \\ | \overline{ack} \cdot recY.(dec_j \cdot \overline{reply}(r, ok) + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot Y)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\} \\ \rightarrow \dots | \overline{inst_{i+1}} | \llbracket I_i \rrbracket | \dots | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}} \\ | r(ok) \cdot \overline{ack} \cdot recX.(dec_j \cdot \overline{zero} \cdot X + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot X)) \\ | recY.(dec_j \cdot \overline{reply}(r, ok) + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot Y)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\} \triangleq D'$$

where D' corresponds to the state $(i + 1, c_1, \dots, c_{j-1}, 1, c_{j+1}, \dots, c_m)$ and clearly each D' in the derivation from D to D' cannot perform a successful transition. Therefore, by Proposition 4.1, $C \rightarrow^* C'$ with $C' \equiv_{ren} D'$.

$i : DecJump(r_j, s)$: suppose again that $r_j = 0$. Then, $(i, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m) \rightarrow_R (s, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m)$.

The contract C corresponding to $(i, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m)$ is:

$$C \equiv_{ren} D \triangleq \overline{inst_i} | \dots | recX.(inst_i \cdot \overline{dec_j} \cdot (ack.(\overline{inst_{i+1}} | X) + zero.(\overline{inst_s} | X))) | \dots | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}} \\ | \{\{r_1, c_1\}\} | \dots | recX.(dec_j \cdot \overline{zero} \cdot X + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot X)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\}$$

and

$$D \rightarrow^* \dots | ack.(\overline{inst_{i+1}} | \llbracket I_i \rrbracket) + zero.(\overline{inst_s} | \llbracket I_i \rrbracket) | \dots | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}} \\ | \overline{zero} \cdot recX.(dec_j \cdot \overline{zero} \cdot X + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot X)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\} \\ \rightarrow^* \overline{inst_s} | \dots | \llbracket I_i \rrbracket | \dots | inst_{n+1} \cdot \sqrt{\{\{r_1, c_1\}\} \dots \{\{r_m, c_m\}\}} \\ | recX.(dec_j \cdot \overline{zero} \cdot X + inc_j \cdot \overline{unit_j} \cdot invoke(u_j, ok \cdot \overline{ack} \cdot X)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\} \triangleq D'$$

where D' corresponds to the state $(s, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m)$ and clearly each D' in the derivation from D to D' cannot perform a successful transition. Again, by Proposition 4.1, $C \rightarrow^* C'$ with $C' \equiv_{ren} D'$.

Suppose now that $r_j \neq 0$, e.g. let $r_j = 1$. Then

$$(i, c_1, \dots, c_{j-1}, 1, c_{j+1}, \dots, c_m) \rightarrow_R (i + 1, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m).$$

The contract C corresponding to $(i, c_1, \dots, c_{j-1}, 1, c_{j+1}, \dots, c_m)$ is:

$$C \equiv_{ren} D \triangleq \overline{inst}_i | \dots | recX.(inst_i.\overline{dec}_j.(ack.(\overline{inst}_{i+1}|X) + zero.(\overline{inst}_s|X))) | \dots | inst_{n+1}.\checkmark \\ | \{\{r_1, c_1\}\} | \dots \\ | r(ok).\overline{ack}.recX.(dec_j.\overline{zero}.X + inc_j.\overline{unit}_j.invoke(u_j,ok.\overline{ack}.X)) \\ | recY.(dec_j.reply(r,ok) + inc_j.\overline{unit}_j.invoke(u_j,ok.\overline{ack}.Y)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\}$$

and

$$D \rightarrow^* \dots | ack.(\overline{inst}_{i+1}|\llbracket I_i \rrbracket) + zero.(\overline{inst}_s|\llbracket I_i \rrbracket) | \dots | inst_{n+1}.\checkmark | \{\{r_1, c_1\}\} | \dots \\ | r(ok).\overline{ack}.recX.(dec_j.\overline{zero}.X + inc_j.\overline{unit}_j.invoke(u_j,ok.\overline{ack}.X)) \\ | reply(r,ok) | UNIT_j | \dots | \{\{r_m, c_m\}\} \\ \rightarrow \dots | ack.(\overline{inst}_{i+1}|\llbracket I_i \rrbracket) + zero.(\overline{inst}_s|\llbracket I_i \rrbracket) | \dots | inst_{n+1}.\checkmark | \{\{r_1, c_1\}\} | \dots \\ | r(ok).\overline{ack}.recX.(dec_j.\overline{zero}.X + inc_j.\overline{unit}_j.invoke(u_j,ok.\overline{ack}.X)) \\ | \bar{r}(ok) | UNIT_j | \dots | \{\{r_m, c_m\}\} \\ \rightarrow \dots | ack.(\overline{inst}_{i+1}|\llbracket I_i \rrbracket) + zero.(\overline{inst}_s|\llbracket I_i \rrbracket) | \dots | inst_{n+1}.\checkmark | \{\{r_1, c_1\}\} | \dots \\ | \overline{ack}.recX.(dec_j.\overline{zero}.X + inc_j.\overline{unit}_j.invoke(u_j,ok.\overline{ack}.X)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\} \\ \rightarrow \dots | \overline{inst}_{i+1}|\llbracket I_i \rrbracket | \dots | inst_{n+1}.\checkmark | \{\{r_1, c_1\}\} | \dots \\ | recX.(dec_j.\overline{zero}.X + inc_j.\overline{unit}_j.invoke(u_j,ok.\overline{ack}.X)) \\ | UNIT_j | \dots | \{\{r_m, c_m\}\} \triangleq D'$$

where D' corresponds to the state $(i + 1, c_1, \dots, c_{j-1}, 0, c_{j+1}, \dots, c_m)$ and clearly each D' in the derivation from D to D' cannot perform a successful transition. As before, by Proposition 4.1, $C \rightarrow^* C'$ with $C' \equiv_{ren} D'$. \square

As a corollary we get that client-service compliance is undecidable.

Corollary 4.3. Consider a RAM R with instructions I_1, \dots, I_n and registers r_1, \dots, r_m . Consider the client contract $C = \overline{inst}_1|\llbracket I_1 \rrbracket | \dots | \llbracket I_n \rrbracket | inst_{n+1}.\checkmark$ and the service contract $S = \{\{r_1, 0\}\} | \dots | \{\{r_m, 0\}\}$. We have that C is compliant with S if and only if R terminates.

Proof. The proof proceeds by proving that both directions hold in the most general case: $C|S \equiv_{ren} \overline{inst}_1|\llbracket I_1 \rrbracket | \dots | \llbracket I_n \rrbracket | inst_{n+1}.\checkmark | \{\{r_1, 0\}\} | \dots | \{\{r_m, 0\}\}$.

(\Leftarrow) Suppose R terminates. Theorem 4.2 can be applied to guarantee that C and S are compliant. The proof proceeds by induction on the number n of steps needed by R to terminate.

Suppose $n = 0$, hence R has terminated. In this case $C \equiv_{ren} \overline{inst}_{n+1}|\llbracket I_1 \rrbracket | \dots | \llbracket I_n \rrbracket | inst_{n+1}.\checkmark$, hence (Proposition 4.1) there exists exactly one computation from $C|S$ as below.

$$C|S \rightarrow_{\equiv_{ren}} \llbracket I_1 \rrbracket | \dots | \llbracket I_n \rrbracket | \checkmark | S \xrightarrow{\checkmark} .$$

Clearly, this computation guarantees the compliance of C and S .

Suppose now $n > 0$ and $(i, c_1, \dots, c_m) \rightarrow_R (i', c'_1, \dots, c'_m) \triangleq R'$. Theorem 4.2 and Proposition 4.1 guarantee that there exists $l > 0$ such that $C|S \rightarrow D_1 \rightarrow \dots \rightarrow D_l$ and $D_l \equiv_{ren} \overline{inst}_{i'}|\llbracket I_1 \rrbracket | \dots | \llbracket I_n \rrbracket | inst_{n+1}.\checkmark | \{\{r_1, c'_1\}\} | \dots | \{\{r_m, c'_m\}\}$ and that $D_k \not\xrightarrow{\checkmark}$, for any $1 \leq k \leq l$. By looking at the proof of the theorem, it is also clear that any D_k cannot originate other transitions, apart from that already considered in the computation above. Therefore, D can only evolve into D_l and then, by applying the induction hypothesis to R' , it follows that C and S are compliant.

(\Rightarrow) Suppose now that C and S are compliant. To prove that R terminates it is sufficient to suppose, by contradiction, that it is not the case. By Theorem 4.2, this implies that there exists an infinite (hence maximal) computation $C|S \rightarrow$

$D_1 \rightarrow \dots \rightarrow D_l \rightarrow \dots$ where $D_k \not\xrightarrow{\checkmark}$, for any k . This, by Theorem 4.2, contradicts the hypothesis that C and S are compliant. \square

5. Mutual compliance

In Section 2, we have introduced a notion of compliance based on client's satisfaction: whenever the client reaches a success state the whole system is successful. Hence, the success of the system is established by ignoring what happens on the service side. This could leave the service in an inconsistent state. Let us consider for example a service demanding an additional confirmation from the client before executing the required task. In case the client decides to abandon the session before sending this final approval the (current instance of the) service is blocked. This situation could require the usage of

timeout mechanisms, especially in the presence of recursive services having at most only one active instance at a time (this could be the case e.g. when the service accesses critical data). A concrete example could be an e-banking service demanding the executive password of the client before performing any operation, e.g. bank transfer, shares purchase, . . . , as below.

$$\begin{aligned}
C &\triangleq \text{invoke}(e - \text{bank}, \text{ok}, \text{recreply}(\text{login}, \text{log_data}, C')) \\
C' &\triangleq \text{invoke}(\text{transfer}, \text{ok}, \text{recreply}(\text{send_data}, \text{tran_data}, \surd)) \\
B &\triangleq \text{recreply}(e - \text{bank}, \text{ok}, \text{invoke}(\text{login}, \text{log_data}, B')) \\
B' &\triangleq \text{recreply}(\text{transfer}, \text{ok}, \text{invoke}(\text{send_data}, \text{tran_data}, \\
&\quad \text{invoke}(\text{confirm}, \text{pw}) + \text{recreply}(\text{abort}, \text{ok}))) \\
&\quad + \text{recreply}(\text{other}, \text{ok}, B'') \\
B'' &\triangleq \dots
\end{aligned}$$

It is easy to see that when the client decides to “abandon” the request without notifying the service ($\text{tran_data}, \surd$), a pending session rests on the service side waiting for the confirmation ($\text{invoke}(\text{confirm}, \text{pw})$) or the abort ($\text{recreply}(\text{abort}, \text{ok})$).

In order to avoid such problems, we introduce another notion of compliance, called *mutual compliance*, where the synchronization of client and service’s success actions is mandatory in order to establish the success of the whole system.

In this section, we modify the syntax and semantics of WSCL contracts to distinguish between clients and services’ successes and introduce the notion of mutual compliance. We then prove that this new notion of compliance is still decidable, by slightly modifying the reasoning of Section 3. We do not consider BPEL contracts as the undecidability result proved in Section 4 easily extends also to mutual compliance. In fact, it is easy to reduce mutual compliance to the notion of client-service compliance in Section 4 that considers only the client’s success action \surd : it is sufficient to model mutual success as a synchronization between the client and the service, and only after such a synchronization the client executes its action \surd .

5.1. WSCL contracts and mutual compliance

Guarded contracts are defined as follows:

$$G ::= \text{invoke}\left(a, \sum_{i \in I} b_i \cdot C_i\right) \mid \text{recreply}\left(a, \sum_{i \in I} b_i \cdot C_i\right) \mid \surd_c \mid \surd_s$$

where \surd_c and \surd_s denote the success of the client and the service, respectively.

The *run-time* syntax of contracts extends the syntax introduced in Definition 2.1 in order to take into account the occurred synchronization of \surd_c and \surd_s :

$$C ::= \dots \mid \surd.$$

A *client contract* is a contract C containing at least one occurrence of the guarded contract \surd_c and no occurrences of \surd_s and \surd , while a *service contract* is a contract S containing at least one occurrence of the guarded contract \surd_s and no occurrences of \surd_c and \surd .

The operational semantics of contracts is extended, as expected, by adding to the rules in Definition 2.3 the following ones:

$$\begin{array}{c}
\surd_c \xrightarrow{\surd_c} \mathbf{0} \quad \surd_s \xrightarrow{\surd_s} \mathbf{0} \quad \frac{C \xrightarrow{\surd_c} C' \quad S \xrightarrow{\surd_s} S'}{C|S \xrightarrow{\tau} C'|S'|\surd}
\end{array}$$

Mutual compliance is defined in the same way as client-service compliance over the modified transition system and, as before, it makes sense only in case of dyadic communications and cannot be applied in a multi-party setting.

Definition 5.1 (Mutual Compliance). A client contract C and a service contract S are *mutually compliant* if for every maximal computation $C|S \longrightarrow D_1 \longrightarrow \dots \longrightarrow D_l \longrightarrow \dots$ there exists k such that $D_k \xrightarrow{\surd}$.

Notice that, with the new semantics, $D_k \xrightarrow{\surd}$ in the definition above implies a previous synchronization in the computation of \surd_c and \surd_s .

Example 5.2 (An e-bank Service). Consider the e-bank process B introduced at the beginning of this section and another version of the client, D , that confirms the execution of the bank transfer before exiting the session. The two processes are

reported below.

$$\begin{aligned}
D &\triangleq \text{invoke}(e - \text{bank}, \text{ok}, \text{recreply}(\text{login}, \text{log_data}, D')) \\
D' &\triangleq \text{invoke}(\text{transfer}, \text{ok}, \text{recreply}(\text{send_data}, \text{tran_data}, \text{recreply}(\text{confirm}, \text{pw}, \sqrt{c}))) \\
B &\triangleq \text{recreply}(e - \text{bank}, \text{ok}, \text{invoke}(\text{login}, \text{log_data}, B')) \\
B' &\triangleq \text{recreply}(\text{transfer}, \text{ok}, \text{invoke}(\text{send_data}, \text{tran_data}, \\
&\quad \text{invoke}(\text{confirm}, \text{pw}, \sqrt{s}) + \text{recreply}(\text{abort}, \text{ok}, \sqrt{s}))) \\
&\quad + \text{recreply}(\text{other}, \text{ok}, B'').
\end{aligned}$$

There is a sole computation from $D|B$, reported below, which guarantees mutual compliance of the two.

$$\begin{aligned}
D|B &\longrightarrow^* \text{recreply}(\text{login}, \text{log_data}, D') \mid \text{invoke}(\text{login}, \text{log_data}, B') \\
&\longrightarrow^* D' \mid B' \\
&\longrightarrow^* \text{recreply}(\text{send_data}, \text{tran_data}, \text{recreply}(\text{confirm}, \text{pw}, \sqrt{c})) \\
&\quad \mid \text{invoke}(\text{send_data}, \text{tran_data}, \text{invoke}(\text{confirm}, \text{pw}, \sqrt{s}) + \text{recreply}(\text{abort}, \text{ok}, \sqrt{s})) \\
&\longrightarrow^* \text{recreply}(\text{confirm}, \text{pw}, \sqrt{c}) \mid \text{invoke}(\text{confirm}, \text{pw}, \sqrt{s}) + \text{recreply}(\text{abort}, \text{ok}, \sqrt{s}) \\
&\longrightarrow^* \sqrt{c} \mid \sqrt{s} \\
&\longrightarrow \sqrt{\quad} \longrightarrow.
\end{aligned}$$

5.2. Decidability of mutual compliance

As before, decidability is obtained by translating WSCL contracts into Petri nets. The definition of the Petri net associated to a contract is essentially the same as in Section 3, except for the presence of new places for \sqrt{c} and \sqrt{s} and of a new transition allowing the synchronization of the two and leading to the success state labelled by $\sqrt{\quad}$, as below.

$$\left\{ \sum_{i \in I} G_i, \sum_{j \in J} G_j \right\} \Rightarrow \{\sqrt{\quad}\} \quad \text{if } G_k = \sqrt{c} \text{ and } G_l = \sqrt{s} \text{ for some } k \in I \text{ and } l \in J.$$

Example 5.3. Consider the e-bank service B and the client D from Example 5.2. The marked net $\text{Net}^m(B|D)$ is depicted in Fig. 1.

In the remaining part of the section, we prove that soundness and completeness of the translation still hold and that mutual compliance is preserved by the translation.

The relationship between markings and contracts introduced in Lemma 3.8 needs to be modified by adding a fourth item as below.

Lemma 5.4 (Extension of Lemma 3.8). *Let C be a WSCL contract system and suppose $\text{dec}(C) = m$. The following holds:*

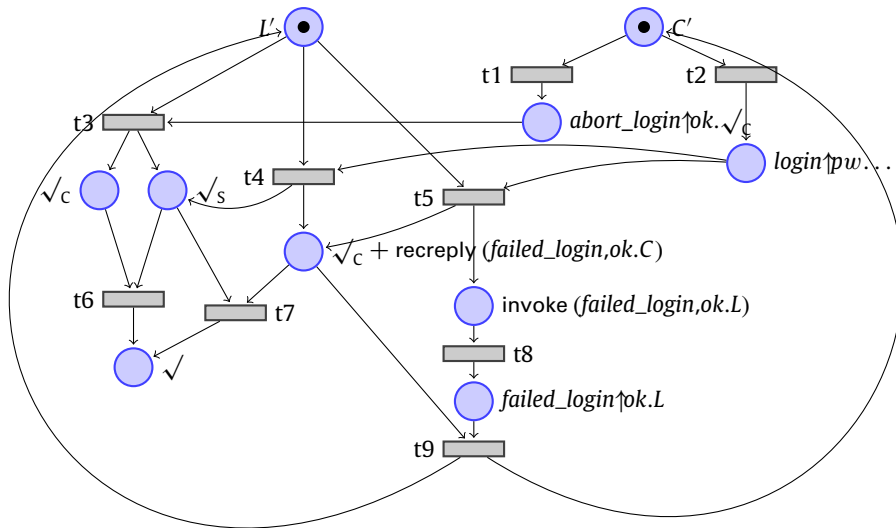
- (1) if $m = \{\sum_{i \in I} G_i\} \oplus m'$ then $C \equiv \sum_{i \in I} G_i \mid D$, for some D such that $\text{dec}(D) = m'$;
- (2) if $m = \{a \uparrow \sum_{j \in J} b_j.C_j\} \oplus m'$ then $C \equiv \sum_{j \in J} r \langle b_j \rangle.C_j \mid \bar{a}(r) \mid D$, for some D and r such that $r \notin \text{Names}(D)$ and $\text{dec}(D) = m'$;
- (3) if $m = \{c \downarrow \sum_{j \in J} b_j.C_j\} \oplus m'$ then $c \neq b_j$ for each $j \in J$ and $C \equiv \sum_{j \in J} r \langle b_j \rangle.C_j \mid \bar{r}(c) \mid D$, for some D and r such that $r \notin \text{Names}(D)$ and $\text{dec}(D) = m'$;
- (4) if $m = \{\sqrt{\quad}\} \oplus m'$ then $C \equiv \sqrt{\quad} \mid D$, for some D such that $\text{dec}(D) = m'$.

The remaining propositions and theorems are still valid; little changes in the proofs of Proposition 3.9 and Theorem 3.10 are needed. In case of Proposition 3.9, it is sufficient to extend the proof of (\Rightarrow) by adding another item guaranteeing the absence of synchronization of \sqrt{c} and \sqrt{s} . In case of Theorem 3.10, it is necessary to extend the proof by considering the new kind of net transition. In both cases, the changes are minimal and easy to adjust, and the whole proofs are omitted.

The following version of Corollary 3.12 carries over.

Corollary 5.5 (Mutual Compliance Preservation). *Let C and S be respectively a WSCL client and service contract, as defined in Section 5.1. We have that C and S are mutually compliant if and only if in the marked Petri net $\text{Net}^m(C|S)$ all the maximal computations traverse at least one success marking.*

This result is essentially the same as that reported in Corollary 3.12; therefore the reasoning introduced in Section 3.2 applies to the new notion of compliance: the algorithm introduced in Table 4 together with Theorem 3.15 guarantee the decidability of mutual compliance.

Fig. 2. $\text{Net}^m(L'|C')$.

model, even if unboundedly many instances of client and services can be spawned, it is not possible to generate a chain of nested interactions of unbounded depth.

This paper is in the line of recent research dedicated to the formal analysis of service behavioural contracts exploiting process calculi. To the best of our knowledge, though, only one-way operations have been considered so far. An initial theory of contracts for client-service interaction has been proposed by Carpineti et al. [13] and then independently extended along different directions by Bravetti and Zavattaro (see e.g. [8]) by Laneve and Padovani [24], and by Castagna et al. [14]. The main objective of those papers was to define a *subcontract* relation suitable to check the replaceability of one service with another one without affecting the correctness of a modelled system. The approach in [13] considers a notion of system correctness similar to the one used in this paper and inspired by must-testing. A corresponding *subcontract* relation, enhancing the must-testing preorder, is defined in [24]. By making use of explicit *interfaces* indicating the operations used by one service to interact with the external environment, both *in-width* and *in-depth* refinements are admitted: a subcontract can have additional behaviour, available either as new choices in branches or as longer continuations, but only if this additional behaviour is activated by actions on operations that are not in the interface. A notion of correctness in which all the involved partners should eventually reach successful completion (similar to the mutual compliance considered in this paper) has been presented in [8], where a corresponding subcontract relation is also introduced.

Similar to contracts, session types [20] are devoted to the analysis of the dialogues among clients and servers, or more generically, among the participants of any interaction. Several works on session types consider dyadic interactions (see e.g. [32] and references therein), in such cases the so called *duality* of types guarantees the correctness of the communications, hence the compliance of the two partners. Other works (see e.g. the series of work starting from [21] which is discussed below) consider multiparty interactions, i.e. interactions involving multiple participants. In this case, correctness of the protocol is guaranteed by a *projection* function, which will be discussed in the following. In both dyadic and multiparty session types, a *subtyping* relation plays the same role as the subcontract one: it guarantees the replaceability of a participant with another one without affecting the interaction in a sensible way.

The global completion approach is particularly appropriate for systems where there is no clear distinction between clients and services as in the so called *service choreographies*. A formal study of the relationship between choreography languages (such as WS-CDL [33]) and process calculi has been performed by Carbone, Honda and Yoshida. In [12], they introduce a π -calculus like choreography language in which the basic atom is the interaction among two distinct partners, and basic atoms can be combined with the usual sequential, choice and composition operators. By exploiting the theory of session types, they prove the correctness of a projection algorithm that extracts from a global choreographic description the behaviour of each involved partner expressed in a calculus more similar to the π -calculus in which there are two kinds of basic atoms representing input and output actions separately. In [21], they extend their work by considering asynchronous communication, thus moving to a setting closer to ours. Different from our calculi, in their language the full power of the π -calculus communication paradigm can be used, thus allowing for the generation and communication of fresh channels. In our calculi, on the contrary, this possibility is constrained by the request–response pattern.

The relationship between contract theories and choreography languages has been investigated in [10]. In all the above theories, the defined subcontract relation is influenced by the operations that a service can use to interact with the external environment, as invocations on these operations could activate the additional behaviour available in refinements. A different approach is taken in [14], where dynamic filters are automatically synthesized in order to guarantee that such an additional behaviour cannot be wrongly activated. A slightly different approach, reflecting the choreography language BPEL4Chor [15], has been considered by researchers coming from the Petri nets area (see, for instance, [1]): each participant is represented

by an open workflow net (a special class of Petri nets) representing only the *public view* of the partner behaviour, and a choreography is obtained as the composition of the descriptions of the involved partners. Despite the different modelling approach, a notion of correctness similar to our *mutual compliance* is considered in which all the partners should reach a final state. Moreover a contract refinement theory is defined (and named *accordance*) to check whether the *private view* of a service conforms with the public view of a choreography participant. In [1], different from our setting, there is no direct representation of the request–response pattern and of the corresponding ability to run multiple instances of the same partner.

It is worth mentioning that similar decidability and undecidability results have been proved also in the context of coordination languages, see [11] for a survey about these results. Coordination languages allow concurrent processes to interact through shared data spaces in which messages can be inserted and retrieved via coordination primitives. This communication paradigm is radically different from Web Services, where interaction is via uni-directional and bi-directional invocations of operations. Random Access Machines and Petri nets have been used also in the context of coordination languages, but rather different reduction techniques have been used. Concerning Random Access Machines, some coordination languages includes explicit test-for-absence operations that makes the encoding of test-for-zero trivial. In languages without test-for-absence primitives, very specific nondeterministic encodings have been defined using, for instance, publish–subscribe coordination mechanisms. Also the encodings into Petri nets are rather different as coordination languages simply require the representation of shared data as tokens in corresponding places, without requiring the representation of the binding between a client and the corresponding service instance.

As for future work, we plan to investigate the complexity of our decision procedure. Given that we rely on results based on the well-quasi order theory, a high (exponential) complexity is expected. We also plan to investigate the (un)decidability of other definitions of compliance present in the literature. In fact, the must-testing approach – the one that we consider in this paper – has been adopted in early works about service compliance (see e.g. [13]). More recent papers consider more sophisticated notions. For instance, the should-testing approach [31] (adopted, e.g., in [8] in the context of process calculi and in [28] for Petri nets) admits also infinite computations if in every reached state there is always at least one path leading to a success state.

It would be interesting to apply the techniques presented in this paper to more sophisticated orchestration languages, like the recently proposed calculi based on the notion of *session* [7,6]. For instance, in [3], a type system is presented ensuring a client progress property – basically, absence of deadlock – in a calculus where interaction between (instances of) the client and the service is tightly controlled via session channels. It would be interesting to check to what extent the decidability techniques presented here apply to this notion of progress. Also connections with *behavioural types* [22,2] deserve attention. In the setting of process calculi, these types are meant to provide behavioural abstractions that are in general more tractable than the original process. In the present paper, the translation function of WSCL contracts into Petri nets can be seen too as a form of behavioural abstraction. In the case of tightly controlled interactions (sessions) [3], BPP processes, a proper subset of Petri nets featuring no synchronization [17], have been seen to be sufficient as abstractions. For general pi-processes, full CCS with restriction is in general needed. One would like to undertake a systematic study of how communication capabilities in the original language (unconstrained interaction vs. sessions vs. request–response vs....) trade off with tractability of the behavioural abstractions (CCS vs. BPP vs. Petri nets vs....).

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