Available online at www.sciencedirect.com
SCIENCE DIRECT®
PHYSICS LETTERS B
Physics Letters B 578 (2004) 443–448
www.elsevier.com/locate/physletb

Newton law in DGP brane-world with semi-infinite extra dimension

D.K. Park [a], S. Tamaryan [b], Yan-Gang Miao [c]

[a] Department of Physics, Kyungnam University, Masan 631-701, South Korea
[b] Theory Department, Yerevan Physics Institute, Yerevan-36 375036, Armenia
[c] Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

Received 7 September 2003; accepted 17 October 2003

Editor: P.V. Landshoff

Abstract

Newton potential for DGP brane-world scenario is examined when the extra dimension is semi-infinite. The final form of the potential involves a self-adjoint extension parameter α, which plays a role of an additional mass (or distance) scale. The striking feature of Newton potential in this setup is that the potential behaves as seven-dimensional in long range when α is non-zero. For small α there is an intermediate range where the potential is five-dimensional. Five-dimensional Newton constant decreases with increase of α from zero. In the short range the four-dimensional behavior is recovered. The physical implication of this result is discussed in the context of the accelerating behavior of universe.

Although the extra-dimensional theories have their own long history [1–3], the recent activities on this field seem to be motivated from string theories [4]. Much cosmological implications are investigated and established from the recent brane-world scenario. Especially Randall–Sundrum (RS) scenario [5,6], one of the recent brane-world scenario developed by making use of the warped extra dimensions, provides an clue for the nature of $1/r$-type Newton potential in our universe.

The original RS computation of Newton potential is extended and developed from various aspects [7–11]. Especially, Refs. [10,11] derived Newton potential arising due to the confined gravity on the brane when bulk space is a single copy of AdS_5 from an aspect of the singular quantum mechanics (SQM). In this case the gravitational fluctuation equation is treated as an usual Schrödinger equation with a singular potential and it should be solved with incorporation of the self-adjoint extension technique [12]. The real parameter, say ξ, introduced in the course of the self-adjoint extension parametrizes the boundary condition (BC) the gravitational fluctuation obeys on the brane.

The SQM approach is, more recently, applied to the RS scenario when $4d$ induced gravity is involved [13,14]. The physical origin of the $4d$ induced gravity is one-loop quantum effect [15–17]. When $\xi = 1/2$ which makes a singular brane to be usual RS brane, the $4d$ induced term generates an intermediate range in which the $5d$ potential $1/r^2$ emerges. Furthermore,

E-mail addresses: dkpark@hep.kyungnam.ac.kr (D.K. Park), sayat@moon.yerphi.am (S. Tamaryan), miao@het.phys.sci.osaka-u.ac.jp (Y.-G. Miao).

0370-2693 © 2003 Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2003.10.058

the other singular branes corresponding to different ξ may trap a massive graviton, leading to Yukawa-like gravitational behavior.

Recently, the brane-world scenario with Minkowski bulk has attracted attention, which is often referred as DGP model [18,19]. The model also involves a $4d$ induced term and recently applied to the cosmological constant hierarchy and the accelerating universe [20–24]. In this Letter we will examine Newton law assigning on the general 3-brane in DGP scenario when the extra dimension is infinite briefly and semi-infinite in detail.

For the case of the infinite extra dimension it is well known that Newton potential is five-dimensional at long range and four-dimensional at short range [18]. We will reproduce this result by applying SQM to the fluctuation equation. For the case of the semi-infinite extra dimension the final form of Newton potential involves a self-adjoint extension parameter α, which makes an additional distance scale when $\alpha \neq 0$. When $\alpha = 0$, Newton potential is similar to that for the case of the infinite extra dimension. However, the $5d$ Newton constant G_5' becomes $G_5' = 2G_5$ where G_5 is $5d$ Newton constant derived when the extra dimension is infinite. The most striking result occurs in the long-range behavior of the potential when $\alpha > 0$. In this range Newton potential becomes seven-dimensional. In the intermediate range the potential is five-dimensional with smaller Newton constant than $2G_5$. The four-dimensional potential is recovered at short range.

Let us start with the Einstein–Hilbert action

$$S = M^3 \int d^4x\, dy \sqrt{-G}\, \tilde{R} + M_p^2 \int d^4x \sqrt{-g}\, R, \quad (1)$$

where M and M_p are $5d$ and $4d$ Planck scale, respectively. The curvature scalars $\tilde{R}$ and R are, respectively, five-dimensional one and four-dimensional one constructed by $5d$ metric tensor $G_{MN}(x, y)$ and $4d$ metric tensor $g_{\mu\nu}(x) \equiv G_{\mu\nu}\ (x, y = 0)$. Of course, the second term in Eq. (1) represents the induced term generated by one-loop quantum effect [15–17]. The equation derived from action (1) is

$$\left(\tilde{R}_{MN} - \frac{1}{2} G_{MN} \tilde{R}\right) + \lambda \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) \delta^\mu_M \delta^\nu_N \delta(y) = 0, \quad (2)$$

where $\lambda = M_p^2/M^3$. One can easily show that when $\lambda = 0$ (or ∞), Eq. (2) reduces to the usual $5d$ (or $4d$) Einstein field equation without the energy–momentum tensor. This fact makes us expect to derive the usual $5d$ $1/r^2$-type and $4d$ $1/r$-type gravitational potential when $\lambda = 0$ and $\lambda = \infty$, respectively. The expectation is correct when the extra dimension is infinite. For the case of the semi-infinite extra dimension, however, the solution of the fluctuation equation is dependent on the BCs, which introduces two different distance scales. Thus our expectation is not valid in the latter case.

Since there is no contribution from matter, the flat metric $G_{MN} = \eta_{MN}$ is a trivial solution of Eq. (2). In order to examine the behavior of the fluctuation around the trivial solution, we define

$$G_{MN} = \eta_{MN} + H_{MN} \quad (3)$$

with assumption $|H_{MN}| \ll 1$. Inserting Eq. (3) into Eq. (2) one can derive the fluctuation equation

$$H''_{\mu\nu} + \Box^{(4)} H_{\mu\nu} + \lambda \Box^{(4)} H_{\mu\nu} \delta(y) = 0, \quad (4)$$

where the prime denotes a differentiation with respect to y and $\Box^{(4)} \equiv \partial_\mu \partial^\mu$. When deriving Eq. (4) we have used the traceless and transverse gauge

$$H_{55} = H_{\mu 5} = H^\mu_\mu = \partial^\mu H_{\mu\nu} = 0 \quad (5)$$

to ignore the tensor structure of H_{MN} for simplicity. Defining $\psi(y)$ as $\psi(y) \equiv H_{\mu\nu}(x, y) e^{-ipx}$ changes the fluctuation equation (4) as a Schrödinger-type equation

$$\hat{H} \psi(y) = E \psi(y), \quad (6)$$

where

$$\hat{H} = -\frac{1}{2} \partial_y^2 - \lambda E \delta(y) \quad (7)$$

and $E = m^2/2 \equiv -p^2/2$.

The remarkable feature of $\hat{H}$ from the aspect of SQM is the fact that the coupling constant of the singular potential $\delta(y)$ is dependent on the energy eigenvalue. Similar potential in the fluctuation level arises when the bulk is AdS_5 [13,14]. In this case the fixed-energy amplitude can be constructed explicitly by applying Schulman procedure.[1] If, for example, the

[1] Schulman procedure is in detail explained in Ref. [11].

Hamiltonian is $H = H_V(\vec{p}, \vec{r}) + \hat{v}(E)\delta(\vec{r})$, the fixed-energy amplitude $\hat{G}[\vec{r}_1, \vec{r}_2; E]$ for total Hamiltonian can be constructed from the fixed-energy amplitude $\hat{G}_V[\vec{r}_1, \vec{r}_2; E]$ for $H_V(\vec{p}, \vec{r})$ as following;

$$\hat{G}[\vec{r}_1, \vec{r}_2; E] = \hat{G}_V[\vec{r}_1, \vec{r}_2; E] - \frac{\hat{G}_V[\vec{r}_1, \vec{0}; E]\hat{G}_V[\vec{0}, \vec{r}_2; E]}{\frac{1}{\hat{v}(E)} + \hat{G}_V[\vec{0}, \vec{0}; E]}. \quad (8)$$

Once the fixed-energy amplitude for total Hamiltonian system is constructed, Newton potential on the brane is directly computed as following [14];

$$V(r) = \frac{1}{2\pi^2 M^3 r} \int_0^\infty dm\, m \sin mr\, \hat{G}\left[0, 0; \frac{m^2}{2}\right] \quad (9)$$

if the brane is located at $y = 0$. Thus the main problem is converted to the construction of the fixed-energy amplitude.

If H_V is simply $1d$ free case as Eq. (7), the construction is considerably easy if extra dimension y is infinitely flat. If, however, the extra dimension is semi-infinite ($y \geqslant 0$) and the 3-brane is located at the end point of it, the construction of $\hat{G}_V$ for H_V is not relatively simple. In this case $\hat{G}_V$ should be derived by incorporating the self-adjoint extension. In this Letter we will examine Newton potential on the brane for both cases.

Firstly, let us discuss the case of the infinite flat extra dimension for brevity. In this case the fixed-energy amplitude $\hat{G}_V[\vec{r}_1, \vec{r}_2; E]$ in Eq. (8) is simply replaced by an amplitude for usual $1d$ free-particle:

$$\hat{G}_V[\vec{r}_1, \vec{r}_2; E] \to \hat{G}_F[a, b; E] = \frac{e^{-\sqrt{2E}\,|a-b|}}{\sqrt{2E}}. \quad (10)$$

Inserting Eq. (10) into Eq. (8) with letting $\hat{v}(E) = -\lambda E$ enables us to compute the fixed-energy amplitude in this set-up:

$$\hat{G}[a, b; E] = \frac{e^{-\sqrt{2E}\,|a-b|}}{\sqrt{2E}} - \frac{1}{\sqrt{2E} - \frac{2}{\lambda}} e^{-\sqrt{2E}\,(|a|+|b|)}. \quad (11)$$

To compute Newton potential in this case, let us insert

$$\hat{G}\left[0, 0; \frac{m^2}{2}\right] = \frac{-\frac{2}{\lambda}}{m(m + \frac{2}{\lambda})} \quad (12)$$

into Eq. (9). Then, the potential on the brane reduces to

$$V(r) = -\frac{1}{\pi^2 M_p^2 r} \int_0^\infty dm \frac{\sin mr}{m + \frac{2}{\lambda}}. \quad (13)$$

When deriving Eq. (12) we identified m as $\sqrt{2E} = -m$ to derive a correct sign in Newton potential, which seems to correspond to a choice of the retarded Green's function.

Before computing Newton potential $V(r)$, let us consider two special cases. If $M = 0$, λ becomes infinity and $V(r)$ simply reduces to four-dimensional:

$$V_{d=4}(r) = -\frac{G_4}{r}, \quad (14)$$

where

$$G_4 = \frac{1}{2\pi M_p^2}. \quad (15)$$

If $M_p = 0$, $V(r)$ in Eq. (13) is changed into

$$V(r) = -\frac{1}{2\pi^2 M^3 r} \int_0^\infty dm \sin mr. \quad (16)$$

Since the integral in Eq. (16) is not well-defined, we should adopt an appropriate regularization. For the regularization we introduce a damping factor $e^{-\epsilon m}$ as following:

$$\int_0^\infty dm \sin mr \to \int_0^\infty dm \sin mr e^{-\epsilon m}. \quad (17)$$

Then, the final form of Newton potential is purely five-dimensional as expected

$$V_{d=5}^{(\mathrm{Reg})}(r) = -\frac{G_5}{r^2}, \quad (18)$$

where

$$G_5 = \frac{1}{2\pi^2 M^3}. \quad (19)$$

Thus our previous expectation is exactly recovered at the level of Newton potential with a correct sign.

The general Newton potential is obtained by carrying out the integral of Eq. (13):

$$V(r) = -\frac{1}{\pi^2 M_p^2 r}\left[\mathrm{ci}\left(\frac{2r}{\lambda}\right)\sin\left(\frac{2r}{\lambda}\right) - \cos\left(\frac{2r}{\lambda}\right)\mathrm{si}\left(\frac{2r}{\lambda}\right)\right], \quad (20)$$

where $\mathrm{ci}(z)$ and $\mathrm{si}(z)$ are usual sine and cosine integral functions. Using the asymptotic behaviors and the short-range expansions of these special functions, it is straightforward to show that the long-range behavior ($r \gg \lambda/2$) of Newton potential is five-dimensional

$$V(r) = -\frac{G_5}{r^2}\left(1 - \frac{2r_0^2}{r^2}\right) \tag{21}$$

and the short-range behavior ($r \ll \lambda/2$) is four-dimensional

$$V(r) = -\frac{G_4}{r}\left[1 + \frac{2r}{\pi r_0}\left(\gamma - 1 + \ln\frac{r}{r_0}\right)\right], \tag{22}$$

where $r_0 \equiv \lambda/2$ and γ is an Euler's constant. If, of course, $\lambda = 0$ (or ∞), Eq. (21) (or (22)) exactly coincides with $5d$ (or $4d$) Newton potential given in Eqs. (14) and (18).

Now, let us discuss Newton potential when the extra dimension is semi-infinite. With this set-up the fixed-energy amplitude should be computed by incorporating an half-line constraint. From the viewpoint of quantum mechanics the constraint should be chosen to maintain the unitarity for physical reason. This implies that the BC we should adopt must yield the vanishing probability current at $y = 0$, i.e., $(\psi^*\partial_y\psi - \partial_y\psi^*\psi)|_0 = 0$. This requirement can be ensured if all states in the domain of the definition of the Hamiltonian obey

$$\frac{\partial}{\partial y}\psi\bigg|_0 = \alpha\psi\big|_0, \tag{23}$$

where α is an arbitrary real number called as a 'self-adjoint extension parameter'.

The fixed-energy amplitude for free particle living in half-line compatible with BC (23) is computed long ago in Refs. [25,26], whose explicit form is

$$\begin{aligned}\hat{G}_V[\vec{r}_1, \vec{r}_2; E] &\to \hat{G}_\alpha[a, b; E] \\ &= \frac{1}{\sqrt{2E}}\left(e^{-\sqrt{2E}\,|a-b|} + \frac{\sqrt{2E}-\alpha}{\sqrt{2E}+\alpha}e^{-\sqrt{2E}\,(a+b)}\right).\end{aligned} \tag{24}$$

Particular attention is paid to $\alpha = 0$ and $\alpha = \infty$ cases:

$$\hat{G}_{\alpha=0}[a, b; E] \equiv \hat{G}^N[a, b; E] = \frac{1}{\sqrt{2E}}\left(e^{-\sqrt{2E}\,|a-b|} + e^{-\sqrt{2E}\,(a+b)}\right),$$

$$\hat{G}_{\alpha=\infty}[a, b; E] \equiv \hat{G}^D[a, b; E] = \frac{1}{\sqrt{2E}}\left(e^{-\sqrt{2E}\,|a-b|} - e^{-\sqrt{2E}\,(a+b)}\right). \tag{25}$$

One can show easily that $\hat{G}^D$ and $\hat{G}^N$ obey the usual Dirichlet and Neumann BCs at $y = 0$.

Inserting Eq. (24) into (8) with $\hat{v}(E) = -\lambda E$, it is easy to compute the fixed-energy amplitude for total Hamiltonian system. Then, the fixed-energy amplitude on the brane simply reduces to

$$\hat{G}\left[0, 0; \frac{m^2}{2}\right] = -\frac{2}{\lambda m^2 + m - \alpha}. \tag{26}$$

Thus, combining Eq. (9) and Eq. (26) one can express the gravitational potential $V_\alpha(r)$ as following:

$$V_\alpha(r) = -\frac{1}{\pi^2 M_p^2 (m_+ - m_-) r} \times \int_0^\infty dm \left(\frac{m_+}{m + m_+} - \frac{m_-}{m + m_-}\right)\sin mr, \tag{27}$$

where

$$m_\pm = \frac{1}{2\lambda}\left[1 \pm \sqrt{1 + 4\lambda\alpha}\,\right]. \tag{28}$$

At this stage it is worthwhile noting that our potential $V_\alpha(r)$ involves two mass (or distance) scales for arbitrary non-zero α. This means Newton potential for the case of semi-infinite extra dimension can be completely different from that for the case of the infinite extra dimension. Shortly, we will show that the extremely long-range behavior of the potential in this case is seven-dimensional when $\alpha > 0$.

For simplicity let us consider $\alpha = 0$ case first where only one distance scale emerges. Since $m_+ = 1/\lambda$ and $m_- = 0$, Newton potential $V_{\alpha=0}(r)$ can be obtained straightforwardly from Eq. (27):

$$V_{\alpha=0}(r) = -\frac{1}{\pi^2 M_p^2 r}\left[\mathrm{ci}\left(\frac{r}{\lambda}\right)\sin\left(\frac{r}{\lambda}\right) - \cos\left(\frac{r}{\lambda}\right)\mathrm{si}\left(\frac{r}{\lambda}\right)\right]. \tag{29}$$

Thus, the long-range behavior ($r \gg \lambda$) is five-dimensional

$$V_{\alpha=0}(r) = -\frac{2G_5}{r^2}\left(1 - 2\frac{\lambda^2}{r^2}\right) \tag{30}$$

and the short-range behavior ($r \ll \lambda$) is four-dimensional

$$V_{\alpha=0}(r) = -\frac{G_4}{r}\left[1 + \frac{2r}{\pi\lambda}\left(\gamma - 1 + \ln\frac{r}{\lambda}\right)\right]. \tag{31}$$

Thus, the global structure of Newton potential when $\alpha = 0$ is very similar to that in the case of the infinite extra dimension. However, $5d$ Newton constant in the long-range behavior becomes $G_5' = 2G_5$ while $4d$ Newton constant in the short-range behavior remains unchanged. This seems to imply that the change of the fifth dimension does not affect the $4d$ quantity.

Next let us consider $\alpha > 0$ case. Since $m_+ > 0$ and $m_- < 0$ in this case we define two different distance scales $r_\pm = \pm 1/m_\pm$. The explicit computation of the integrals in Eq. (27) makes Newton potential $V_{\alpha>0}(r)$ to be

$$\begin{aligned} V_{\alpha>0}(r) &= -\frac{1}{\pi^2 M_p^2(\frac{1}{r_+} + \frac{1}{r_-})r}\left[\frac{1}{r_+}\left\{\mathrm{ci}\left(\frac{r}{r_+}\right)\sin\left(\frac{r}{r_+}\right)\right.\right. \\ &\quad \left. - \mathrm{si}\left(\frac{r}{r_+}\right)\cos\left(\frac{r}{r_+}\right)\right\} - \frac{1}{r_-}\left\{\mathrm{ci}\left(\frac{r}{r_-}\right)\sin\left(\frac{r}{r_-}\right)\right. \\ &\quad \left.\left. - \mathrm{si}\left(\frac{r}{r_-}\right)\cos\left(\frac{r}{r_-}\right) - \pi\cos\left(\frac{r}{r_-}\right)\right\}\right]. \end{aligned} \tag{32}$$

Using the asymptotic behaviors of the sine and cosine integral functions one can show that the long-range behavior ($r \gg r_+$, $r \gg r_-$) of $V_{\alpha>0}(r)$ is seven-dimensional as following

$$V_{\alpha>0}(r) = -\frac{1}{\pi^2 M^3 \alpha^2 r^4}\left[1 - \frac{12(r_+^2 + r_-^2)}{r^2}\right]. \tag{33}$$

Mathematically, this pecular behavior arises due to the appearance of two different distance scales and the exact cancellation of the five-dimensional term. However, the physical origin of this seven-dimensional Newton potential is unclear to us. It is interesting to note that $7d$ Newton constant is contributed from G_5 and the self-adjoint extension parameter as $G_7 = 2G_5/\alpha^2$.

Next let us examine the behavior of $V_{\alpha>0}(r)$ in intermediate range ($r_+ \ll r \ll r_-$). Since $r_- - r_+ = 1/\alpha$, this region actually arises only for small α. When, thus, α is comparatively large, this region does not exist. In this range the potential behaves as

$$\begin{aligned} V_{\alpha>0}(r) &= -\frac{4G_5}{(1 + \sqrt{1 + 4\lambda\alpha})r^2} \\ &\quad \times \left[1 + \left\{\frac{\pi}{2}\frac{r}{r_-} - 2\left(\frac{r_+}{r}\right)^2\right\}\right]. \end{aligned} \tag{34}$$

The leading term of the potential is five-dimensional, whose Newton constant G_5'' is less than $2G_5$:

$$G_5'' = \frac{4G_5}{1 + \sqrt{1 + 4\lambda\alpha}} < 2G_5. \tag{35}$$

Thus $5d$ Newton constant in this region is smaller than that in $\alpha = 0$ case. For example, if $\lambda\alpha \ll 1$, G_5'' becomes

$$G_5'' \sim 2G_5(1 - \lambda\alpha + \cdots). \tag{36}$$

The subleading term in this region is not uniquely determined. In the region $r/r_- \gg (r_+/r)^2$, we have

$$V_{\alpha>0}(r) = -\frac{G_5''}{r^2}\left(1 + \frac{\pi}{2}\frac{r}{r_-}\right) \tag{37}$$

and in the region $r/r_- \ll (r_+/r)^2$, we have

$$V_{\alpha>0}(r) = -\frac{G_5''}{r^2}\left[1 - 2\left(\frac{r_+}{r}\right)^2\right]. \tag{38}$$

In the extremely short-range region ($r \ll r_+$, $r \ll r_-$) the potential behaves as four-dimensional

$$\begin{aligned} V_{\alpha>0}(r) &= -\frac{G_4}{r}\left[1 + \frac{2r}{\pi}\left\{\frac{\gamma - 1}{\lambda}\right.\right. \\ &\quad \left.\left. + \frac{\lambda}{\sqrt{1 + 4\lambda\alpha}}\left(\frac{1}{r_+^2}\ln\frac{r}{r_+} - \frac{1}{r_-^2}\ln\frac{r}{r_-}\right)\right\}\right]. \end{aligned} \tag{39}$$

The invariability of $4d$ Newton constant shows again the half-line constraint of the extra dimension does not change the $4d$ quantities.

In this Letter we examined Newton potential for the DGP brane-world scenario when the extra dimension is semi-infinite. The final form of Newton potential involves a self-adjoint extension parameter α, which generates an additional mass (or distance) scale. The potential for non-zero α behaves as seven-dimensional in the extremely long range. In the short range the potential recovers four-dimensional behavior. If α is very small, there is an intermediate range where the potential is five-dimensional. The $5d$ Newton constant for non-zero α is smaller than that for $\alpha = 0$ case.

It seems to be interesting to understand the physical origin of the seven-dimensional behavior in long range.

In Ref. [21] the accelerated behavior of universe is approached using the DGP brane-world picture when the extra dimension is infinite. The origin of the behavior in this picture comes from weakness of the gravitational force in the long range, which is referred as gravity leakage. Since the gravitational force becomes weaker for the case of the semi-infinite extra dimension, the accelerating behavior at ultra large scale may easily occurred. We hope to address this issue in the future.

Acknowledgement

This work was supported by the Kyungnam University Research Fund, 2003.

References

[1] H.C. Lee (Ed.), An Introduction to Kaluza–Klein Theories, World Scientific, Singapore, 1984.
[2] V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 125 (1983) 139.
[3] M. Visser, Phys. Lett. B 159 (1985) 22.
[4] P. Horava, E. Witten, Nucl. Phys. B 460 (1996) 506, hep-th/9510209.
[5] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370, hep-ph/9905221.
[6] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690, hep-th/9906064.
[7] J. Garriga, T. Tanaka, Phys. Rev. Lett. 84 (2000) 2778, hep-th/9911055.
[8] S.B. Giddings, E. Katz, L. Randall, JHEP 0003 (2000) 023, hep-th/0002091.
[9] M.J. Duff, J.T. Liu, Phys. Rev. Lett. 85 (2000) 2052, hep-th/0003237.
[10] D.K. Park, S. Tamaryan, Phys. Lett. B 532 (2002) 305, hep-th/0108068.
[11] D.K. Park, H.S. Kim, Nucl. Phys. B 636 (2002) 179, hep-th/0204122.
[12] M. Reed, B. Simon, Methods of Modern Mathematical Physics, Academic Press, New York, 1975.
[13] D.K. Park, Phys. Lett. B 562 (2003) 316, hep-th/0304056.
[14] E. Jung, S.H. Kim, D.K. Park, Nucl. Phys. B 669 (2003) 306, hep-th/0305156.
[15] D.M. Capper, Nuovo Cimento A 25 (1975) 29.
[16] S.L. Adler, Rev. Mod. Phys. 54 (1982) 729.
[17] A. Zee, Phys. Rev. Lett. 49 (1982) 513.
[18] G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B 485 (2000) 208, hep-th/0005016.
[19] G. Dvali, G. Gabadadze, M. Kolanović, F. Nitti, Phys. Rev. D 64 (2001) 084004, hep-ph/0102216.
[20] C. Deffayet, G. Dvali, G. Gabadadze, A. Lue, Phys. Rev. D 64 (2001) 104002, hep-th/0104201.
[21] C. Deffayet, G. Dvali, G. Gabadadze, Phys. Rev. D 65 (2002) 044023, astro-ph/0105068.
[22] G. Dvali, G. Gabadadze, M. Shifman, Phys. Rev. D 67 (2003) 044020.
[23] V.A. Rubakov, hep-th/0303125.
[24] G. Gabadadze, hep-ph/0308112.
[25] T.E. Clark, R. Menikoff, D.H. Sharp, Phys. Rev. D 22 (1980) 3012.
[26] E. Farhi, S. Gutmann, Int. J. Mod. Phys. A 5 (1990) 3029.