A MARKOV MODEL FOR MODULATION PERIODS IN BRAIN OUTPUT

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ABSTRACT A theoretical model is proposed to explain the modulation of bioelectric brain output and its temporal characteristics. The model assumes a time series based on a Markov process with transition probabilities generated by a negative exponential function. One parameter is estimated. Computer runs of the theoretical model compare well with empirical findings.

INTRODUCTION

If the gross bioelectric output of the brain is put through a narrow passband filter which defines a signal-frequency channel, the channel ouptut is a stationary time series in which amplitude modulation is observed where periods can be defined as the time interval between two successive points of corresponding inflection in the amplitude envelope. It has been found in studies with humans that when such modulation periods are measured and classified in terms of their temporal duration, their distributions are similar among individuals and differ systematically under no-signal and signal conditions (Trehub, 1965). While a number of theories have been formulated for the time-series behavior of single neurons (Gerstein and Mandelbrot, 1964; Stein, 1965), no satisfactory theoretical explanation of the basic multi-unit modulation phenomenon has been presented to date. This paper proposes a theoretical model which attempts to explain the observed distributions of modulation periods in the brain.

THEORY AND MODEL

Since we are concerned with brain activity which falls within the narrow frequency domain defined as the signal channel, we introduce the concept of the neuronal collective. A neuronal collective is a subset of neurons characterized by a temporally coherent discharge pattern. It is useful to distinguish two kinds of neuronal collectives: (1) kinetic collectives (K) and (2) potential collectives (Π). K is a neuronal collective in the process of discharge. The magnitude of K is what is measured in the signal channel. Π is the *maximum* number of neurons in a neural mass which can

coherently discharge under a specified condition. In real neuronal systems, K is almost invariably smaller than Π because the probability of a stimulus discharging any given neuron at any particular time is less than unity.

The bioelectric output of the brain can be treated as an ordinal step time series without the loss of information concerning period modulation when periods are measured in time between successive points of corresponding inflection in the amplitude envelope. This is illustrated in Fig. 1.

Assume a constant interval of time τ between successive steps in an ordinal step time series which is taken as descriptive of the behavior of K.

Assume the magnitude of K changes a unit ordinal position (either $+1$ or -1) at each successive interval τ.

If the probability of transition to a unit increment or decriment were specified at each ordinal level, and τ were also specified, an ordinal step time series could be computed as a Markov process. Since we are interested in comparing the characteristics of a particular theoretical model against the empirical output of a real measurement system, τ is set at 0.5 sec which is approximately the time it takes for our existing measurement system to fully reflect a change in the bioelectric output of K.

Having set τ, we must now specify the procedure for generating sets of transition probabilities in the Markov lattice which is to yield the ordinal step time series.

Assume that the larger the ratio of the kinetic collective to the potential collective (K/π) during any τ, the greater is the probability of a decrease in K in the succeeding τ. Conversely, the smaller K/Π, during any τ, the greater is the probability of an increase in K in the succeeding τ.

Under the foregoing assumptions, it follows that K will vary at a rate proportional to its magnitude relative to Π. This suggests an exponential function as a generator for the set of transition probabilities in our Markov lattice model. Since

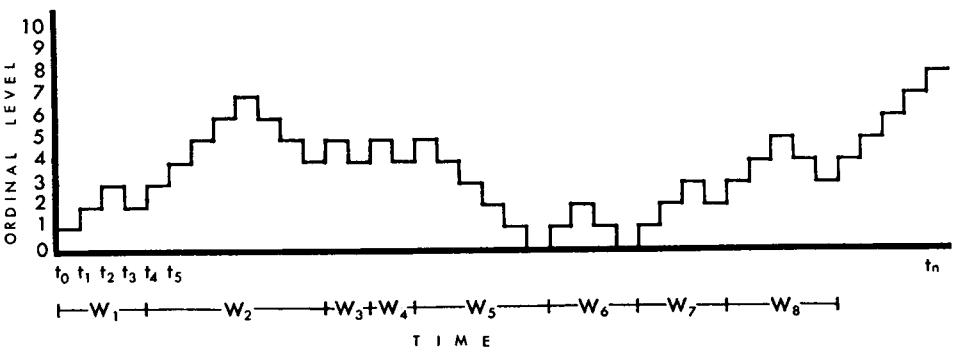

FIGURE 1 Ordinal step representation of brain output. Time interval between successive time points t_1, $t_2 \cdots t_n$ is constant τ. If ouptut at $t_n < t_{n+1}$, a unit step increase in ordinal level is plotted at t_{n+1}. If output at $t_n > t_{n+1}$, a unit step decrease is plotted at t_{n+1}. Periods W_1, $W_2 \cdots W_n$ are defined as the time interval between points of rising inflection at successive troughs in the time series.

there is an inverse relationship between the relative magnitude of K and the probability of increase, a negative exponential generator for the probability of increase is postulated

$$P_k\{+1\} = e^{-ck}$$

and the probability of decrease

$$P_k\{-1\} = 1 - e^{-ck}$$

where c is a coefficient of stimulation (an estimated parameter), and $k = \{0, 1, 2, 3, \cdots n\}$, the ordinal step transformation of K.

A Monte Carlo algorithm for computing the ordinal step time series can now be written.

$$k = \sum_{\epsilon 0}^{\epsilon n} \left[\begin{array}{l|l} \{\epsilon < P_k\} \supset +1 & \left\{ \begin{array}{l} P_{k0} \\ P_{k1} \\ P_{k2} \\ \cdot \\ \cdot \\ P_{kn} \end{array} \right\} \\ \{\epsilon \geq P_k\} \supset -1 & \end{array} \right] \tag{1}$$

and

$$P_k = e^{-ck} \tag{2}$$

where

ϵ is a uniformly distributed random variable within Lim $\{0.00\text{–}0.99\}$

$\{P_{k0}, P_{k1}, P_{k2} \cdots P_{kn}\}$ is a set of transition probabilities to $k + 1$

Computer runs of the algorithm (equation 1) yield time series which can be analyzed for period distributions in the same manner as the actual brain output, and direct comparison can be made between the theoretical model and empirical data.

TEST OF MODEL

In the previously cited study, EEG recordings without stimulation and during 16 fps visual flicker stimulation at a standard moderate intensity were obtained from occipital scalp electrodes in nine adult male subjects. Raw EEG and its 16 cps frequency component were recorded simultaneously. The 16 cps component constituted bioelectric energy in the signal channel defined by passing the amplified cortical output through a fixed 16 cps center frequency, 1 cps passband filter, Grass passive LC type (Grass Instrument Company, Quincy, Mass.). The modulation period

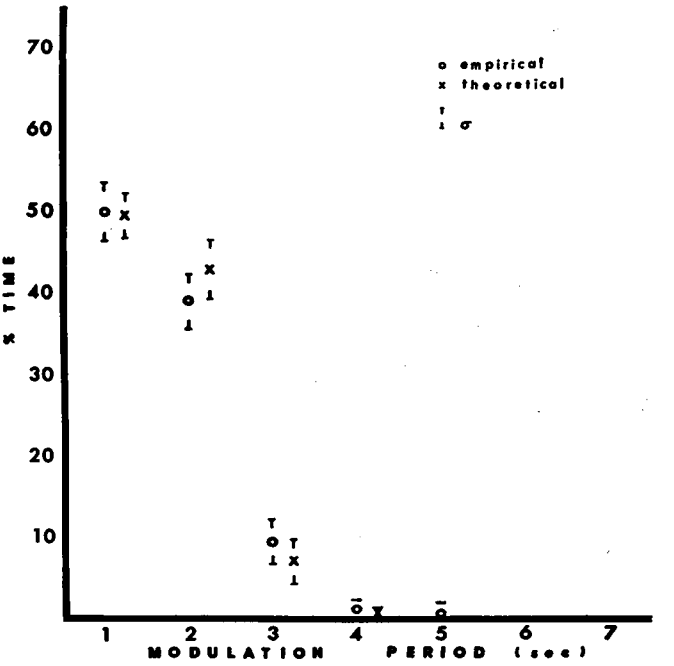

FIGURE 2 Resting condition. Empirical and theoretical plots of per cent time incidence for the indicated modulation periods. Means ± 1 standard deviation shown. $c = 0.98$.

of brain output in the signal channel was determined by measuring the time intervals between successive positive inflections in the amplitude envelope and classifying the duration of each period to its nearest second. Such measurements were made for both resting and stimulus conditions. The number of periods falling in each duration class was multiplied by the duration, and the per cent of total time occupied by each class of cycles was computed for each subject.

Fig. 2 shows the empirical mean distribution of per cent time occupied by each duration class in the sample under the resting condition compared to the mean distribution yielded by nine runs of the theoretical model in a digital computer. Fig. 3 compares the empirical and theoretical computer generated distributions under the condition of a standard, moderately bright photic stimulus. Under the resting condition, the empirical data are fitted by a coefficient approaching 1.00 ($c = 0.98$), while the data obtained under stimulation are best fitted with the coefficient $c = 0.20$.

The functional relationship between the coefficient c and the variable of stimulus intensity remains to be explored.

I am indebted to the Veterans Administration Eastern Research Support Center for computational assistance on the IBM 7094 Digital Computer.

Received for publication 21 October 1968 and in revised form 15 January 1969.

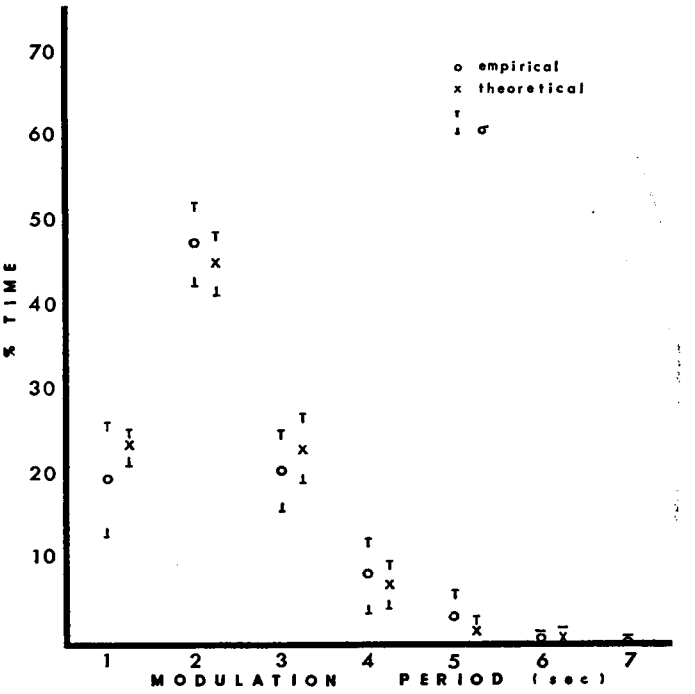

FIGURE 3 Stimulus condition. Empirical and theoretical plots of per cent time incidence for the indicated modulation periods. Means ± 1 standard deviation shown. $c = 0.20$.

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