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Visual Representations in Mathematical Word Problem Solving Among Form Four Students in Malacca

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Abstract

Learning how to solve mathematical problems has been a long withstanding difficulty faced by students and has been given constant focus in mathematical development. Successful problem solvers spend more time analyzing a problem and they will find all the alternatives related to the situation. Therefore many educators (Cai, 2003, National Council of Teachers of Mathematics, 2000; Smith, 1991) agreed that teaching mathematics through problem solving hold great promise. Furthermore, according to the Malaysian Curriculum Development Centre (2003), one of the important objectives in teaching mathematics is to develop students' mathematical problem solving skills. Mathematical problems are not simply computational tasks type but are problems which also require appropriate selection of strategies and decisions that lead to logical solutions. Word problem solving is one of the important components of mathematics problem solving which incorporate real-life problems and applications. However, many researches revealed that students express great difficulties in handling a word or story problem. This study examined the use of visual representations which posed great difficulties in most mathematical tasks. Secondary students were given the 15Mathematical Processing Instrument (MPI) adapted from Hegarty and Kozhevnikov (1999). This study examined 381 students from eight secondary schools in three different districts in Malacca. Results indicated that less than two percent of the problems were solved using pictorial representation and most of the students preferred to used schematics solutions.

Keywords: Mathematics; Visualization; Word problem

1. Introduction

Learning how to solve mathematical problems has been a long withstanding difficulty faced by students and has been given constant focus in mathematical development. Successful problem solvers spend more time analyzing a problem and they will find all the alternatives related to the situation. Therefore many educators (Cai, 2003, National Council of Teachers of Mathematics [NCTM], 2000; Smith, 1991) agreed that teaching mathematics through problem solving hold great promise. Furthermore, according to the Malaysian Curriculum Development Centre (2003), one of the important objectives in teaching mathematics is to develop students' mathematical

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problem solving skills. Mathematical problems are not simply computational tasks type but also require appropriate selection of strategies and decisions that lead to logical solutions.

Word problem solving is one of the important components of mathematics problem solving which incorporate real-life problems and applications. However, many researches revealed that students express great difficulties in handling a word or story problem (Gagne, 1983; Orton, 1992, Ekbia & Alamolhodaei, 2000; Takahashi 2000; Alamolhodaei, 2002; Nickson 2004). Word problem is actually a story problem and the students normally have to relate between the known and the unknown. As Gagne (1983) suggested, in the process of mathematical word problem solving, student should be able to translate the concrete to the abstract and the abstract to the concrete. Therefore the mathematical word problem exam is more unique and challenging task than the ordinary mathematics task (Alamolhodaei, 2002). Thus, students normally find difficulty in solving word problems initially from translating the word representations into mathematical representation. Related problem to this was students' difficulty in deriving 'mental images' which then followed by operative actions "transforming in to dynamic images, invoking images of formulae from memory" (Dorfler, 1991; Presmeg, 197). Presmeg (1997) pointed out that concrete imagery needs be coupled with rigorous analytical thought processes to be effectively used in mathematics problem solving. Hence "It is argued that the status of visualization in mathematics education should and can be upgraded from that of a helpful learning aid" (Dreyfuss, 1991 pg. 30). Dreyfuss also pointed out that students basically were reluctant to use visualization in mathematics and this may be attributed to low status accorded to visual aspects of mathematics in the classroom. However, he also posed many effective examples demonstrating the powerful use visualization in mathematical reasoning and learning.

The uses of visual representations in mathematical word problem are very useful. Hegarty and Kozhevnikov (1999) has approved that the use of schematic spatial representations was associated with success in mathematical problem solving, whereas use of pictorial representations was negatively correlated with success. Lean and Clements (1981) also found that there are different types of visual representational strategies used by students when solving mathematical problems by separating student-generated imagery into five categories: concrete imagery, pattern imagery, kinesthetic imagery, dynamic imagery, and memory of formula. However Presmeg (1986) argued that concrete imagery (vivid pictorial images of objects contained in mathematical problems) may actually focus the reasoning on irrelevant details and distract the 'solver" from the main element of the problem.

This study adapted the method and instruments used by Hegarty and Kozhevnikov (1999). The research questions of this study were as follows: i) What is the level of achievement in mathematical word problem solving among Form Four students in Malacca? ii) What visualization strategy do the Form Four students use in attempting mathematical word problem solving? iii) What is the relationship between the type of visualization strategy used by the students in solving word problems and their mathematical achievement?

2. Methodology

2.1 Population and sample

The target population for this study is Form Four students in the state of Malacca in Peninsular Malaysia. A total of 381 students took part in this research using the multi-stage cluster sampling. The subjects of study were from eight different schools (second cluster) in Melaka chosen from three different districts (first cluster) in Melaka, namely Jasin, Melaka Tengah and Alor Gajah. Table 1 shows the demographic descriptions.

Table 1: Demographic information on the participating students

Gender	Male	166
	Female	215
Race	Malay	272
	Chinese	100
	Indian	7

	Others	2
Mathematics Grade in PMR	A	231
	B	96
	C	54

2.2 The Research Materials

The research material is a set of paper and pencil test named Mathematical Processing Istrument (MPI). The test comprised of 15-items mathematical word problem solving adapted from Hegarty and Kozhevnikov (1999). The word problems were also translated into Malay language and posed alongside the English version. This would help the students who have the difficulty in understanding English language. Table 2 below shows the word problems. Forty-five minutes were given to the students to answer the questions (with the maximum of 3 min per question). The students' responses were analyzed. The first score was the total number of correct answers or work solutions shown on the test set. Each correct answer or work solution were awarded one mark whereas the incorrect answer or work solution were awarded zero mark. Further analyses on the type and frequency of visualization used by the students for solving the mathematical word problem were conducted. Four categories of visualizations were used: no answer, pictorial, schematic and the combination of pictorial and schematic. The first category included students did not attempt to answer the word problems and the second category included students' solution involving drawing pictures or diagrams as formation of "mental images" related to the word problems. The third category included students' solutions involving schematics (objects and/or parts of an object described in the problem object) and the fourth category included students' solution involving the combination of pictorial and schematics.

Table 2 : Fifteen mathematical word problems

No	Mathematical word problems	No of words	No of sentences
1	At the two ends of a straight path, a man planted a tree and then every 5 m along the path he planted another tree. The length of the path is 15 m. How many trees were planted?	38	3
2	On one side of the scale, there is a 1kg weight and half a brick. On the other side, there is one full brick. The scale is balanced. What is the weight of the brick?	36	4
3	A balloon first rose 200 m from the ground, then moved 100 m to the east, then dropped 100 m. It travelled 50 m to the east, and finally dropped straight to the ground. How far was the balloon from its original starting point?	44	3
4	In an athletic race Jim is 4 m ahead of Tom and Peter is 3 m behind Jim. How far is Peter ahead of Tom?	25	2
5	A square (A) has an area of 1 square meter. Another square (B) has sides twice as long. What is the area of B?	24	3
6	From a long stick of wood, a man cut six short sticks, each 2 feet long. He then found he had piece of 1 foot long left over. Find the length of the original stick?	35	3
7	The area of a rectangular field is 60 square meters. If its length is 10 m, how far would you have travelled if you walked the whole way around the field?	31	2
8	Jack, Paul, and Brian all have birthdays on the 1st of January, but Jack is 1 year older than Paul and Jack is 3 years younger than Brian. If Brian is 10 years old, how old is Paul?	38	2
9	The diameter of a can of peaches is 10 cm. How many cans will fit in a box 30 cm by 40 cm (one layer only)?	26	2
10	Four young trees were set out in a row 10 m apart. A well was located by the last tree. A bucket of water is needed to water two trees. How far would a gardener have to walk altogether if he had to water the four trees using only	51	4

	one bucket?		
11	A hitchhiker set out on a journey of 60km. He walked the first 5km then got a lift from a truck driver. When the driver dropped him off he still had half of his journey to travel. How far had he travelled in the truck?	47	4
12	How many picture frames 6-cm long and 4-cm wide can be made from a piece of framing 200-cm long?	22	1
13	On one side of a scale, there are three pots of jam and a 100 oz. weight. On the other side, there are a 200g and a 500g weight. The scale is balanced. What is the weight of a pot of jam?	44	4
14	A ship was Northwest. It made a turn of 90 degrees to the right. An hour later it made a turn through 45 degrees to the left. In what direction was it then travelling?	34	4
15	15. There are eight animals on a farm. Some of them are hens and some are rabbits. Between them they have 22 legs. How many hens and how many rabbits are on the farm?	33	4

3. Results

Table 3 shows the frequencies and percentage of students' response to the MPI questions. An overall view of the correct answers indicated that a large percentage of students (96.1%) were able to answer MPI-6 question. The most difficult question was MPI-10, with only 44 students (11.5%) able to answer the question correctly whilst the other 184 pupils failed to answer the question correctly. Questions 4, 6, 8, 11, 13, and 15 were solved correctly by a large percentage of students; 95%, 96.1%, 84.8%, 74%, 83.2% and 72.2% respectively. MPI-3, MPI-7 and MPI-14 questions were solved correctly by more than half of the students, 58.3%, 62.2% and 63.8% respectively. Overall more than half (59.3%) of the questions were answered correctly and less than 8% of the questions left unanswered.

Table 3: Frequencies and percentage of students response to the questions.

Questions	Correct answer (n=381)		Incorrect answer (n=381)		Omission (no answer) (n=381)	
	F	%	f	%	F	%
1	88	23.1	290	76.1	3	0.8
2	174	45.7	163	42.8	44	11.5
3	222	58.3	133	34.9	26	6.8
4	362	95.0	18	4.7	1	0.3
5	173	45.4	157	41.2	51	13.4
6	366	96.1	15	3.9	0	0
7	237	62.2	115	30.2	29	7.6
8	323	84.8	52	13.6	6	1.6
9	131	34.4	184	48.3	66	17.3
10	44	11.5	309	81.1	28	7.3
11	282	74.0	81	21.3	18	4.7
12	153	40.2	191	50.1	37	9.7
13	317	83.2	35	9.2	29	7.6
14	243	63.8	95	24.9	43	11.3
15	275	72.2	58	15.2	48	12.6

Table 4 shows the frequencies and percentages of visual representations shown by the students in answering the 15 MPI questions. Most of the students solved the questions using a schematic representation, with an average of 66.5% of the students attempted the schematic approach. As low as 38.3% to as high as 81.6% of the students showed schematic solution mode hence preferred non-visual mode in solving the word problems. An average of 1.4% of the students used pictorial representation in answering the 15 MPI problems. However, nearly a quarter (average of 26.4%) of the questions was answered using the combination of pictorial and schematics representations. The following figure shows examples of students' combination of pictorial and schematics representations.

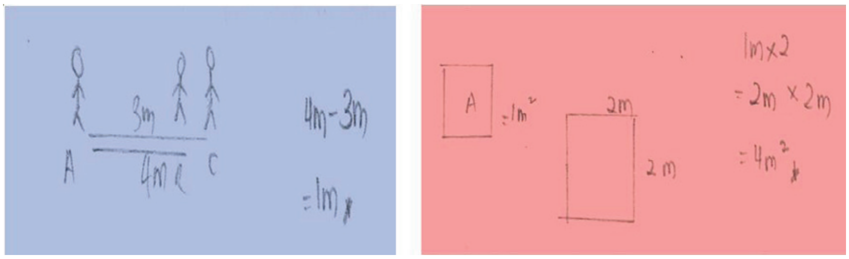

Figure 1: Examples of the combination of pictorial and schematics representation for MPI-4 and MPI-5

Table 4: Frequencies and percentages of types of visual representations

Questions	No answer (n=381)		Pictorial (n=381)		Schematics (n=381)		Combination (n=381)	
	f	%	f	%	f	%	F	%
1	3	0.8	1	0.3	299	78.5	78	20.5
2	44	11.5	9	2.4	267	70.1	61	16.0
3	26	6.8	7	1.8	236	61.9	112	29.4
4	1	0.3	1	0.3	237	62.2	142	37.3
5	51	13.4	7	1.8	245	64.3	78	20.5
6	0	0	0	0	306	80.3	75	19.7
7	29	7.6	6	1.6	201	52.8	145	38.1
8	6	1.6	1	0.3	311	81.6	63	16.5
9	66	17.3	8	2.1	230	60.4	77	20.2
10	28	7.3	8	2.1	193	50.7	152	39.9
11	18	4.7	0	0	310	81.4	53	13.9
12	37	9.7	8	2.1	25	65.9	85	22.3
13	29	7.6	1	0.3	272	71.4	79	20.7
14	43	11.3	22	5.8	146	38.3	170	44.6
15	48	12.6	2	0.5	297	78.0	34	8.9
Total	429	**7.5**	81	**1.4**	3801	**66.5**	1404	**24.6**

Generally, findings indicated that students preference in solving the word problems were mixed or in combination form. This may suggest that this group of students may have formed pictorial images during the problem solving but their mental images were not fully concrete or diagrammatize hence does not lend support to learners during the problem solving sessions. Ironically, all students took the effort to answer MPI-6 and more than 80% of them used schematics representations. The most frequent used of pictorial representation (22 pupils) was on MPI-16. This may be due to the characteristics of question which encouraged or lead students to draw the direction of the ship. On the other hand, students did not show any use of such representations on MPI-6 and MPI-11 hence indicating that students preferred to use schematic representation involving arithmetic and algebra solutions. Overall, these findings indicated that learners' preference in visualization of word mathematical problems is schematic rather than pictorial or combination of pictorial and schematic.

Questions MPI-4 and MPI-10 showed weak relationships between visualization and MPI achievement for both questions. The other MPI questions showed low and moderate relationship with correlation index (point-biserial) ranging from .34 to .58. This indicated that students mathematical achievement is associated with ability to visualize (pictorial representations) or otherwise (non-visualize). This may suggest that students normally choose any strategy that the easiest and less time consuming to solve the problem without prior analyzing the problems visually.

4. Conclusion

The aim of this study is to identify the level of achievement of mathematical word problem solving and the students' representations or visualization of the problems in the process of deriving solutions. In the two questions, majority of the students preferred to use schematics representations rather than pictorial representations. In conclusion, students should be given the opportunity to utilize different types of representations in order to help them to understand any mathematical word problem solving. Students' preference mostly depends on what their teacher have taught them in class. Therefore teachers also must be alert of the representations that can help them to understand and solve the mathematical word problems.

This finding is additional support for past research suggesting visualization or imagery does not really helps the processes of problem solving and specifically in word problem solving (Hewitt, 2003; Herman et al., 2004). Much has to be done in this area of how visualization can be integrated in teaching and learning of mathematics in the classrooms. As Woolner (2004) pointed out, effective pedagogy that can enhance the use and power of visualization in mathematics education is perhaps a pressing concern and need further emphases. Ways or strategies to facilitate visualization in mathematics teaching and learning should be given concern.

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