Shape memory alloys behaviour: A review

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Abstract

Shape memory alloys are used in a variety of fields, such as medical or aeronautical. Other fields of knowledge have been researching these materials, attracted by their capacity to dissipate energy through high-strain hysteretic cycles without significant residual strains. Because of these interesting properties for seismic protection, an example of the possible beneficiaries of these materials are civil engineering structures. This paper reports a bibliographic review on the characteristics and uniaxial macroscale constitutive models for shape memory alloys, of interest for a significant number of applications, most often based on wires and bars. The constitutive model assessment focuses on mechanical and kinetic laws, as well as on the energy balance law, of relevance for dynamic loadings. Some characteristics of these materials are still not sufficiently well known, especially those related to ageing. With regard to behaviour prediction, the most frequently used uniaxial constitutive models result in similar responses.

Keywords: Shape memory alloys; Ageing; Residual strain; Uniaxial models; Self-heating

1. Introduction

Shape memory alloys (SMAs) may be found in two state phases: austenite and martensite. Austenite is called the parent phase, from which a transformation process occurs into the softer product phase, martensite. This solid-solid transformation is referred to as non-diffusive, a reference to the transformation behaviour of the crystalline structure, with the atoms taking small-length ordered displacements, the opposite of diffusive. Transformation from the parent phase may be induced by a temperature reduction or a stress increase, each resulting in a characteristic type of martensite: twinned or multi-variant martensite in the first case and detwinned or single-variant in the latter. This terminology results from the crystallographic geometry of the formed martensite. Macroscopically, SMAs behaviour is characterised by the superelasticity and by the shape memory effect. The former is associated with the recovery from loading-originated strains of up to 8\% or 10\%, without significant residual strains. The latter effect is characterized by the capacity of recovering residual strains developed after cyclic loading with a temperature variation.

Stress-free, SMAs are characterized by four transition temperatures: $M_f$, $M_s$, $A_s$, and $A_f$, from the lowest to the highest. $M$ refers to martensitic and $A$ to austenitic state phases. \textit{f} and \textit{s} are references to the finish and start

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temperatures of the transformation process. When the temperature, $T$, is lower than $M_f$, martensite is stable. For $T$ values higher than $A_f$, the stable phase is austenite. For temperature values between $A_s$ and $M_f$, both phases are stable. From the start to the finish temperatures, martensitic, forward, and austenitic, reverse, transformations occur. There are SMAs that do not follow this order [1], however, those materials, which do not belong to the "Type I" group, are not considered here. If the temperature is lower than $M_f$, single-variant martensite may be obtained from multi-variant martensite by the application of stress, which occurs due to a reorientation of the martensite crystals. After unloading, significant residual strains remain. In order to recover multi-variant martensite, it is necessary to transform the material into austenite, by raising the temperature of the SMA higher than $A_f$, and then back to single-martensite as the temperature is lowered. The superelastic effect is characterized by the single-variant martensite transformation from the parent phase due to loading at a temperature higher than $A_f$. This effect begins with elastic austenite, after which the transformation into single-variant martensite is initiated at a critical stress value. The elastic loading of martensite begins after the transformation is completed. Increasing loading leads to the SMA plastification. Unloading the elastic martensite initiates the reverse transformation, because at that range of $T$ values only austenite is stable in the stress-free state. The unloading path presents lower stresses at the transformation phase than the loading one, what allows for energy dissipation associated with cyclic loadings. After complete unloading, the SMA presents virtually null residual strains. For temperatures higher than $M_f$, SMA materials present an elastic-plastic behaviour.

Although SMAs are highly sensitive to variations in material composition and thermomechanical treatments, they can generally be divided in nickel-titanium (NiTi) and Cu-based alloys. Near-equiatomic NiTi with cold working and annealing treatments are considered most suitable for a number of engineering applications, given its better superelastic properties, higher temperature variations stability and higher resistance to corrosion and fatigue [2–4].

The behaviour of SMAs can be predicted with constitutive models that are available, which should be adapted in order to take into account all the characteristics of the specific material to be applied. Even though three-dimensional macroscale constitutive models for SMAs exist, one-dimensional models are adequate for the simulations involving wires and bars, which are the majority of cases. Also, the majority of the constitutive laws are phenomenological in nature. Most uniaxial constitutive models of SMAs have a mechanical law, relating stress, strain and martensite fraction, and a kinetic law governing the martensitic transformation as a function of temperature and stress. These models are known as temperature dependent models. If an energy balance law is added, it is possible to obtain a strain rate dependent constitutive law. SMA uniaxial constitutive relations are discussed in Chapter 3.

2. Characterization of shape memory alloys

Miyazaki et al. [2] pointed out the change in the stress-strain curve with loading cycles as the most important factor inhibiting the use of NiTi SMAs. Based on 0.5 mm diameter wire of Ti-50.5 at.%Ni testing, they reported that residual strains result from dislocations in the martensitic phase, which develop a stress field at the grain boundaries, reducing the externally applied-stress required to commence the transformation [2]. The same authors found that, in order to stabilize the superelasticity, it is necessary to raise the critical stress for slip, attainable by ageing treatments and by annealing. The combination of these treatments accompanied by training was found to be highly efficient for this purpose. The benefits of cyclic training was advocated by other authors [5–7]. Auguet et al. [8], based on test of 2.46 mm diameter wires, reaching a deformation of more than 8% concluded that SMAs residual strain is due to both a recoverable and a permanent contribution. McCormick et al. [9] tested coupons extracted from the center and the edge of 25.4, 12.7 and 6.35 mm diameter bars of hot rolled Ti-50.90 at.%Ni, and also tested a 12.7 and a 6.35 mm bar. These authors concluded that the decrease in bar diameter seems to be related to a decrease in the residual strain. A different trend had been reported previously [10]. NiTi wires of 0.254 mm and bars of 12.7 mm diameter were tested. The residual strains of the wires were close to 1% after 6% strain cycles, while the bars registered values close to 0.2%. The chemical composition of the materials were not reported and the specimens were subjected to different treatments, which impairs the evaluation of the results. Nevertheless, the test of NiTi bars with very low residual strain is noted. Residual strain development is accompanied by a reduction in the energy dissipated in each hysteresis loop [5,9]. This is a core issue with regard to damping applications. Researchers have come to conclusions which, in some cases, are contradictory. This may result from the limited number of specimens analysed and to different chemical compositions and treatments of the tested materials. Even though efficient thermomechanical treatments for stabilizing the superelasticity of Ni-rich alloys have been reported [2], this is an issue that requires further research.
Differences in tension/compression behaviour have been found in tests on martensitic and austenitic NiTi SMAs, due to different microstructural deformation mechanisms [11,12]. DesRoches [12] tested austenitic Ti-55.95 wt.%Ni bars and reported good superelastic behaviour, with a flag shape hysteresis and good re-centring capacity, under both compression and tension. These tests showed different transformation plateau stress and initial elastic modulus, of about 25% and 30% lower in tension, respectively. Jalaeefar and Asgarian [13] also studied the tension/compression behaviour of superelastic Ti-53.5 wt.%Ni bars, of 8 mm diameter, subjected to cycles of up to 6% strain. Almost similar stress-strain curves in tension and compression were observed, with higher stresses for the compression loadings, following the trend reported by the previously cited authors. An increased initial elastic modulus was not reported. Despite the above, the hypothesis that the same behaviour in tension and compression is followed is usually assumed [10,14,15]. Nevertheless, if, for a given SMA, this matter is found to be relevant, different material parameters may be used for tension and compression [13]. Moreover, the review of the bibliography indicates that this is an issue that depends on the material composition and one that should be further investigated.

Dolce et al. [3] reported significant variations in stress-strain profile, energy dissipation and stiffness of SMAs due to temperature changes. Reductions of the equivalent damping ratio close to 40% for temperature variations from 0 °C to 40 °C were reported. Noting that temperature variations related to annual and daily cycles are inevitable and that high-rate loadings, because of self-heating, discussed below, induce significant temperature changes, this relationship, defined by the Clausius-Clapeyron equation, becomes important. It follows experimental evidence, which demonstrates that the transformation stresses increase linearly with increasing temperature, and it is characterized by the Clausius-Clapeyron coefficient (CCC), usually assumed to be equal for both martensitic and austenitic transformation (\( \frac{\Delta C}{\Delta T} = C_{M} = CCC \)) and constant throughout the entire temperature range [1]. Previous reports claimed that NiTi and other frequently used SMAs were essentially strain rate independent [16], but later research indicates otherwise [17,18]. DesRoches and Smith [19] noted that research on this subject has led to contradictory conclusions on the effects of strain rate. There are reports which demonstrate that the dissipation capacity increases with the increase of the strain rate and others that show the exact opposite. Different responses were found for bars and wires [10]. This may be due to the use of different strain ranges, sample sizes, testing conditions and properties of the materials [19]. Self-heating, due to the latent heat originating at the solid-solid transformation phases [20], and, with less significance, to hysteretic dissipation, has been identified as the main source of SMA strain rate dependency. Because the superelastic behaviour is temperature dependent, when the temperature of the material changes so does the critical transformation stresses, \( \sigma_f^A, \sigma_f^M, \sigma_s^A \) and \( \sigma_s^M \) (the subscript and superscripts definitions are presented above). This modifies the damping capacity of SMAs, with boundary cases for adiabatic, typical of high-strain rate loadings, and isothermal conditions, associated with low-strain rate loadings response.

Although the atomic displacements associated with the solid-solid transformations of SMAs are essentially displacive, Auguet et al. [21] reported measurements of diffusive effects in NiTi. They focused on temperature effects on ageing, involving tests at constant strain and at room temperature for periods of nearly one month. At the parent phase, the stress needed to proceed with cycling was increased roughly proportionally to the previously applied stress. On this matter, Torra et al. [7] also concluded, based on wire tests, that for low prestress of about 200 MPa in the parent phase, even for temperatures resulting from direct sun exposure, the variation of stress cannot be considered as relevant. At the coexistence phase, the stress required to proceed with the transformation was also increased (prior relaxation was observed), with this stress step reaching 20 MPa [8], due to an increase in the austenite fraction [21]. This test was also conducted at room temperature and with only one month duration, which led Torra et al. [7] to state later that for significant periods of times this variation may become highly relevant. For the martensite phase, the strain was maintained for different time intervals, after which an inner loop was imposed [8]. The most relevant observation is that, after the period in which the strain was fixed, the martensite transformation stress of the inner loop was lower than the initial transformation stress and this difference increased with longer fixed strain time intervals. Dolce et al. [3] conducted 20 days time interval tests and reported negligible relaxation effects. Auguet et al. [4] studied NiTi wires of 0.5 and 2.46 mm diameter, involving ageing at 100 °C for time intervals of up to 270 days. This research showed that NiTi cannot be exposed to direct sunlight for long periods of time without occurring progressive and spontaneous increase in the transformation stresses, of over 90 MPa. Furthermore, Torra et al. [7] evaluated the effect of eight months ageing at 100 °C of 2.46 mm NiTi wires subjected to a high prestrain of 6.8%. The results show that the flag shape of the hysteresis cycle changed to a narrow “S” shape curve and that the maximum stress for the cyclic tests with maximum strain of about 8% changed from about 600 MPa to close to 1000 MPa. These relevant
changes in SMAs under temperature and stress require further investigation [7]. With the current state of knowledge, the use of permanently pre-stressed NiTi may be dangerous, given the previously mentioned ageing effects, which may result in unpredictable changes in the stress-strain response [21].

Structural fatigue, also known as classical mechanical fatigue, is related to the formation of microcracks, inducing material rupture [22]. A series of cyclic loading tests, with 2.46 mm diameter wires, was carried out [23]. The results indicate that high-cycle fatigue is significant, associated with the vibration due to rain and/or wind actions. Given the small number of cycles expected in earthquakes damping - less than 200 -, the decay of the energy dissipation due to this issue does not go beyond 2% [23]. Dolce et al. [3], based on experimental tests, also reported considerable low-cycle fatigue resistance of NiTi alloys, of hundreds of 6% to 8% strain cycles. Therefore, unlike for other cases, for seismic applications substitution of SMAs is not required [24].

3. Uniaxial constitutive models for shape memory alloys

A significant number of the current macroscale models are based on the work by Tanaka et al. [25]. This model combines a mechanical and a kinetic law which governs the martensitic fraction of the material. It is a temperature dependent phenomenological model with the crystallographic structural changes taken into account through an internal state variable, the martensite fraction, \( \xi \), characterizing the extent of the transformation. Although a general theory was developed, Tanaka et al. [25] applied it to the uniaxial behaviour description of alloys in the process of stress-induced transformation, thus modelling the superelasticity effect, resulting in

\[
\dot{\sigma} = E \dot{\varepsilon} + \Theta \dot{\Omega} + \Omega \dot{\xi}, \quad \text{with} \quad E = E_A + (E_M - E_A)\xi \quad \text{and} \quad \Theta = \Theta_A + (\Theta_M - \Theta_A)\xi
\]

where \( \sigma \) is the normal stress, \( \varepsilon \) represents the strain, \( \Omega \) is the phase transformation tensor, \( E \) is the elasticity modulus and \( \Theta \) is the modulus of thermoelasticity. The subscripts \( A \) and \( B \) are a reference to the pure austenitic and martensitic phases. An overdot denotes a time derivative. When integrated, taking \( E \) and \( \Theta \) as constants, the previous equation becomes \( \Delta \sigma = E \Delta \varepsilon + \Theta \Delta \Omega + \Omega \Delta \xi \). Using the concept of maximum residual strain, \( \varepsilon_L \), which reflects the residual strain of a SMA with 100% austenite and with temperature lower than \( A_t \), that is completely transformed into martensite through loading and after it is unloaded, the relation \( \Omega = -\varepsilon_L E \) was obtained. This model is strain rate independent.

Liang and Rogers [26] developed a model based on the previous work. For uniaxial behaviour, taking the material functions as constants, this model is the same as the former. Similar to the Tanaka et al. model, the Liang and Rogers model cannot predict the shape memory effect if any temperature-induced martensite exists. On this matter, Brinson [1] presented a constitutive model considering the martensite fraction composed of one part related to the martensite induced by temperature and another part related to the stress-induced martensite, given by \( \xi = \xi_s + \xi_T \). \( S \) and \( T \) are references to the stress-induced and temperature-induced martensite, respectively. Moreover, experimental evidence shows that taking the material functions, \( E, \Omega \) and \( \Theta \), as constants is not accurate. In particular, the Young’s modulus shows a significant dependence on the martensite fraction [1]. Thus, this author adopted the relation expressed in the second equation in (1), which may be obtained with the Voigt scheme. If the Reuss scheme had been used, the mechanical law obtained would have been identical [25]. Also, the relation \( \Omega(\xi) = -\varepsilon_L E(\xi) \) was derived. Given its insignificance for the overall response, presenting a value five orders of magnitude lower than \( E(\xi) \) [1], \( \Theta(\varepsilon, \xi, T) \) was assumed constant.

The constitutive relation with material functions varying linearly with \( \xi \), which allows for good agreement with experimental observations [1], delivers

\[
\sigma - \sigma_o = E(\xi)e - E(\xi_o)e_o + \Omega(\xi)\xi_S - \Omega(\xi_o)\xi_{So} + \Theta(T - T_o)
\]

The subscript \( o \) is a reference to the initial conditions. This mechanical law is the backbone of most current SMA constitutive models. It is a temperature dependent model, capable of tracing both the superelastic and the shape memory effect at all temperatures, but it is strain rate independent.

Lubliner and Auricchio [27] developed a fully three-dimensional model which was later specialized to model SMAs behaviour [16,27,28]. The one-dimensional mechanical law derived [16,27] is the same adopted by Brinson [1] considering only single-variant martensite and isothermal conditions, \( \sigma = E\varepsilon \), in which the \( \varepsilon \) superscript is a reference to the elastic strain, given by \( \varepsilon^e = \varepsilon - \xi_T \xi_S \), which may be obtained by the substitution in Equation (2) of null initial values, \( \sigma_o = 0, \varepsilon_o = 0 \) and \( \xi_{So} = 0 \), assuming that the material always starts with 100% austenite.
with \( T = T_0 \), or simply neglecting the contribution of this term, knowing that it is significantly smaller than the transformation strain [29]. This mechanical law is equal to the simplified constitutive equation presented by Brinson and Huang [29]. Furthermore, the Young’s modulus was considered constant [27]. Auricchio and Sacco [30] proposed a uniaxial model capable of simulating only the superelastic effect, but they also addressed the \( E \) variation with the transformation process. The mechanical law adopted is the same as proposed previously [16,27]. For the evolution of the elastic modulus, three homogenization methods were addressed: the Mori-Tanaka scheme, the Reuss scheme and the Voigt scheme. A similar model was presented by Fugazza [15], with the same mechanical and linear kinetic law, differing only in the adoption of a constant Young’s modulus. Later, Auricchio and Lubliner [28] proposed an extension to their previous work, consisting of a more general uniaxial constitutive model for SMAs, making it possible to reproduce the shape memory effect and the superelastic effect, with the inclusion of two internal variables, as did Brinson [1]. The same phase transformation zones presented by Brinson [1] were adopted and the flow rules were defined including parameters that measure the rates at which the transformations occur.

Piecewise linear constitutive relations have been proposed for superelastic analysis [31]. These proposals aim to reduce the computational effort, but their application is limited, regarding the consideration of constant temperature. Other more complex models, such as the micromechanical models proposed by Patoor et al. [32] and by Mirzaeifar et al. [33], for studying the superelastic response of polycrystalline SMAs, in which the transformation strain is related to the crystallographic data, have been developed. Their review is beyond the scope of this paper.

There are numerous possibilities for the kinetic law. The most relevant seem to be the exponential law adopted by Tanaka et al. [25], the cosine laws developed by Liang and Rogers [26] and adapted by Brinson [1], and the linear flow rule adopted by Auricchio and Sacco [30]. Reports from these authors show that any of these flow rules are able to satisfactorily model the behaviour of SMA materials.

The cosine laws deliver, in the case of superelastic SMAs,

\[
\dot{\xi} = \frac{1}{2} \cos \left[ a_M \left( T - M_f - \frac{\sigma}{C_M} \right) \right] + \frac{1 - \xi_o}{2} \quad \text{and} \quad \dot{\xi} = \frac{\dot{\xi}_o}{2} \cos \left[ a_A \left( T - A_s - \frac{\sigma}{C_A} \right) \right] + 1
\]

for forward and reverse transformation, respectively. \( \dot{\xi}_o \) is the fraction of martensite existing before the current transformation, \( a_M = \pi/(M_s - M_f) \) and \( a_A = \pi/(A_f - A_s) \). Note that, as in the Brinson [1] kinetic law, the \( \dot{\xi}_o \), memory term has to be maintained for the other rules, because the transformation is dependent on the previous martensite fraction. Taking this parameter into consideration, small cycles are intrinsically modelled. Thus, the exponential flow law adopted by Tanaka et al. [25], expressed by Koistinen and Marburger [34] and Wang and Inoue [35], delivers

\[
\dot{\xi} = (\dot{\xi}_o - 1)e^{a_M(M_s - T) + b_M\sigma} + 1 \quad \text{and} \quad \dot{\xi} = \dot{\xi}_o e^{a_A(A_f - T) + b_A\sigma}
\]

for forward and reverse transformations, with \( a_M = -2\ln(10)/(M_s - M_f) \), \( b_M = a_M(M_s - M_f) / (\Delta M_s) \), \( a_A = 2\ln(10)/(A_f - A_s) \) and \( b_A = a_A(A_f - A_s) / (\Delta A_s) \), in which \( \Delta M_s \) and \( \Delta A_s \) are the width along the stress axis of the martensitic and austenitic transformation strips. Given the nature of the exponential function, for the determination of the previous constants the transformation was considered completed when \( \dot{\xi} \) is equal to 0.99, which is a common criterion in metallurgy [25]. This results in a discontinuity in the constitutive law, which is not compatible with nonlinear equations solution methods that may be employed. It is resolved by dividing the right-hand member of the kinetic rule by the above mentioned value. This does not change the nature of the kinetic function and has no relevant influence on the solution. The linear kinetic law is defined by

\[
\dot{\xi} = (1 - \dot{\xi}_o) \frac{[\sigma] - \sigma^M_f}{\sigma^M_s - \sigma^M_f} + \dot{\xi}_o \quad \text{and} \quad \dot{\xi} = \dot{\xi}_o \frac{[\sigma] - \sigma^A_s}{\sigma^A_f - \sigma^A_s}
\]

Given the dependency of the superelasticity effect on the temperature, a more complex model is required for general behaviour prediction. If an energy balance law is added, it is possible to obtain a strain rate dependent constitutive model, capable of tracing the behaviour of SMAs for both quasi-static and dynamic loading [17,20]. The heat equation can be written as \( \rho c_T \dot{T} + \text{div} (q) + \gamma(T - T_0) = b \), in which \( \rho \) is the density of the material, \( c_T \) is the specific heat, \( \text{div} \) is the divergence operator, \( q \) is the heat flux, \( \gamma \) is a heat exchange parameter linking the temperature difference between the surface of the element and the surroundings and \( b \) is the heat source [20,36].

Given the high thermal conductivity of SMAs and the uniform distribution of heat sources, for elements with small sections, such as wires and bars, a uniform temperature inside the body may be assumed and conduction resistance
may be neglected [15,20,36]. The validity of this assumption is verified by the calculation of the Biot number, defined as the ratio between the conductive and the convective resistance, in which case it should be lower than 0.10 [17].

\( b \) is due to two contributions: the heat production associated with internal friction, i.e., hysteretic dissipation, and the contribution from the transformation latent heat. It may be defined as
\[
b = c_L \rho \dot{\xi} + W,
\]
in which \( c_L \) is the latent heat and \( W \) is the energy dissipated by hysteresis [17,18,20], whose influence may be neglected when compared to latent heat contribution [20,37]. The energy balance law can then be written as
\[
-\rho c_p V \dot{T} + h A_i (T - T_0) = b V,
\]
in which \( V \) is the specimen volume, \( h \) is the convection heat transfer coefficient and \( A_i \) is the interface area of the SMA [17,20,37].

The results obtained with different macro-scale models depend greatly on the kinetic rule adopted, because the mechanical laws are essentially similar, with any differences being associated with the Young’s modulus of the transforming material. Nearly identical results were obtained with different models based on the schemes of Voigt and Reuss when the same kinetic law was used. Because the transformation strain is high when compared to the elastic strain, the differences between mechanical laws are mainly lost [29].

As seen, the majority of the uniaxial mechanical laws can be derived from the Tanaka et al. rate model [29]. For superelastic behaviour, this mechanical law, in the time continuous form, is written as
\[
\sigma = E(\xi) (\varepsilon - \varepsilon_L) + \Theta (T - T_0) \quad \text{with} \quad E(\xi) = E_A + (E_M - E_A) \xi
\]
This equation is written with \( \sigma \) in an implicit form, thus an iterative procedure must be applied, in which case the complete Newton-Raphson method may be chosen. The mechanical law may be written in the time discrete evolutive form as
\[
\varepsilon^{i+1}_n = \varepsilon^n + \frac{\partial \varepsilon(\sigma_n)}{\partial \sigma} (\sigma^{i+1}_n - \sigma^n)
\]
in which \( n \) refers to the instant and \( j \) refers to the iteration step. In the transformation zone, the derivative present in Equation (7) has to be computed according to the kinetic law adopted. The kinetic laws can then be treated separately.

Figure 1 shows a generic isothermal stress-strain curve for each of the kinetic laws considered.

The heat balance law may now be coupled with the model. It can be integrated with regard to time, by means of the backward Euler method [17,38], and introducing the iterations regarding the fixed-point method results in
\[
T^{i+1}_n = T_{n-1} + \Delta t \left[ \frac{b_n}{\rho c_p} - \frac{h A_i (T^n_i - T_0)}{\rho c_p V} \right]
\]
where \( \Delta t \) is the time interval. Thus, before performing the material state determination, the temperature of the material must be computed. As a result, according to the Clausius-Clapeyron equations, different critical stresses are considered for each time instant. If the strain steps are sufficiently small, \( \dot{\xi}_n \) may be taken approximately equal to \( \dot{\xi}_{n-1} \), with insignificant error in the case of the cosine and the exponential laws and without error in the case of the linear kinetic law. It is not possible to generally define an analytical equation relating strain with time, thus a numerical procedure, such as the finite difference approximations method, may be adopted to obtain the martensitic fraction derivative with respect to time.

Figure 2 shows a general response of a NiTi SMA subjected to two complete cycles of 6.5% strain, considering coupling between the mechanical, the kinetic and the heat balance equations.
4. Conclusions

A bibliographic review of the behaviour of SMAs has been presented. The review focused on the characterization of SMAs and on uniaxial models for prediction of the response of these materials. It was concluded that residual strains may be of relevance for real design projects. The extent of this effect has shown to be a function of the composition of the alloy, the specimen size and the thermomechanical treatments, that resulted in contradictory reports. Nevertheless, research indicates that ageing treatments and annealing together with cyclic training are highly efficient in stabilizing the superelasticity of NiTi, commonly considered the most suitable SMA for a number of engineering solutions. Even though different tension/compression behaviour of SMAs has been observed, researchers most often consider the same SMA behaviour in tension/compression. The temperature dependence of SMAs was also addressed. Together with self-heating, it should be accounted for in the response prediction. Prestress and temperature ageing may also be important for actual design. Research has shown that significant changes in the material behaviour may occur for SMAs subjected to permanent prestress. Structural fatigue is not of concern for seismic applications, unlike for rain and/or wind vibration control. Generally, it was found that there is a deficit in research in aspects of the behaviour of SMAs fundamental to many applications. The constitutive models used for predicting the behaviour of SMAs were also reviewed. The focus on uniaxial models was due to the fact that the majority of applications are based on wires and bars, in which case these models are adequate. They generally combine a mechanical and a kinetic law, which governs the martensitic fraction of the material. The majority of the mechanical laws are similar, with differences arising from the Young’s modulus considered, which nevertheless are limited. Differences result mainly from the adopted kinetic rule. Nevertheless, implementation indicates that the cosine and the linear laws deliver very similar stress-strain curves. Even though the exponential kinetic law is the one which differs the most from the previous two, the differences are moderate. A strain rate dependent model is obtained when the mechanical and kinetic laws are coupled with an energy balance law. The general response of a SMA dynamic test is also presented.

References


