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Distributed State Fusion Kalman Predictor Based on WLS Algorithm

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Abstract

Based on the weighted least square (WLS) algorithm, a unified multisensor distributed state fusion estimation criterion is presented in this paper, which includes the optimal weighted fusion algorithms weighted by matrices, scalars and diagonal matrices as the special cases. Furthermore, the corresponding optimal distributed state fusion Kalman predictor is presented for the multisensor linear discrete time-varying stochastic control systems with different measurement matrices and correlated noises. A simulation example for a tracking system with 3-sensor shows its effectiveness.

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Keywords: Multisensor information fusion; optimal information fusion criterion; distributed state fusion; Kalman predictor

1. Introduction

Multisensor information fusion has received considerable attentions with the development of high-tech fields such as military, defense and information war in recent years [1]. For Kalman filtering-based fusion, two basic fusion methods are centralized and distributed fusion methods, depending on whether raw data are used directly for fusion or not [2]. The state fusion methods also include the centralized and distributed Kalman filtering methods [3]. The centralized fusion method can give the globally optimal state estimation by directly combining the local measurement data, but its disadvantage is that it may require a larger computational burden. The distributed fusion method can give the globally suboptimal state estimation by weighting the local state estimators. This method has considerable advantages that it can facilitate fault detection and isolation more conveniently, and can reduce the computational burden.

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Some results about the distributed state fusion algorithm have been presented over the past few years. The linear minimum variance optimal fusion estimation formula weighted by matrices was most early presented by Carlson [4], which considered the case that the estimation errors of each local sensors are uncorrelated. Hereafter, the maximum likelihood optimal fusion estimation formula was presented by Kim [5], with the consideration of correlated estimation errors of local sensors. However, it is required that the estimation errors obey normal distribution. Based on the Lagrange multiplier method and matrix differential calculation, the same result as that in [5] was presented by [6], considering the correlation between estimation errors and not requiring the normal distribution of the estimation errors. Based on the weighted least square (WLS) method, the unified distributed state fusion criterion is presented in this paper, which includes the results in [6] as the special cases. Furthermore, the corresponding optimal distributed state fusion N steps ahead Kalman predictor is presented for the multisensor linear discrete time-varying stochastic control systems with different measurement matrices and correlated noises.

2. Problem Formulation

Consider the multisensor linear discrete time-varying stochastic control systems

$$x(t+1) = \Phi(t)x(t) + B(t)u(t) + \Gamma(t)w(t) \quad (1)$$

$$y_i(t) = H_i(t)x(t) + v_i(t), \quad i = 1, \dots, L \quad (2)$$

where t is the discrete time, $x(t) \in R^n$ is the state, $y_i(t) \in R^{m_i}$ is the measurement of the i th sensor, $u(t) \in R^p$, $\Phi(t)$, $B(t)$, $\Gamma(t)$ and $H_i(t)$ are known matrices with compatible dimensions, $w(t)$ is the input noise, and $v_i(t)$ is the measurement noise of the i th sensor.

Assumption 1 $w(t) \in R^r$ and $v_i(t) \in R^{m_i}$ are correlated white noises with zero means:

$$E \left\{ \begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix} \begin{bmatrix} w^T(k) & v_i^T(k) \end{bmatrix} \right\} = \begin{bmatrix} Q(t) & S_i(t) \\ S_i^T(t) & R_i(t) \end{bmatrix} \delta_{ik}, \quad i = 1, \dots, L \quad (3)$$

where E is the mathematical expectation, the superscript T denotes the transpose, and $\delta_{tt} = 1$, $\delta_{ik} = 0 (t \neq k)$, the variance and covariance matrices for $w(t)$ and $v_i(t)$ are $Q(t)$, $R_i(t)$ and $S_i(t)$, respectively.

Assumption 2 The measurement noises $v_i(t)$ and $v_j(t)$ are correlated, i.e.

$$E \left\{ \begin{bmatrix} v_i(t) \\ v_j(t) \end{bmatrix} \begin{bmatrix} v_i^T(k) & v_j^T(k) \end{bmatrix} \right\} = \begin{bmatrix} R_i(t) & R_{ij}(t) \\ R_{ij}^T(t) & R_j(t) \end{bmatrix} \delta_{ik}, \quad i = 1, \dots, L \quad (4)$$

with the definition $R_i(t) = R_{ii}(t)$.

Assumption 3 $x(0)$ is correlated with $w(t)$ and $v_i(t)$, $i = 1, \dots, L$, and $Ex(0) = \mu$, $\text{cov } x(0) = P$, where cov is the covariance.

Assumption 4 $u(t)$ is the known input.

The problem is to find the local and distributed state fusion N steps ahead Kalman predictors $\hat{x}_i(t+N|t)$, $N \geq 1$, $i = 1, \dots, L$ and $\hat{x}(t+N|t)$ based on the measurements $(y_i(t), y_i(t-1), \dots, y_i(0))$ and $(u(t-1), u(t-2), \dots, u(0))$.

3. Local Kalman Predictors

Lemma 1 [3] For the multisensor system (1) and (2) with Assumptions 1-4, the local N steps ahead Kalman predictor for the i th sensor subsystem is given as follows

$$\hat{x}_i(t+N|t) = \Phi(t+N, t+1)\hat{x}_i(t+1|t) + \sum_{s=t+2}^{t+N} \Phi(t+N, s)B(s-1)u(s-1), \quad N > 1 \tag{5}$$

$$\hat{x}_i(t+1|t) = \Psi_{pi}(t)\hat{x}_i(t|t-1) + B(t)u(t) + K_{pi}(t)y_i(t) \tag{6}$$

$$\Psi_{pi}(t) = \Phi(t) - K_{pi}(t)H_i(t) \tag{7}$$

$$K_{pi}(t) = [\Phi(t)P_i(t|t-1)H_i^T(t) + \Gamma(t)S_i(t)]Q_{ei}^{-1}(t) \tag{8}$$

$$Q_{ei}(t) = H_i(t)P_i(t|t-1)H_i^T(t) + R_i(t) \tag{9}$$

with the definition

$$\Phi(t, s) = \Phi(t-1)\Phi(t-2)\cdots\Phi(s), \quad t > s, \quad \Phi(s, s) = I_n \tag{10}$$

The optimal N steps ahead prediction error variance matrix $P_i(t+N|t) = E[\tilde{x}_i(t+N|t)\tilde{x}_i^T(t+N|t)]$ is given by

$$P_i(t+N|t) = \Phi(t+N, t+1)P_i(t+1|t)\Phi^T(t+N, t+1) + \sum_{j=2}^N \Phi(t+N, t+j)\Gamma(t+j-1) \times Q(t+j-1)\Gamma^T(t+j-1)\Phi^T(t+N, t+j) \tag{11}$$

with the one step ahead prediction error variance matrix $P_i(t+1|t)$ satisfying the following Riccati equation

$$P_i(t+1|t) = \Phi(t)P_i(t|t-1)\Phi^T(t) - [\Phi(t)P_i(t|t-1)H_i^T(t) + \Gamma(t)S_i(t) \times [H_i(t)P_i(t|t-1)H_i^T(t) + R_i(t)]^{-1}[\Phi(t)P_i(t|t-1)H_i^T(t) + \Gamma(t)S_i(t)]^T + \Gamma(t)Q(t)\Gamma^T(t)] \tag{12}$$

with the initial values $\hat{x}_i(0|0) = \mu, \quad P_i(0|0) = P$.

The N steps ahead prediction error covariance matrix is given by

$$P_{ij}(t+N|t) = \Phi(t+N, t+1)P_{ij}(t+1|t)\Phi^T(t+N, t+1) + \sum_{j=2}^N \Phi(t+N, t+j)\Gamma(t+j-1) \times Q(t+j-1)\Gamma^T(t+j-1)\Phi^T(t+N, t+j) \tag{13}$$

with the one step ahead prediction error variance matrix $P_{ij}(t+1|t)$ given as follows

$$P_{ij}(t+1|t) = \Psi_{pi}(t)P_{ij}(t|t-1)\Psi_{pj}^T(t) + \Gamma(t)Q(t)\Gamma^T(t) - K_{pi}(t)S_i^T(t)\Gamma^T(t) - \Gamma(t)S_j(t)K_{pj}^T(t) + K_{pi}(t)R_{ij}(t)K_{pj}^T(t) \tag{14}$$

where $i, j = 1, \dots, L, \quad i \neq j$, the initial value is $P_{ij}(0|0) = P$.

4. Unified Optimal Distributed State Fusion Kalman Predictor

Setting the local N steps ahead prediction error and variance matrix as

$$\tilde{x}_i(t+N|t) = x(t+N) - \hat{x}_i(t+N|t) \tag{15}$$

$$P_{ij}(t+N|t) = E[\tilde{x}_i(t+N|t)\tilde{x}_j^T(t+N|t)] \tag{16}$$

then applying (15) yields the extended dimension form of local predictors as follows

$$\hat{x}^{(0)}(t+N|t) = x^{(0)}(t+N) - \tilde{x}^{(0)}(t+N|t) \tag{17}$$

where we define

$$\hat{x}^{(0)}(t+N|t) = \begin{bmatrix} \hat{x}_1(t+N|t) \\ \vdots \\ \hat{x}_L(t+N|t) \end{bmatrix}, \quad x^{(0)}(t+N) = \begin{bmatrix} x_1(t+N) \\ \vdots \\ x_L(t+N) \end{bmatrix}, \quad \tilde{x}^{(0)}(t+N|t) = \begin{bmatrix} \tilde{x}_1(t+N|t) \\ \vdots \\ \tilde{x}_L(t+N|t) \end{bmatrix} \tag{18}$$

and $\tilde{x}^{(0)}(t+N|t)$ has the prediction error variance matrix $P^{(0)}(t+N|t)$ as follows

$$P^{(0)}(t+N|t) = \begin{bmatrix} P_{11}(t+N|t) & \cdots & P_{1L}(t+N|t) \\ \vdots & & \vdots \\ P_{L1}(t+N|t) & \cdots & P_{LL}(t+N|t) \end{bmatrix} \quad (19)$$

Eq. (17) can be considered as L unbiased estimations for $x(t+N)$, with the local estimation error $\tilde{x}_i(t+N|t)$ and the local estimation error covariance matrix $P_{ij}(t+N|t)$. So we have the following measurement model

$$\hat{x}^{(0)}(t+N|t) = ex(t+N) - \tilde{x}^{(0)}(t+N|t) \quad (20)$$

with the definition $e^T = [I_n, I_n, \dots, I_n]$. We have the WLS estimator (Gauss-Markov estimator) [7, 8] for $x(t)$ as

$$\hat{x}(t+N|t) = (e^T P^{(0)-1}(t+N|t)e)^{-1} e^T P^{(0)-1}(t+N|t) \hat{x}^{(0)}(t+N|t) \quad (21)$$

where we define $P^{(0)-1}(t+N|t) = [P^{(0)}(t+N|t)]^{-1}$. Applying the definitions of e^T and $x^{(0)}(t+N)$, we have

$$\hat{x}(t+N|t) = \sum_{i=1}^L A_i(t) \hat{x}_i(t+N|t), \quad [A_1(t), \dots, A_L(t)] = (e^T P^{(0)-1}(t+N|t)e)^{-1} e^T P^{(0)-1}(t+N|t) \quad (22)$$

then we can easily obtain the optimal fusion error variance matrix $P(t+N|t) = E[\tilde{x}(t+N|t)\tilde{x}^T(t+N|t)]$ as follows

$$P(t+N|t) = (e^T P^{(0)-1}(t+N|t)e)^{-1} e^T P^{(0)-1}(t+N|t) P^{(0)}(t+N|t) P^{(0)-1}(t+N|t) e (e^T P^{(0)-1}(t+N|t)e)^{-1} = (e^T P^{(0)-1}(t+N|t)e)^{-1} \quad (23)$$

It is easily proved that [3]

$$P(t+N|t) \leq P_i(t+N|t), \quad i=1, \dots, L \quad (24)$$

The above derivation can be summarized as the following theorem:

Theorem 1 Based on the L known unbiased estimations $\hat{x}_i(t+N|t)$, $i=1, \dots, L$ for the stochastic vectors $x(t) \in R^n$ of L sensors and the known estimation error covariance matrices $P_{ij}(t+N|t)$, $i, j=1, \dots, L$, using the WLS method, the distributed state fusion N steps ahead Kalman predictor is given by

$$\hat{x}(t+N|t) = (e^T P^{(0)-1}(t+N|t)e)^{-1} e^T P^{(0)-1}(t+N|t) \hat{x}^{(0)}(t+N|t) \quad (25)$$

with the optimal fusion error variance matrix as

$$P(t+N|t) = (e^T P^{(0)-1}(t+N|t)e)^{-1} \quad (26)$$

and we have the relationship as follows

$$P(t+N|t) \leq P_i(t+N|t), \quad i=1, \dots, L \quad (27)$$

Remark 1 Theorem 1 can directly yield the same form as that of the fusion criterion weighted by matrices in [6]. Under the condition of Theorem 1, the measurement error variance matrix in measurement model (20) is taken as

$$P^{(0)}(t+N|t) = \begin{bmatrix} trP_{11}(t+N|t) & \cdots & trP_{1L}(t+N|t) \\ \vdots & & \vdots \\ trP_{L1}(t+N|t) & \cdots & trP_{LL}(t+N|t) \end{bmatrix} \quad (28)$$

Then we can obtain the same form as that of the fusion estimation criterion weighted by scalars in [6].

Under the condition of Theorem 1, the measurement error matrix in measurement model (20) is taken as

$$P^{(0)}(t+N|t) = \begin{bmatrix} P_{11}^{(ii)}(t+N|t) & \cdots & P_{1L}^{(ii)}(t+N|t) \\ \vdots & & \vdots \\ P_{L1}^{(ii)}(t+N|t) & \cdots & P_{LL}^{(ii)}(t+N|t) \end{bmatrix} \tag{29}$$

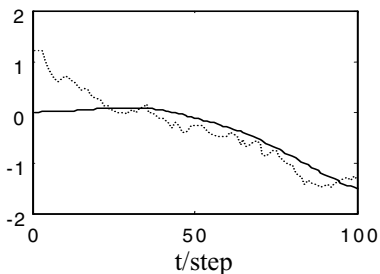
where $P_{kj}^{(ii)}(t+N|t)$ is the (i,i) th diagonal element of $P_{kj}(t+N|t)$. Then we can obtain the same form as that of the fusion estimation criterion weighted diagonal matrices in [6]. So we can obtain the same conclusion as that in [3] according to the information quality of the chosen error matrix: the accuracy of the fusion estimation weighted by matrices is higher than that of the fusion estimation weighted by scalars, and the accuracy of the fusion estimation weighted by diagonal matrices is between both of them.

5. Simulation Example

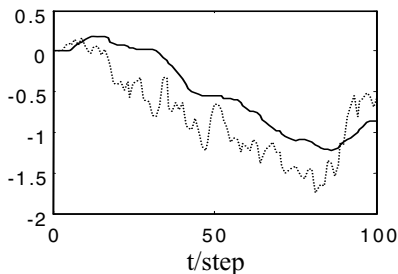
Consider the 3-sensor target tracing system (1) and (2), where $\Phi = \begin{bmatrix} 1 & T_0 & 0.5T_0^2 \\ 0 & 1 & T_0 \\ 0 & 0 & 1 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$,

$H_1 = [1 \ 0 \ 0]$, $H_2 = [0 \ 1 \ 0]$, $H_3 = [0 \ 0 \ 1]$, $T_0 = 0.03$ is the sampled period, $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T$, and $x_1(t)$, $x_2(t)$ and $x_3(t)$ respectively are the position, velocity and acceleration at time tT_0 , $w(t)$, $v_i(t)$ are correlated white noise with zero mean and variances as $Q_w = 0.16$, $R_1 = 1$, $R_2 = 2$, $R_3 = 3.16$, $R_{ij} = 0$, $i, j = 1, \dots, L$, $i \neq j$, $S_1 = S_2 = 0$, $S_3 = 0.16$. The problem is to find the local and optimal distributed state fusion two steps ahead Kalman predictors $\hat{x}_i(t+2|t)$, $i = 1, \dots, L$ and $\hat{x}(t+2|t)$.

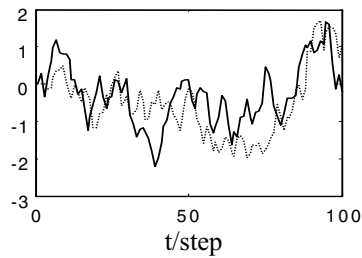
The simulation results are shown in Fig. 1 and Fig.2. The comparison curves between the state $x(t)$ and the distributed state fusion two steps ahead Kalman predictor $\hat{x}(t+2|t)$ are given by Fig. 1, where the lines denote the true value, and the curves denote the estimates. The comparison curves of traces between the local and distributed state fusion estimation error variance matrices are given by Fig. 2. They show that the accuracy of the distributed state fusion Kalman predictor is higher than that of each local predictor.



(a) The comparison curves for position



(b) The comparison curves for velocity



(c) The comparison curves for acceleration

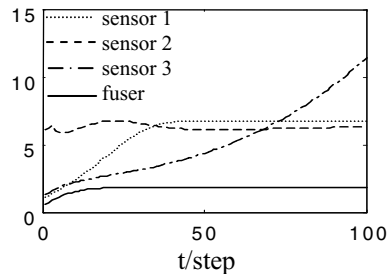
Fig. 1. The comparison curves between the state $x(t)$ and the distributed state fusion Kalman predictor $\hat{x}(t+2|t)$ 

Fig. 2. The comparison curves of traces between the local and distributed state fusion Kalman prediction error variance matrices

6. Conclusions

The unified and universal distributed state fusion criterion is presented in this paper based on the weighted least square method, which includes the results in [6] as the special cases. It avoids the Lagrange multiplier method and matrix differential calculation. Meanwhile, the corresponding optimal distributed state fusion Kalman predictor is presented for the multisensor discrete linear time-varying stochastic control systems with different measurement matrices and correlated noises, based on the proposed optimal fusion criterion. The simulation example shows that the accuracy of the optimal distributed state fusion Kalman predictor is higher than that of each local predictor.

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