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A bibliography on roots of polynomials *

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Abstract

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A categorized bibliography on roots of polynomials is presented, covering the period from the earliest times until late 1990. The actual bibliography is on a computer diskette accompanying this issue, together with programs designed to print parts of the main work, such as entries belonging to a certain category.

Keywords: Polynomial roots; nonlinear equations; zeros; bibliography.

Introduction

The calculation of roots of polynomials is one of the oldest of mathematical problems. The solution of quadratics was known to the Arab scholars of the early Middle Ages, the most famous of them being Omar Khayyam. The cubic was first solved in closed form by G. Cardano in the mid-16th century, and the quadratic soon afterwards. However, N.H. Abel in the early 19th century showed that polynomials of degree five or more could not be solved by a formula involving roots of expressions in the coefficients, as those of degree up to four could be. Since then (and for some time before in fact) researchers have concentrated on numerical (iterative) methods such as the famous Newton's method of the 17th century, Bernoulli's method of the 18th, and Graeffe's

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method of the early 19th century. Of course there have been a plethora of new methods in the 20th century, especially since the advent of electronic computers. These include the Jenkins–Traub method, Larkin's method, Muller's method, and several methods for simultaneous approximation of all the roots, starting with the Durand–Kerner method.

Polynomial roots have many applications. For one example, in control theory we are led to a linear differential equation with constant coefficients:

$$a_0 \frac{\mathrm{d}^n y}{\mathrm{d}x^n} + a_1 \frac{\mathrm{d}^{n-1} y}{\mathrm{d}x^{n-1}} + \cdots + a_{n-1} \frac{\mathrm{d}y}{\mathrm{d}x} + a_n y = 0.$$

As a trial solution we set $y = C\,\mathrm{e}^{\lambda x}$, where λ is as yet unknown. This gives

$$(a_0\lambda^n + a_1\lambda^{n-1} + \cdots + a_{n-1}\lambda + a_n)C\,\mathrm{e}^{\lambda x} = 0.$$

For a nontrivial solution ($C \neq 0$) the expression in the brackets must be zero, leading to a polynomial equation.

Another application arises in certain financial calculations, e.g., to compute the rate of return on an investment where a company buys a machine for, (say) \$100,000. Assume they rent it out for 12 months at \$5000/month, and for a further 12 months at \$4000/month. It is predicted that the machine will be worth \$25,000 at the end of this period. The solution goes as follows. The present value of \$1 received n months from now is $1/(1+i)^n$, where i is the monthly interest, as yet unknown. Hence,

$$100{,}000 = \sum_{j=1}^{12} \frac{5000}{(1+i)^j} + \sum_{j=13}^{24} \frac{4000}{(1+i)^j} + \frac{25{,}000}{(1+i)^{24}}.$$

Hence,

$$100{,}000(1+i)^{24} - \sum_{j=1}^{12} 5000(1+i)^{24-j} \sum_{j=13}^{24} 4000(1+i)^{24-j} - 25{,}000 = 0,$$

a polynomial equation in $1 + i$.

In coding theory, there arise polynomials over finite fields, although the details will not be described here.

In this work we present a comprehensive bibliography on roots of polynomials, covering (hopefully) most published work between the "Dawn of History" and mid-1990. This could be of help to anyone contemplating research into methods of calculating polynomial roots. To keep the length manageable we have excluded theses, unpublished reports, and works in very obscure journals or very obscure languages.

The general subject of polynomial root finding has been divided into 29 categories, and each entry has been allocated one or more category numbers. A list of category numbers and their meanings appears in Appendix A.

The actual bibliography is not printed here, but is contained (as a LaTeX file (bibliog.tex) consisting of some 3500 entries) on a $3\frac{1}{2}$ inch diskette suitable for reading on an IBM PC or compatible. This disk is attached to the back cover of this journal issue.

The disk contains a program (bibliog.exe) to extract information from the main bibliography file (bibliog.tex), and the files espart.sty and espart12.sty. (The LaTeX documentstyle `espart`, developed by Elsevier Science Publishers, is a preprint style which is available for authors who want to submit a paper as an electronic file to one of the Elsevier journals. This style is used in the output of the program.)

Before starting the program, copy the files on the disk to your working directory on your hard disk. On an IBM PC or compatible the program is now ready to use by entering

```
bibliog
```

(The C source of the program is available for other computers, see Appendix B.)

After starting the program, you may choose one of the following options:

- `c` *range* selects categories in the range *range* (see the examples);
- `g` produces the actual selection (after a `c` and/or `s` command);
- `i` *file* changes the input file to *file* (default bibliog.tex) to perform additional selections;
- `l` gives a list of the available categories (see also Appendix A);
- `o` *file* changes the output file to *file* (default sel.tex);
- `s` *string* selects a string/keyword;
- `x` exit;
- `?` help.

Examples

`c2` gives all references of category 2: Newton's method;

`c3,4,7-12,18` gives all references that are in (at least) one of the following categories: 3, 4, 7, 8, 9, 10, 11, 12 and 18.

`s J. Comput. Appl. Math.` gives papers on roots of polynomials published in this journal;

`s Cauchy` gives all references in which the word "Cauchy" appears, e.g., Cauchy's own papers.

The program ignores LaTeX codes in the strings. For instance, `s equation` also finds all entries containing (the French) équation.

For selecting all entries which belong to both category 22 and 24, do the following:

`c22` (To select all entries from category 22.)

`o cat22.tex` (To change the output file to, for instance, cat22.tex instead of sel.tex, in order that this file can be used as input file for further selection (instead of the complete file bibliog.tex). Input and output file must have different names!)

`g` (To perform the actual selection.)

The selection is now written to the file cat22.tex. Now again run the program, and continue as follows.

`i cat22.tex` (Use the file cat22.tex, which contains all references of category 22, as input file for further selection.)

`c24` (To select all entries from category 24 that are in cat22.tex.)

`g`

The output file sel.tex now contains all references belonging to both category 22 and 24.

Appendix A. Available categories

1. Bracketing methods (real roots only).
2. Newton's method.
3. Simultaneous root-finding methods.
4. Graeffe's method.
5. Integral methods, esp. Lehmer's.
6. Bernoulli's and QD method.
7. Interpolation methods such as secant, Muller's.
8. Minimization methods.
9. Jenkins–Traub method.
10. Sturm sequences, greatest common divisors, resultants.
11. Stability questions (Routh–Hurwitz criterion, etc.).
12. Interval methods.
13. Miscellaneous.
14. Lin and Bairstow methods.
15. Methods involving derivatives higher than first.
16. Complexity, convergence and efficiency questions.
17. Evaluation of polynomials and derivatives.
18. A priori bounds.
19. Low-order polynomials (special methods).
20. Integer and rational arithmetic.
21. Special cases such as Bessel polynomials.
22. Vincent's method.
23. Mechanical devices.
24. Acceleration techniques.
25. Existence questions.
26. Error estimates, deflation, sensitivity, continuity.
27. Roots of random polynomials.
28. Relation between roots of a polynomial and those of its derivative.
29. *N*th roots.

Appendix B. The source code of the program

The disk supplied with this issue contains the ready-to-use C program bibliog.exe for use on IBM PCs or compatible. The source code of this program (consisting of one "c" and one "h" file) is also available:

\ansi\bibliog.c \ansi\texstr.h	for ANSI C compilers,	\unix\bibliog.c \unix\texstr.h	for Unix C compilers.

Copy the relevant two source files to your working directory and type

```
cc bibliog.c
```

(if `cc` is the name of your C compiler). This should work immediately; consult your system management if compilation fails.