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The performances optimization of finger seal based on fuzzy game theory

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Abstract

Leakage and abrasion are two key performances of finger seals (FS). They not only contradict each other in FS design but also relate to many design parameters. Moreover, in the multi-objective optimization progress, the problems of optimizing results decision and preference requirement for optimization objectives are still challenge to researcher. So far, they are still important influence factors for advanced FS design. Therefore, the current work presents a new multi-objective optimization method by introducing game theory and fuzzy comprehensive evaluation theory. The optimizing results are compared to that of the general optimization method and finite element method (FEM). The study show that the FS, which is obtained by presented optimization method, has good performances. Compared respectively with the general optimization method and FEM, the computational results indicate that the presented method can effectively reflect the different response requirements of optimization objectives. Furthermore, the decision-making difficulty for multi-objective optimization of FS performances is significantly reduced.

Keywords: game theory, fuzzy comprehensive evaluation, FS, multi-objective optimization, FEM

1. Introduction

FS is a revolutionary technology in air-to-air sealing for secondary flow control and gas path sealing in gas turbine engines. It has been demonstrated to be considerably cheaper than a brush seal but the sealing

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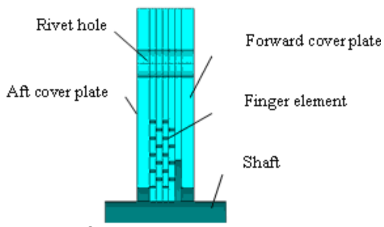
performance is similar to brush seal and better than labyrinth seal. The cost to produce FS is estimated to be 40 to 50 percent of the cost to produce brush seals [1]. Therefore it is promising in improving aero-engine performance, increasing operation life and decreasing operation cost.

Research indicates that increasing stiffness can decrease the hysteresis and leakage of FS, while the contact pressure between finger feet and rotor increases synchronously and leads to the reduction of operation life for FS[2]. This contradiction is a key problem for FS application. Furthermore, the sealing performance and operation life perhaps have different response requirements in FS application. Sometimes the requirement, called "preference", is unclear. For the FS multi-objective optimization with unclear preference, it is found that the results gained by general optimization methods are still unsatisfactory in preference and decision-making. Therefore, it is necessary to present an effective method to solve the above problems for FS.

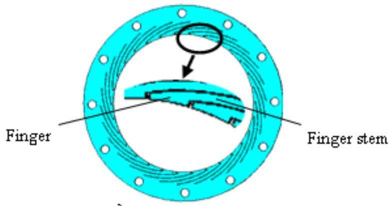
For the FS multi-objective optimization, the leakage (Q) and operation life (expressed by using wear ratio V generally) are taken as the two conflicting objectives. By combining fuzzy theory and the game theory, the multi-objective optimization results of different preference requirement for FS could be gained.

2. FS Structures

The structure of FS is shown in Fig 1. The FS is to process a set of finger beams in the thin slice and make the finger slices staggered close together to cover the adjacent interstice. The multiple finger slices and two cover plates are assembled with the rivet tightly. The seal is fitted over the rotating shaft or rotor with a small amount of clearance or interference, depending on the application. The fluid through the seal is impeded by the staggered fingers/pads as well as the radial contact between the rotor and the FS feet. From the FS geometric characteristics, we find that the angle of finger stems (φ), thickness of the finger element (s), finger length (The finger length is control by the radius of base circle (r) if the shape-curve of finger stems is involute), the height of finger foot (h) and the angle between finger stems (δ) are the main parameters for FS.



a) Axial structure of FS



b) The shape of FS

Figure 1 Structure of FS

The fingers' compliance allows radial adjustment to rotor excursions when operation. With the runout of shaft, the fingers move out radially but do not recover to their original position, since the friction between the aft cover plate and fingers is greater than the restoring force in the fingers. Then the seal fails to work with appearance of leakage. The previous research work indicates that increasing the finger stiffness can reduce leakage. However, the increase of stiffness leads to the rise of contact pressure between the finger pad and the rotor. Thereby, it worsens the wear and decreases the operation life.

3. Performance optimization model for FS based on fuzzy game theory

3.1 Optimization model based on fuzzy game theory

Nash equilibrium is a game theory first proposed by J. F. Nash in 1951[3]. It was initially developed to solve problems in economics. For an optimization problem with N-objectives, a Nash strategy consists of N players. Each player is in charge of one objective and has its own strategy group and criterion. To improve its criterion, each player searches its own best strategy in the search space. And now all the criteria of other players are fixed. At the end of each game circulation, the players exchange their best strategies. Game proceeds until no one can further improve its criterion, and the game state is at a situation called Nash equilibrium. For the two-player Nash equilibrium, the process is presented as follows[4-5].

Suppose W_1 as the strategy space of the first player, W_2 as the strategy space of the second player, then the strategy pair $(w^*_{1-}, w^*_{2-}) \in W_1 \cdot W_2$ can be a Nash equilibrium only when

$$f_{W1}(w^*_{1-}, w^*_{2-}) = \inf_{x \in W_1} f_{W2}(w_{1-}, w^*_{2-})$$

$$f_{W2}(w^*_{1-}, w^*_{2-}) = \inf_{y \in W_2} f_{W1}(w^*_{1-}, w_{2-}) \quad (1)$$

In the game, suppose the player set is $N = \{N_1, N_2, \cdots, N_L\}$, where, L is the number of players; the strategy space for the ith player is $W_i = \{w_{i1}, w_{i2}, \cdots, w_{iki}\}$, where, ki is the strategy number of the ith player; the payoff function of the ith player is $f_{Wi} = f_{Wi}(w_1, w_2, \cdots, w_L, \xi)$, $w_i \in W_i$, where, ξ is the stochastic factors of the payoff function. When the players' payoff requires different response to objectives, the strategy model for fuzzy game can be expressed as follows

$$G = \{ W_1, W_2, \cdots, W_L ; f_{W1}, f_{W2}, \cdots, f_{WL} ; \xi \} \quad (2)$$

For the convenience of analysis, the payoff functions can be transformed as

$$F_{W1}(f_{W1}, f_{W2}, \xi) = (a - f_{W1} + b f_{W2} + \xi)(f_{W1} - f_{c1})$$

$$F_{W2}(f_{W1}, f_{W2}, \xi) = (a - f_{W2} + b f_{W1} + \xi)(f_{W2} - f_{c2}) \quad (3)$$

Where, a is a constant, b is the assumed influence factor between the leakage and operation life. f_{c1} and f_{c2} are the smallest payoffs of leakage and operation life, respectively. Their values relate to the preference requirement. The stochastic factor ξ influencing to the payoff functions follows normal school. The expectation payoff and payoff variance are respectively gained from Eqs.3.

$$E(F_{W1}) = (a - f_{W1} + b f_{W2})(f_{W1} - f_{c1})$$

$$E(F_{W2}) = (a - f_{W2} + b f_{W1})(f_{W2} - f_{c2})$$

$$D(F_{W1}) = (f_{W1} - f_{c1})^2$$

$$D(F_{W2}) = (f_{W2} - f_{c2})^2 \quad (4)$$

For the multi-objective optimization of FS, the player set $N = \{Q, V\}$. Where Q is the player leakage; V is the abrasion of FS. f_{W1} is the payoff function of Q and f_{W2} is the payoff function of V. In engineering, the leakage and operation life are supposed to be as small as possible, while their corresponding payoffs are expected to increase. So the payoff functions of the two players can be respectively expressed as

$$f_{w1}(\varphi^*, \delta^*, s^*, r, h) = 1 - Q/[Q]$$

$$f_{w2}(r^*, h^*, \varphi, \delta, s) = 1 - V/[V] \quad (5)$$

Where，Q and V can be calculated by the method from references [6] and [7]; $[Q]$ is the limited leakage; [V] is the limited abrasion. The Nash equilibrium state can be reached only when the strategy group pair (w^*_{1-}, w^*_{2-}) belongs to the strategy space $(W_1 \cdot W_2)$ and satisfies the fuzzy comprehensive evaluation. Where, w^*_{1-} is the selected strategy of the player leakage, $w^*_{1-} = \{\varphi^*, \delta^*, s^*\}$; w^*_{2-} is the selected strategy of the second player operation life, $w^*_{2-} = \{r^*, h^*\}$.

3.2 The comprehensive evaluation method for fuzzy game of FS

When the strategies of other players are fixed for the optimization with preference, the evaluation vector of the player can be gained through making comprehensive evaluation for the strategy space. Then the Nash equilibrium state for this preference requirement can be achieved.

When the ith player selects the jth （$j = 1, 2, \cdots, k_i$） strategy and all of the other players' strategy ($w_{-i} = (w_1, w_2, \cdots, w_{i-1}, w_{i+1}, \cdots, w_L)$) are fixed, $Ef_{WiWj}(w_{ij}, w_{-i})$ and $Df_{WiWj}(w_{ij}, w_{-i})$ are supposed as the expectation payoff and payoff variance respectively. Then the detailed steps in fuzzy game are shown as

(a) Suppose $T = \{Ef, Df\}$ as the main selection criteria of players. Where, Ef and Df are the expectation payoff and payoff variance respectively.

(b) Suppose the evaluation set as the strategy space of the ith player.

$$W_i = \{w_{i1}, w_{i2}, \cdots, w_{iki}\} \quad (6)$$

(c) Build the matrix of single factor evaluation

Since more payoffs and less payoff risk are expected for the players in the game, the single factor evaluation matrix of the ith player is as follows

$$\boldsymbol{R}=\begin{bmatrix} R1_{i1} & R1_{i2} & \cdots & R1_{iki} \\ 1-R2_{i1} & 1-R2_{i2} & \cdots & 1-R2_{iki} \end{bmatrix} \quad (7)$$

Where, $R1_{ij}(w_{ij}, w_{-i}) = Ef_{WiWj}(w_{ij}, w_{-i}) / M_i(w_{-i})$, $M_i(w_{-i}) = \sum_{j=1}^{ki} Ef_{WiWj}(w_{ij}, w_{-i})$; $R2_{ij}(w_{ij}, w_{-i}) = Df_{WiWj}(w_{ij}, w_{-i}) / N_i(w_{-i})$, $N_i(w_{-i}) = \sum_{j=1}^{ki} Df_{WiWj}(w_{ij}, w_{-i})$.

(d) Comprehensive evaluation method. The evaluation of game results is greatly affected by the players' preference. The players' preference requirement is composed of seven types, which are very preferential, relatively preferential, preferential, eclectic, non-preferential, relatively non-preferential and very non-preferential, respectively. Then, the preference requirement corresponds to optimization result. And the weight factors a_k ($k = 1,2,\cdots, 7$), which individually correspond to seven preference types, are given based on the expert marking principle. For example, if the ith player has the kth preference requirement, the weight factors of $R1_{ij}$ and $1- R2_{ij}$ are $1-a_k$ and a_k, respectively. They can be expressed by the weight matrix $A_i = (1-a_k, a_k)$. a_k with smaller value means greater preferential requirement, contrary to the common weighted method. Through distributing the weight, the evaluation vector of the ith player can be obtained.

$$B_i = A_i \, . \, R_i \quad (8)$$

Provided the player leakage select the jth strategy $w_{1j} = \{\varphi^*, \delta^*, s^*\}$ and the fixed strategy of the other player operation life is $w_{-1} = \{r, h\}$. The weight factors corresponding to the preference requirement are decided by using expert marking principle, which are shown in table 1. Afterwards, the weight matrix A_1 and A_2 for the leakage and operation life are gained towards different preference requirement. And the fuzzy comprehensive evaluation vector B_1 and B_2 of the leakage and operation life can be solved through equation (8). Based on the fuzzy comprehensive evaluation vector, the Nash equilibrium state of fuzzy multi-objective optimization for FS is achieved. The optimization model can be solved by gene algorithm (GA).

Table 1 Expert marking principle corresponds to different preference requirement for CFS

Preference requirement	Very preferential	Relatively preferential	Preferential	Eclectic	Non-preferential	Relatively non-preferential	Very non-preferential
Expert marking value	0.2	0.3	0.4	0.5	0.6	0.7	0.8

3.3 The optimization process

The optimization algorithm, based on Nash equilibrium, fuzzy comprehensive evaluation and GA theories, is called fuzzy Nash GA algorithm (FNGA). And its flowchart is shown in Fig 2. From the flowchart, it can be seen that some problems have to be solved before beginning of the optimization process. Firstly, scope of the search spaces W_1 and W_2 must be decided. The second problem is how to solve the fuzzy Nash equilibrium point through GA. Finally, all necessary parameters must be initialed when the calculation begins.

4. Results and Discussion

Fig. 3 shows that the increasing preference of leakage can increase the payoff of leakage and reduce the payoff of operation life. It can be seen in Fig.3 that the tendency of the payoff of operation life is opposite to that of leakage when increasing their preferences, respectively. This is caused by the contradiction between operation life and leakage in CFS. The figure also indicates that the FNGA can satisfy the different preference requirement.

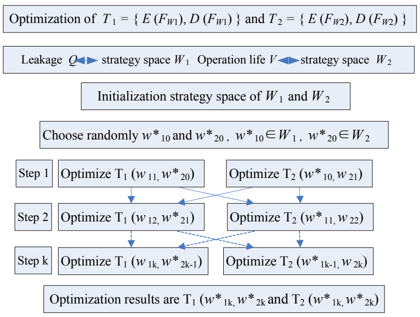


Figure 2 Flowchart of FNGA

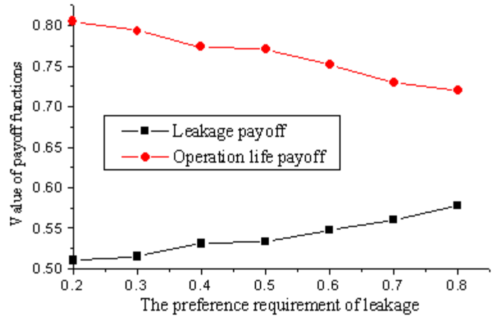


Figure 3 Payoffs of leakage and operation life corresponding to different preference requirements

To study the efficiency of the algorithm, the results from FNGA are compared with those from weighted average optimization algorithm (WAOA). Table 2 shows the results for eclectic preference requirement of leakage and operation life. From table 2, it can be seen that all results from the FNGA are in the scopes of results from WAOA. This indicates that the FNGA is able to satisfy with the preference requirement and the optimization results are reasonable.

Table 2 Optimized results for leakage Eclectic

Method	φ-r-h	Leakage payoff	Operation life payoff	$0.5f_{W1}+0.5f_{W2}$
FNGA	9.75-5.0-0.6	0.53368	0.77062	0.65215
WAOA	12.00-5.0-0.6	0.53118	0.76210	0.64664
	10.50-5.0-0.5	0.51682	0.75243	0.63463

Moreover, it can be seen from table 2 that the optimization result of FNGA is unique. The results from WAOA are an aggregate and how to select the best result from the aggregate is still an intractable problem. However, the FNGA makes decisions following the fuzzy theory and greatly diminishes the errors from manual making-decision. Therefore, the FNGA can facilitate making-decision in optimization results and reduce the difficulty of making-decision. In order to evaluate those results in table 2, the evaluation criterion values are calculated by using the weighted objective function in weighted average method.

The criterion values in table 2 reveal that the criterion value for result from the FNGA is the maximal, even though the evaluation criterion comes from the WAOA. That also means that the result of FNGA is the most optimal in table 2.

In order to validate the FNGA, the FEM is adopted and the results are compared with those from the FNGA. The FEM models of the design parameters according to different preference requirement are built and analyzed by the commercial software ANSYS. In Figure 4, the results from the optimization method are compared with those from FEM under different preference requirement. From this figure, it can be shown that the results of the two methods are matched well. Therefore, the FNGA is validated.

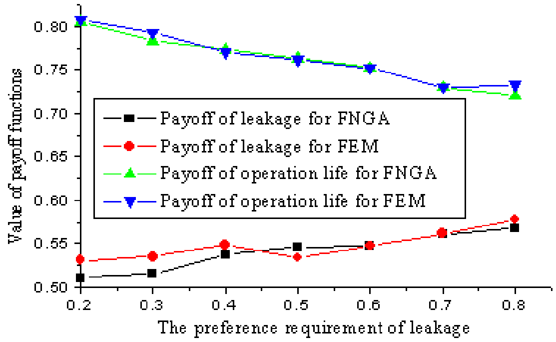


Figure 5 Results comparison analysis of FNGA and FEM

Conclusion

The current work show that FNGA method not only solved fit the FS optimization problem with different preference requirement but also reduced the optimized results decision work. Comparing with general optimization method, the FNGA method has smaller optimizing solution set. So the results decision becomes very easy and the optimizing efficiency is increased greatly.

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