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Generalized metarationalities in the graph model for conflict resolution

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Abstract

A metarational tree is defined within the graph model for conflict resolution paradigm, providing a general framework within which rational behavior in models with two decision makers (DMs) can be described more comprehensively. A new definition of stability for a DM that depends on the total number, h, of moves and counter-moves allowed is proposed. Moreover, the metarational tree can be refined so that all moves must be unilateral improvements, resulting in a new set of stability definitions for each level of the tree. Relationships among stabilities at various levels of the basic and refined trees are explored, and connections are established to existing stability definitions including Nash stability, general metarationality, symmetric metarationality, sequential and limited-move stability, and policy equilibria.

Keywords: Conflict; Graph model for conflict resolution; Metarational tree; Metarationality; Policy equilibrium

1. Introduction

Social conflicts, and our knowledge of them, may possess certain characteristics that should be incorporated into formal models and their analyses. For example, in many conflict situations only relative preference information may be available, whereby a given decision maker (DM) is known to prefer one scenario over another, but the degree of preference cannot be specified. When DMs strategically interact with one another in a conflict, the specific timing and sequencing of moves and counter-moves are usually not known a priori and hence, many possible sequences should be considered, including the order in which DMs make their moves and the length of allowable sequences of moves. Another feature that often must be taken into account is the modeling of irreversible moves by one or more DMs.

The foregoing and other key characteristics of conflict are incorporated into the design of the graph model for conflict resolution [3]. Within the graph model structure, stability concepts define a range of ways in which DMs may interact when determining which states are stable for a given DM, as well as the possible resolutions. The graph model framework is widely used to analyze real-world conflicts, such as the disputes investigated by Fang et al. [4,5], Kilgour

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et al. [13], Hipel et al. [8] and Noakes et al. [18]. Recently, Zeng et al. [22] introduced the concept of policy stable state to the graph model framework, based on the idea of policy. A policy for a DM is a complete plan for the DM's action from each possible state, similar to a strategy in game theory. The objective of this research is to extend the graph model methodology by providing a general framework for describing a rich variety of stability concepts modeling rational behavior of DMs. The results show how the policy concept systematically links various stability concepts applied in conflict management.

After an overview of the graph model for conflict resolution is furnished in the next section, a metarational tree is constructed and metarationally stable states of horizon h for a two-DM model are defined for any sequence of length h. Theorems show that existing solution concepts including Nash [16,17], general metarationality and symmetric metarationality [9] are special cases of metarationally stability of horizons $h = 1, 2$, and 3, respectively. Moreover, relationships among metarationally stable states of various depths h are derived, as well as the connection of these stability concepts to policy equilibrium states.

A DM may invoke an action to move the conflict to a less preferred state for himself in order to sanction his opponent. However, such an action may be considered to be non-credible because the DM puts himself in a worse situation. Therefore, a refinement may be made to the metarational tree whereby only unilateral improvements by DMs are permitted; associated credible metarational stability can then be defined for any horizon h. Relationships among credible metarational stabilities of various horizons are found and the stability definition called sequential stability [6,7] is shown to be a special instance for the case $h = 2$. The relationships between credible policy equilibrium and credible metarational stability for various horizons are also determined. Finally, when movements and preferences are transitive, it is proven that there is at least one credible metarationally stable state for any even horizon in a two-DM model.

2. The graph model for conflict resolution

The graph model for conflict resolution constitutes a unique methodology for modeling and analyzing real-world conflict [3]. A graph model for a conflict is composed of a directed graph and a preference structure on the set of all states or possible scenarios for each DM who can influence the dispute. Let $N = \{1, 2, \ldots, n\}$ denote the set of DMs and S the set of u states in the model. A finite directed graph $D_i = (S, A_i)$ with the vertex set S and arc set A_i, $i \in N$, records the movements among states that DM i can make in one step. The vertices of each graph represent the possible states of the conflict and hence the vertex set, S, is common to all directed graphs. Note that A_i is the arc set of D_i. If DM i can unilaterally move (in one step) from state s_1 to state s_2, there is an arc with orientation from s_1 to s_2 in A_i and state s_2 is attainable from state s_1 by DM i. For $i \in N$, DM i's reachable list for state $s \in S$ is the set $R_i(s)$ of all states to which DM i can move in one step from state s.

To permit wide applicability to real-world conflict, a cardinal utility function is not used within the graph model structure to express the preference of each DM. Specifically, in the graph model, preference is described in terms of a pair of binary relations $\{\succ_i, \sim_i\}$ on S, where $s_1 \succ_i s_2$, for $s_1, s_2 \in S$, indicates that DM i prefers s_1 to s_2, and $s_1 \sim_i s_2$ means that DM i is indifferent between s_1 and s_2. The following properties for preferences are assumed:

1. $\succ_i$ is *asymmetric*. Therefore, $s_1 \succ_i s_2$ and $s_2 \succ_i s_1$ cannot hold true at the same time, where $s_1, s_2 \in S$.
2. $\sim_i$ is *reflexive*, i.e., $s \sim_i s$ for any $s \in S$, and *symmetric*, i.e., if $s_1 \sim_i s_2$ then $s_2 \sim_i s_1$, where $s_1, s_2 \in S$.
3. $\{\succ_i, \sim_i\}$ is *strongly complete*. In other words, if $s_1, s_2 \in S$, then exactly one of $s_1 \succ_i s_2$, $s_2 \succ_i s_1$ and $s_1 \sim_i s_2$ is true.

Occasionally, the notation $s_1 \succsim_i s_2$ is utilized to indicate that either $s_1 \succ_i s_2$ or $s_1 \sim_i s_2$. Notice that transitivity of preferences is not assumed, so that the main results in this paper are valid for intransitive or transitive preferences. Also keep in mind that under the third assumption, each DM is assumed to have complete information about every DM's preferences. Within the paradigm of the graph model, research has been carried out on the topic of incomplete preference information [15,19].

A unilateral improvement from a particular state for a specific DM is any preferred state to which the DM can unilaterally move. The unilateral improvement list for DM i from state s is denoted as $R_i^+(s) = \{s_1 \in R_i(s) : s_1 \succ_i s\}$. Likewise, define $R_i^-(s) = \{s_1 \in R_i(s) : s \succ_i s_1\}$ and $R_i^=(s) = \{s_1 \in R_i(s) : s_1 \sim_i s\}$. Clearly $R_i(s) = R_i^+(s) \cup R_i^-(s) \cup R_i^=(s)$.

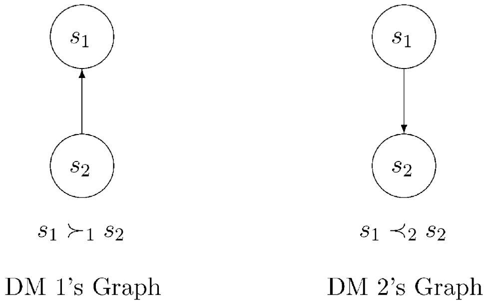

Fig. 1. The bridge conflict in graph form.

Example. To explain the graph model, consider a dispute between two interest groups over the planned building of a bridge across a river. DM 1 represents the drivers of vehicles who will use the bridge and, hence, prefers that the bridge be constructed. On the other hand, DM 2, representing local residents, would prefer that the bridge not be constructed to avoid increased air pollution and noise levels. Fig. 1 displays the two states in this simple conflict consisting of state s_1 in which a bridge is built and state s_2 in which there is no bridge. The preferences of DMs are represented by $s_1 \succ_1 s_2$ and $s_1 \prec_2 s_2$. In DM 1's graph shown on the left in Fig. 1, there are two vertices representing the two states. The direction of the arc connecting them indicates that DM 1 has the ability to build the bridge. In DM 2's graph shown on the right in Fig. 1, the direction of the arc means that DM 2 can destroy the bridge or block its construction. Accordingly, DM 1 can stay at any state or move from s_2 to s_1, while DM 2 can stay at any state or move from s_1 to s_2.

3. Generalized metarationalities

The concepts of reachable lists and policies are used for building a metarational tree for a two-DM conflict consisting of DMs i and j. This construct permits the definitions of metarational stabilities for which interesting relationships among metarational stabilities and existing definitions of stability are derived.

3.1. Metarational tree

In a conflict, a DM may announce in advance what he or she intends to do at each state that could arise. For instance, in a potential military confrontation, one country may declare in advance that it will go to war only if it is invaded, while another may proclaim that it will launch an attack if hostile troops are massed close to its border. Such a declaration, or *policy*, is clearly intended to influence the final result of the conflict. Formally, a policy of DM i is a function $\mathscr{P}_i : S \to S$, such that $\mathscr{P}_i(s) \in R_i(s) \cup \{s\}$. A policy $\mathscr{P}_i$ can be characterized by all $\mathscr{P}_i(s)$ for $s \in S$, and we sometimes denote $\mathscr{P}_i$ by $\{\mathscr{P}_i(s) : s \in S\}$, as is done by Zeng et al. [22]. In words, a policy for a DM specifies what the DM's action will be at each state (stay at that state or move to another state) if that state arises.

It is assumed that no DM can move consecutively. Accordingly, only alternating sequences of DMs are considered. Given an initial state s^*, an originating DM i, i's policy $\mathscr{P}_i$, and j's policy $\mathscr{P}_j$, a sequence of moves–counter-moves is specified as follows [22]:

$$\mathscr{P}_0(s^*) \equiv s^*, \mathscr{P}_i(s^*), \mathscr{P}_j(\mathscr{P}_i(s^*)), \ldots, \mathscr{P}_i(\mathscr{P}_j(\ldots\mathscr{P}_i(s^*)\ldots)), \mathscr{P}_j(\mathscr{P}_i(\ldots\mathscr{P}_i(s^*)\ldots)), \ldots .$$

The above sequence can be rewritten as a series of elements. Each element (s, i) is composed of a state (s) and a DM (i) who moves at that state. If DM i stays at state s, the sequence terminates at element (s, i) and is called a *terminated sequence*.

An element (s, i) in a sequence is said to be *repeating* if the same element (s, i) appeared earlier. If there is a repeating element in a sequence, which is not terminated, then there exists a unique cycle of even length containing the repeating element. Once a sequence encounters the first repeating element, the sequence cycles among all the repeating elements.

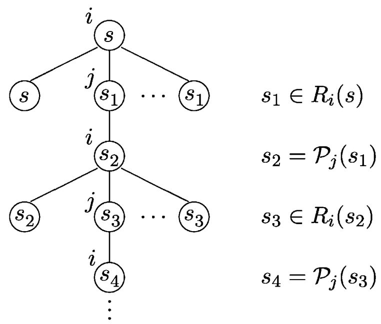

Fig. 2. DM i's metarational tree given $\mathscr{P}_j$.

A sequence having h elements is called a sequence of length h. The result of a sequence of length h or a terminated sequence is the state in the last element. Because the number of all states in a conflict is finite, there must exist a repeating element in a sequence with infinite length. The result of a sequence having infinite length is defined to be the state in the first repeating element. This definition can be justified by considering a move to have an infinitesimal cost as reflected in the inertia assumption [1,14,21]. Therefore, for a sequence of infinite length, the subsequence between the initial element and the second appearance of the first repeating element is important, and we call it a *complete sequence*. The result of any sequence of infinite length is also the *result* of its complete sequence.

Given policies $\mathscr{P}_i, \mathscr{P}_j$, initial state s and first mover i, a sequence of moves is completely determined. Therefore, the result of this sequence is equivalently referred to as the result of corresponding policies $\mathscr{P}_i, \mathscr{P}_j$, with respect to initial state s and first mover i.

Given a policy of DM j, DM i's decision problem at an initial state s is illustrated in the metarational tree displayed in Fig. 2. Initially, DM i can either stay at s or move to a state $s_1 \in R_i(s)$. If DM i moves to s_1, DM i envisions DM j following j's policy. If DM j stays at s_1 according to $\mathscr{P}_j$, the sequence terminates. Otherwise, DM i has the opportunity to respond to j's move and the sequence of moves and counter-moves continues as shown in Fig. 2. Here, $s_1, s_2, \ldots,$ represent the states as they appear in sequence, and are not all different. Furthermore, although s_{2l+1} is always followed by $\mathscr{P}_j(s_{2l+1})$, only one representative s_{2l+1} is connected to s_{2l+2} in the tree. Similar conventions are applied in other figures later. A metarational tree is now formally defined.

Definition 1. A metarational tree for DM i with respect to status quo state s and policy $\mathscr{P}_j$ of DM j is a tree whose vertices and arcs are determined as follows by induction. First, the root is s, and DM i controls any possible movement away from s. A vertex s_{2h} of depth $2h$ ($h = 0, 1, \ldots$ with $s_0 = s$), from which a movement is controlled by DM i, is connected by arcs in A_i to vertices of depth $2h + 1$, which are states in $R_i(s_{2h})$. In addition, s_{2h} is also connected to a terminating vertex s_{2h} to indicate the action of staying. A non-terminating vertex s_{2h+1} of depth $2h + 1$ ($h = 0, 1, \ldots$) is connected to vertex $\mathscr{P}_j(s_{2h+1})$ of depth $2h + 2$, to which DM j moves.

Sometimes, the words “with respect to status quo state s” are omitted if this is not confusing. A tree is finite and of depth k if the depths of all vertices are at most k, and at least one of them is of depth k.

A vertex in a metarational tree consists of a state and a DM who controls the possible moves away from the state. Therefore, it is actually an element in a sequence as defined before. For each vertex in a metarational tree, there is a path from the root to the vertex, and the vertices in the path form a sequence.

3.2. Metarationally stable states of horizon h

Consider DM i's decision problem in the metarational tree in Fig. 2 with depth h. If all of the resulting states of all sequences in the metarational tree of depth h are less than or equally preferred by DM i to the original state s, then DM i has no incentive to move away from s. It is assumed that DM i acts conservatively. Therefore, if there exists a policy $\mathscr{P}_j$ such that DM i cannot move to a more preferred state, DM i is deterred from moving away from the original state. Thus, state s is called metarationally stable with horizon h (MR_h) for DM i and $s \in S_i^{\mathrm{MR}_h}$.

Definition 2. State s is MR_h *stable* for DM i, denoted by $s \in S_i^{\mathrm{MR}_h}$, iff there exists a policy $\mathscr{P}_j$ of DM j with $\mathscr{P}_j(s) = s$, such that for all sequences of length h and all terminated sequences of length shorter than h in the metarational tree for DM i, the results of those sequences are not more preferred to s by DM i. Otherwise, state s is called MR_h *unstable* for DM i. A state is called an MR_h resolution, denoted by $s \in S^{\mathrm{MR}_h}$, iff it is MR_h stable for both i and j.

In the metagame analysis [9], conflict analysis [6,7] and graph model [3] literature, a state which is stable for every DM according to a given stability definition is called an *equilibrium*. To avoid confusion with the game-theoretic concept of an equilibrium, an equilibrium for a graph model is called a *resolution* in this paper.

It is evident that state s is MR_h unstable for DM i iff given any policy $\mathscr{P}_j$ of DM j with $\mathscr{P}_j(s) = s$, there is a sequence of length h or a terminated sequence of length which is shorter than h, such that the result of the sequence is more preferred to s by DM i.

3.3. Relationships among stability concepts

In this section, it is proven that metarational stabilities of horizons 1–3 are equivalent to Nash stability, general metarationality, and symmetric metarationality, respectively. Additionally, relationships among metarational stabilities for various horizons h are established.

Definition 3. A state $s \in S$ is Nash stable for DM i, denoted by $s \in S_i^{\mathrm{Nash}}$, iff $R_i^+(s) = \emptyset$. A state s is called a Nash resolution iff it is Nash stable for all DMs.

This definition is adapted from Nash [16,17] for use with the graph model [2,3]. Fig. 3 depicts the metarational tree of depth one. According to Definition 2, in order for state s to be MR_1 stable for DM i, all $s_1 \in R_i(s)$ must be less than or equally preferred by DM i to s. This is equivalent to Definition 3.

Definition 4. A state $s \in S$ is general metarational (GMR) for DM i, denoted by $s \in S_i^{\mathrm{GMR}}$, iff for every $s_1 \in R_i^+(s)$, there exists at least one $s_2 \in R_j(s_1)$ such that $s \succcurlyeq_i s_2$. A state is called a GMR resolution iff it is GMR for all DMs.

Howard [9] provides the original definition for GMR stability while Fang et al. [2,3] furnish the definition within the graph model paradigm.

Theorem 1. MR_2 *stability is equivalent to general metarationality.*

Proof. Given $s \in S_i^{\mathrm{GMR}}$, it can also be proved that $s \in S_i^{\mathrm{MR}_2}$ as follows. Since s is GMR for DM i, for each $s_1 \in R_i^+(s)$, there exists $s_2 \in R_j(s_1)$, such that $s \succcurlyeq_i s_2$. A metarational tree of depth 2 is shown in Fig. 4. For any $s_1 \in R_i^-(s)$ or

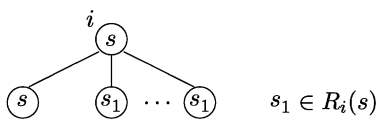

Fig. 3. Metarational tree of depth 1.

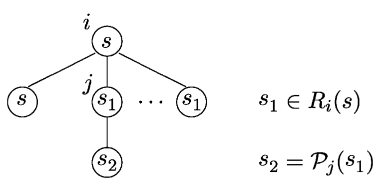

Fig. 4. Metarational tree of depth 2.

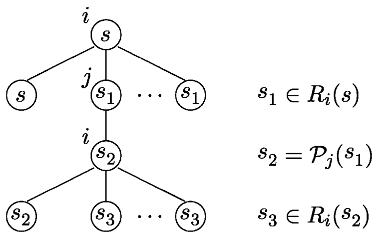

Fig. 5. Metarational tree of depth 3.

$R_i^{=}(s)$, one can choose $\mathscr{P}_j(s_1)=s_1$, so that $s \succcurlyeq_i s_1$. For any $s_1 \in R_j^{+}(s)$, one can set $\mathscr{P}_j(s_1)=s_2$ so that $s \succcurlyeq_i s_2$. A similar argument shows that a MR_2 stable state is also GMR. □

Definition 5. A state $s \in S$ is symmetric metarational (SMR) for DM i, denoted by $s \in S_i^{\mathrm{SMR}}$, iff for every $s_1 \in R_i^{+}(s)$, there exists $s_2 \in R_j(s_1)$ such that $s \succcurlyeq_i s_2$ and $s \succcurlyeq_i s_3$ for all $s_3 \in R_i(s_2)$. A state is called a SMR resolution iff it is SMR for all DMs.

This definition for SMR stability was originally proposed by Howard [9] and later given within the graph model context by Fang et al. [2,3].

Theorem 2. MR_3 *stability is equivalent to symmetric metarationality.*

Proof. Given $s \in S_i^{\mathrm{SMR}}$, it can also be proved that $s \in S_i^{\mathrm{MR}_3}$ as follows. Since s is SMR for DM i, for each $s_1 \in R_i^{+}(s)$, there exists $s_2 \in R_j(s_1)$, such that $s \succcurlyeq_i s_2$ and $s \succcurlyeq_i s_3$ for all $s_3 \in R_i(s_2)$. A metarational tree of depth 3 is shown in Fig. 5. For any $s_1 \in R_i^{-}(s)$ or $R_i^{=}(s)$, one can choose $\mathscr{P}_j(s_1)=s_1$, so that the sequence terminates and hence $s_1 \preccurlyeq_i s$. For any $s_1 \in R_j^{+}(s)$, one can set $\mathscr{P}_j(s_1)=s_2$, so that $s \succcurlyeq_i s_2$ and $s \succcurlyeq_i s_3$ for all $s_3 \in R_i(s_2)$. Therefore, the results of all sequences of length 3 and all terminated sequences of length shorter than 3 are not more preferred by DM i to s. A similar argument shows that a MR_3 stable state is also SMR. □

Limited-move stability of horizon h constitutes another useful stability concept that has been tailored for use within the graph model for conflict resolution [3,12]. Appendix A provides an overview of the definition for limited-move stability of horizon h as well as its relationship to MR_h stability. The key finding is that L_h stability is a subset of MR_h stability.

Theorem 3. (1) *If a state s is* MR_{2k+1} *unstable for DM i, then it is also* $\mathrm{MR}_1, \mathrm{MR}_3, \ldots,$ *and* MR_{2k-1} *unstable for i. Furthermore, if a state s is not an* MR_{2k+1} *resolution, then it is also not an* $\mathrm{MR}_1, \mathrm{MR}_3, \ldots,$ *or* MR_{2k-1} *resolution.*

(2) *If a state s is* MR_{2k} *stable for DM i, then it is also* $\mathrm{MR}_2, \mathrm{MR}_4, \ldots,$ *and* MR_{2k-2} *stable for i. Furthermore, if a state s is an* MR_{2k} *resolution, then it is also an* $\mathrm{MR}_2, \mathrm{MR}_4, \ldots,$ *and* MR_{2k-2} *resolution.*

Proof. (1) If state s is MR_{2k+1} unstable for DM i, then for any given policy $\mathscr{P}_j$ of DM j, there is a policy $\mathscr{P}_i$ of DM i and a sequence of length $2k+1$ or shorter, such that the result of the sequence is more preferred to s by DM i. Given $l<k$, if the sequence is terminated at length shorter than or equal to $2l+1$, then s becomes MR_{2l+1} unstable for i. Otherwise, let

$$\{(s_0=s,i),(s_1^m,j),\ldots,(s_{2l}^m,i),(s_{2l+1}^m,j),\ldots,(s_{2k+1}^m,j)\}_{m=1}^{M} \tag{1}$$

be all the sequences of length greater than $2l+1$, whose results are more preferred to s by DM i: $s_{2k+1}^m \succ_i s$. Then one claims that $s_{2l+1}^m \succ_i s$ for at least one m. Otherwise, $s_{2l+1}^m \preccurlyeq_i s$ for all $m=1,2,\ldots,M$, and then DM j can change policy $\mathscr{P}_j$ to stay at states s_{2l+1}^m for all $m=1,2,\ldots,M$. Subsequently, the result of the mth sequence in (1) becomes s_{2l+1}^m, which is not more preferred to s by DM i for all m. Therefore, s becomes MR_{2k+1} stable for i, which contradicts the given condition. Thus, there is at least one sequence $(s_0,i),(s_1^m,j),\ldots,(s_{2l+1}^m,j)$ of length $2l+1$ whose result is more preferred to s by DM i. Hence, s is also MR_{2l+1} unstable for DM i.

(2) Let s be an MR_{2l} unstable state for DM i and for some $l \in \{1, 2, \ldots, k-1\}$. Then, given any policy $\mathscr{P}_j$ of DM j, there is a sequence $(s_0 = s, i), (s_1, j), \ldots, (s_{2l}, i)$ of length $2l$, or a terminated sequence with length shorter than $2l$ whose result is more preferred to s by DM i. If the sequence is a terminated sequence, then s becomes MR_{2k} unstable for DM i. Otherwise, let DM i choose a policy to stay at s_{2l}. Then sequence $(s_0 = s, i), (s_1, j), \ldots, (s_{2l}, i)$ is terminated of length $2l < 2k$. Therefore, s is MR_{2k} unstable for DM i, which contradicts the condition. □

The relationships established in Theorem 3 are shown in Fig. 8.

4. Credible metarationalities

A policy of a DM may contain some moves going to less preferred states. A DM's policy is deemed to be credible, if he always moves to a more preferred state. Incredible moves are excluded in the refinement of Nash equilibria in game theory, such as the subgame perfect equilibrium of Selten [20]. Hence, a credible policy is defined as $\mathscr{P}_i^c(s) \in R_i^+(s) \cup \{s\}$. By requiring a policy to be credible, one obtains a credible MR_h denoted by MR_h^c. The credible metarational tree shown in Fig. 6 is constructed in a fashion that is similar to the metarational tree in Fig. 2.

Definition 6. State s is MR_h^c *stable* for DM i, denoted by $s \in S_i^{\mathrm{MR}_h^\mathrm{c}}$, iff there exists a policy $\mathscr{P}_j^\mathrm{c}$ of DM j with $\mathscr{P}_j^\mathrm{c}(s) = s$, such that for any policy of DM i, all sequences of length h and all terminated sequences of length shorter than h, the results of those sequences are not more preferred to s by DM i. Otherwise, state s is called MR_h^c *unstable* for DM i. A state is called an MR_h^c resolution, denoted by $s \in S^{\mathrm{MR}_h^\mathrm{c}}$, iff it is MR_h^c stable for both i and j.

The concept of credible MR_1 stability is equivalent to the MR_1 stability, or the Nash stability. The concept of credible MR_2 stability is equivalent to the sequential stability of Fraser and Hipel [6,7].

Definition 7. For a two-DM conflict having the set of DMs $N = \{i, j\}$, a state $s \in S^{\mathrm{SEQ}}$ is sequentially stable for DM i iff for every $s_1 \in R_i^+(s)$ there exists $s_2 \in R_j^+(s_1)$ with $s \succsim_i s_2$. A state is a sequentially resolution iff it is sequentially stable for both DMs.

Similar to Theorems 1 and 3, the following two theorems can be derived.

Theorem 4. MR_2^c *stability is equivalent to sequential stability.*

The credible metarational tree of depth 2 is displayed in Fig. 7.

Theorem 5. (1) *If a state s is* $\mathrm{MR}_{2k+1}^\mathrm{c}$ *unstable for DM i, then it is also* $\mathrm{MR}_1^\mathrm{c}, \mathrm{MR}_3^\mathrm{c}, \ldots,$ *and* $\mathrm{MR}_{2k-1}^\mathrm{c}$ *unstable for DM i. Furthermore, if a state s is not an* $\mathrm{MR}_{2k+1}^\mathrm{c}$ *resolution, then it is also not an* $\mathrm{MR}_1^\mathrm{c}, \mathrm{MR}_3^\mathrm{c}, \ldots,$ *or* $\mathrm{MR}_{2k-1}^\mathrm{c}$ *resolution.*

(2) *If a state s is* $\mathrm{MR}_{2k}^\mathrm{c}$ *stable for DM i, then it is also* $\mathrm{MR}_2^\mathrm{c}, \mathrm{MR}_4^\mathrm{c}, \ldots,$ *and* $\mathrm{MR}_{2k-2}^\mathrm{c}$ *stable for DM i. Furthermore, if a state s is an* $\mathrm{MR}_{2k}^\mathrm{c}$ *resolution, then it is also an* $\mathrm{MR}_2^\mathrm{c}, \mathrm{MR}_4^\mathrm{c}, \ldots,$ *and* $\mathrm{MR}_{2k-2}^\mathrm{c}$ *resolution.*

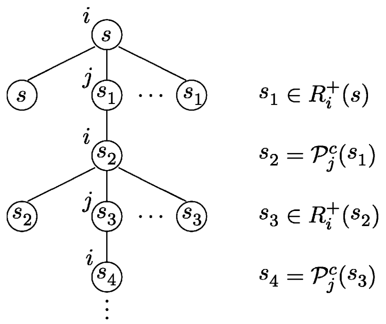

Fig. 6. DM i's credible metarational tree given $\mathscr{P}_j^\mathrm{c}$.

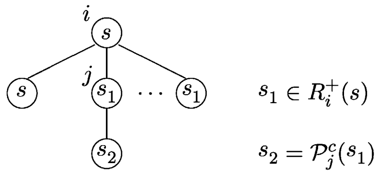

Fig. 7. Credible metarational tree of depth 2.

5. Policy equilibrium

Zeng et al. [22] define the concept of a policy stable state (PSS) for two-DM conflicts as follows.

Definition 8. Policies $\mathscr{P}_1, \mathscr{P}_2$ form a policy equilibrium with respect to status quo state s^* if:

(1) $\mathscr{P}_i(s^*) = s^*$ holds for both DM $i = 1, 2$,
(2) $\forall i = 1, 2, \forall \mathscr{P}'_i$ such that $\mathscr{P}'_i(s^*) \neq s^*$, the result of any terminated sequence or any complete sequence is not preferred to s^* by DM i.

A state s^* satisfying the above two conditions is called a PSS. Denote the set of all PSSs by S^{PSS}.

Theorem 6. (1) *If* $s \in S^{\mathrm{MR}_{2h-1}}$ *for a positive integer* h, *then* $s \in S^{\mathrm{PSS}}$;
(2) *If* $s \in S^{\mathrm{PSS}}$, *then* $s \in S^{\mathrm{MR}_{2h}}$ *for every positive integer* h.

Proof. (1) Contrary to the conclusion, let state s^* be an MR_{2h-1} resolution, but not a PSS. By definition of PSS, for an arbitrarily given policy $\mathscr{P}^\sharp_j$ of DM j, there exists a policy of DM i which moves away from s^* and a sequence whose result is $\bar{s}$ and

$$\bar{s} \succ_i s^*. \tag{2}$$

Specifically, let $\mathscr{P}^\sharp_j$ stay at any state which is not more preferred to s^* by DM i. Rename the sequence as $(s_0 = s^*, i), (s_1, j), (s_2, i), \ldots, (s_{2h-1}, j), (s_{2h}, i), \ldots$. Assume that s_{2h-1} is not more preferred to s^* by DM i for some h. Then DM j stays at s_{2h-1} according to his policy $\mathscr{P}^\sharp_j$. Hence, the sequence becomes terminated and the result is $s_{2h-1} \preccurlyeq_i s^*$, which contradicts (2). Therefore, one concludes that state s^* is MR_{2h-1} unstable for any positive integer h.

(2) Contrary to the conclusion, suppose that there is a state s^* which is not an MR_{2h} resolution, say MR_{2h} unstable for DM i, for a positive integer h, but s^* is a PSS with policies $\mathscr{P}^*_i$ and $\mathscr{P}^*_j$. Since s^* is MR_{2h} unstable for DM i, with respect to $\mathscr{P}^*_j$ there is a policy $\mathscr{P}^\sharp_i$ and a sequence $(s_0 = s^*, i), (s_1, j), \ldots, (s_{2h}, i)$ of length $2h$, or a shorter terminated sequence, such that the result of the sequence is more preferred to s^* by DM i: $s_{2h} \succ_i s^*$. Consider three possible cases.

- If the sequence is terminated, then the result of this sequence is also the result of policies $\mathscr{P}^\sharp_i$ and $\mathscr{P}^*_j$ with respect to the original state s^* and first mover i. Therefore, given DM j's policy $\mathscr{P}^*_j$, DM i obtains a more preferred result by changing from policy $\mathscr{P}^*_i$ to $\mathscr{P}^\sharp_i$, which contradicts the condition that s^* is a PSS with policies $\mathscr{P}^*_i$ and $\mathscr{P}^*_j$.
- If the sequence $(s_0, i), (s_1, j), \ldots, (s_{2h}, i)$ is not terminated and element (s_{2h}, i) is not repeating, then DM i can change his policy from $\mathscr{P}^*_i$ to $\mathscr{P}^{\sharp 1}_i$, which is the same as $\mathscr{P}^\sharp_i$ except choosing to stay at s_{2h}. In this way, state s_{2h} becomes the result of policies $\mathscr{P}^{\sharp 1}_i$ and $\mathscr{P}^*_j$ with respect to the original state s^* and first mover i. Therefore, given DM j's policy $\mathscr{P}^*_j$, DM i obtains a more preferred result by changing from policy $\mathscr{P}^*_i$ to $\mathscr{P}^{\sharp 1}_i$, which contradicts the condition that s^* is a PSS with policies $\mathscr{P}^*_i$ and $\mathscr{P}^*_j$.
- Finally, consider the case when the sequence $(s_0, i), (s_1, j), \ldots, (s_{2h}, i)$ is not terminated and element (s_{2h}, i) is a repeating version of element (s_{2l}, i) $(l < h)$. Without loss of generality, suppose that element (s_{2l}, i) is not repeating. Then $s_{2h} = s_{2l} \succ_i s_0$. DM i can change his policy from $\mathscr{P}^*_i$ to $\mathscr{P}^{\sharp 2}_i$, which is the same as $\mathscr{P}^\sharp_i$ except choosing to stay at s_{2l}. In this way, state s_{2l} becomes the result of policies $\mathscr{P}^{\sharp 2}_i$ and $\mathscr{P}^*_j$ with respect to the original state s^* and first

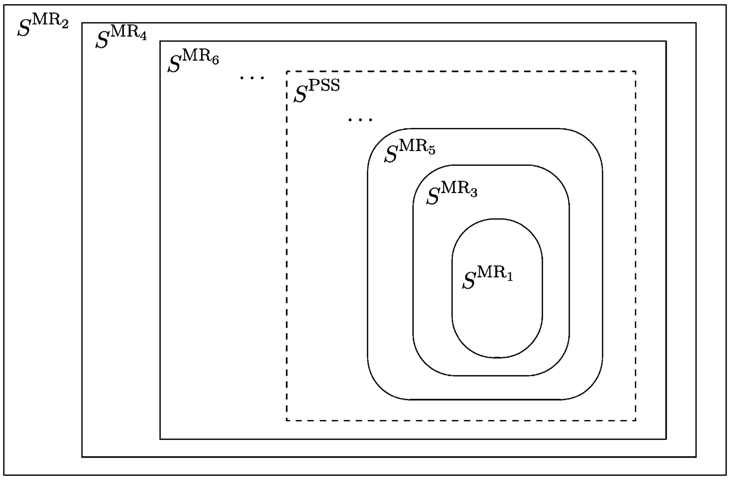

Fig. 8. Relationships among S^{MR_h} and S^{PSS}.

	L	R
U	$(2,3)$	$(4,1)$
D	$(3,2)$	$(1,4)$

Fig. 9. An example in which $\mathrm{MR}_h \neq \mathrm{MR}_{h+1}$.

mover i. Therefore, given DM j's policy $\mathscr{P}_j^*$, DM i obtains a more preferred result by changing from policy $\mathscr{P}_i^*$ to $\mathscr{P}_i^{\sharp 2}$, which contradicts the condition that s^* is a PSS with policies $\mathscr{P}_i^*$ and $\mathscr{P}_j^*$. $\square$

Relationships among the concepts of MR_h resolutions and PSSs are shown in Fig. 8. The following conclusion follows immediately.

Corollary 1. *$S^{\mathrm{PSS}} = S^{\mathrm{MR}_{h^*}}$ holds for some h^* iff $S^{\mathrm{MR}_{h^*}} = S^{\mathrm{MR}_{h^*+1}}$.*

In some situations, $S^{\mathrm{MR}_h} = S^{\mathrm{MR}_{h+1}}$ may not be true for any large h. For example, consider the 2×2 conflict shown in Fig. 9, which is No. 44 in the appendix of Brams [1]. Notice that DM 1, the row DM, controls the actions U and D, while DM 2, the column DM, controls the actions L and R. The four cells in the matrix represent the four possible states, while the two numbers in each cell represent the ordinal preferences of DMs 1 and 2, respectively, where a higher number means more preferred. In this conflict, consider the stability of state (U, L) from each DM's viewpoint. Notice that DM 1 has a unilateral improvement from state (U, L) to state (D, L), and DM 2 can respond with a unilateral improvement from state (D, L) to state (D, R). Then DM 1 has a unilateral improvement from state (D, R) to (U, R) and, subsequently, DM 2 can cause the conflict to move from (U, R) to (U, L) by invoking his unilateral improvement. According to Definition 2, for any positive integer h, state (U, L) is MR_{4h} stable for DM 1. Since DM 2 has no unilateral improvement from state (U, L), this state is MR_1 stable for DM 2 and, therefore, MR_{4h} stable for DM 2. Hence, state (U, L) constitutes an MR_{4h} resolution. However, if $4h + 1$ steps are allowed, DM 1 can move from (U, L) to (D, L) without being sanctioned. Therefore, state (U, L) is an MR_{4h} but not MR_{4h+1} resolution.

One may wish to know, what is S^{PSS} when $S^{\mathrm{MR}_{2h}} \neq S^{\mathrm{MR}_{2h+1}}$? The answer is that S^{PSS} may be neither $S^{\mathrm{MR}_{2h}}$ nor $S^{\mathrm{MR}_{2h+1}}$. To illustrate this, consider the graph model for a conflict having three states shown in Fig. 10. Once again, the numbers given in brackets represent the ordinal preferences of the two DMs, i and j, where a higher number means more preferred. For this conflict, one can easily show that $S^{\mathrm{MR}_{2h}} = \{s_1, s_2, s_3\}$, $S^{\mathrm{MR}_{2h+1}} = \emptyset$ and $S^{\mathrm{PSS}} = \{s_2, s_3\}$.

Theorem 6 describes relationships between metarational resolutions and PSSs. In a similar fashion, one can derive relationships between credible metarational resolutions and credible PSSs. Specifically, the set of credible MR_h

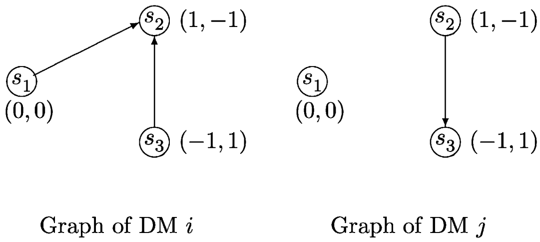

Fig. 10. An example in which $S^{\mathrm{PSS}} \neq S^{\mathrm{MR}_h} \neq S^{\mathrm{MR}_{h+1}}$.

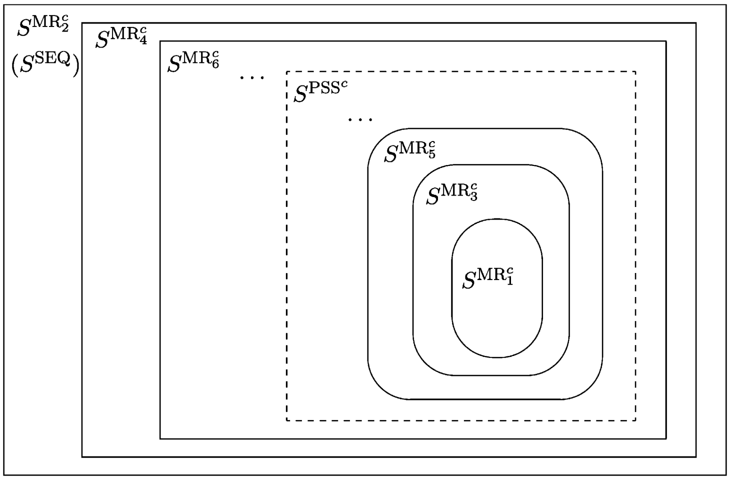

Fig. 11. Relationships among $S^{\mathrm{MR}_h^c}$ and S^{PSS^c}.

resolutions ($S^{\mathrm{MR}_h^c}$) with even h contains the set of all credible PSSs (S^{PSS^c}), which in turn contains the set of all credible MR_h resolutions with odd h, as shown in Fig. 11.

6. Existence of metarationally stable states

Fraser and Hipel [7] prove the existence of a sequential resolution within the context of the conflict analysis approach under the assumption of transitive preferences. The result is generalized as follows for certain metarational resolutions within the graph model paradigm.

Theorem 7. *Given any positive integer h, if movements and preferences are transitive, then there is at least one* MR_{2h}^c *resolution in the graph model for a two-DM conflict.*

Proof. If the theorem is false for a given h, each state is not MR_{2h}^c for at least one DM. Specifically,

$$\text{let } s_0 \text{ be the state that is most preferred by DM } i \text{ among } \mathrm{MR}_{2h}^c \text{ unstable states for DM } i. \tag{3}$$

Then there is at least one sequence of length $2h$ or a shorter terminated sequence, beginning from a move of DM i. Without loss of generality, assume the sequence is not terminated. Otherwise, the following argument holds for a smaller h. Let

$$\{(s_0, i), (s_1^m, j), \ldots, (s_{2h-1}^m, j), (s_{2h}^m, i)\}_{m=1}^M,$$

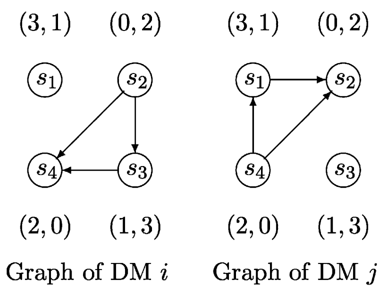

Fig. 12. An example without any PSS.

be all such kinds of sequences, and hence $s_{2h}^m \succ_i s_0$ holds for all $m = 1, 2, \ldots, M$. One can claim that

$$s_{2h-1}^m \succ_i s_0 \quad \text{for some } m = 1, 2, \ldots, M. \tag{4}$$

Otherwise, DM j can use a policy which stays at all s_{2h-1}^m ($m = 1, 2, \ldots, M$), so that there is no sequence of length $2h$ whose result is more preferred to s_0 by DM i and s_0 becomes $\mathrm{MR}_{2h}^{\mathrm{c}}$.

According to (3), s_m^{2h-1} is $\mathrm{MR}_{2h}^{\mathrm{c}}$ stable for DM i for some $m = 1, 2, \ldots, M$ by (4). If s_{2h-1}^m is also $\mathrm{MR}_{2h}^{\mathrm{c}}$ stable for j, then the proof is completed. Otherwise, by use of the transitivity of preference of DM j, one can assume that s_{2h}^m is the most preferred state by DM j among $R_j^+(s_{2h-1}^m)$ without loss of generality. Since $s_{2h}^m \succ_i s_0$, then s_{2h}^m is $\mathrm{MR}_{2h}^{\mathrm{c}}$ stable for DM i by (3) again. If s_{2h}^m is $\mathrm{MR}_{2h}^{\mathrm{c}}$ stable for DM j, then the proof is completed. Otherwise, DM j can move to a better state $\tilde{s}_{2h}^m$. Because of the transitivity of movement, DM j should have a move from s_{2h-1}^m to $\tilde{s}_{2h}^m$ directly, which contradicts the assumption that s_{2h}^m is the most preferred state by DM j. $\square$

According to Theorem 4, sequential stability is the same as $\mathrm{MR}_2^{\mathrm{c}}$. Hence, the existence of a sequential stable resolution within the graph model structure is contained within Theorem 7.

Based on Theorems 6 and 7, the following result is obtained.

Corollary 2. *If movements and preferences are transitive and $S^{\mathrm{PSS}} = S^{\mathrm{MR}_{2h}}$ for some h, then S^{PSS} is non-empty.*

Generally, even when movements and preferences are transitive, it may happen that there is no PSS. Consider the graph model in Fig. 12. Specifically, in this example, if the status quo state is s_1, then no policy of DM i can sanction DM j for deviation, if DM j uses the policy of moving from s_1 to s_2, staying at s_2 and s_3, and moving from s_4 to s_2. If the status quo state is s_2, then no policy of DM j can sanction DM i for deviation, if DM i uses the policy of staying at s_1, moving from s_2 to s_3, and staying at s_3 and s_4. A similar result holds for s_3 and s_4.

7. Conclusions

A general framework for defining human behavior under conflict is developed using the concepts of a metarational tree and its refined version. Theorems are proven to establish interesting relationships among metarational stabilities of various horizons h, previously defined stability concepts, and PSSs. These new developments significantly enhance the graph model methodology for effectively modeling and analyzing real-world conflict.

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Appendix A. Summary of limited-move stability and its relationship to metarational stability

The idea of limited-move stability was proposed by Zagare [21] for strict ordinal 2×2 games while Kilgour [11] extended these limited-move concepts to any finite two-DM game in normal form. Kilgour et al. [12] generalized the limited-move concepts for employment within the graph model for conflict resolution for both two-DM and more than two-DM conflicts.

For limited-move stability with horizon h, DM i's decision problem at the initial state is illustrated in Fig. A.1. This is a finite extensive game of perfect information, and can be solved completely by backward induction analysis. In limited-move stability, transitivity of the binary relations, $\succ_i$ and $\sim_i$, mentioned in Section 2 is assumed. Hence, the preference structure is a weak-order structure and a payoff function can be defined on S where only the ordinal information in the payoff function is employed in the definition of limited-move stability.

A state s is limited move with horizon h stable (denoted by L_h) for DM i if DM i cannot move to a more preferred state in h steps when DM j maximizes his own payoff (if DM j has indifferent choices, he chooses the one minimizing DM i's payoff). State s is called an L_h limited-move resolution if it is L_h limited-move stable for both DMs. Denote the set of all L_h limited-move resolutions by S^{L_h}.

The concept of L_h stability is similar to MR_h stability in the sense that both consider the rationality of horizon h. However, DMs are required to maximize their own payoffs in the definition of L_h, which is not required in MR_h. Therefore, L_h stability for DM i implies MR_h stability for DM i. This can be simply proved as follows. Given an L_h stable state for DM i, construct $\mathscr{P}_j$ by maximizing his own payoff in h steps. Then, for any policy $\mathscr{P}_i$ of DM i, any sequence of length h, and any terminated sequence of length shorter than h, the results of the sequences are not more

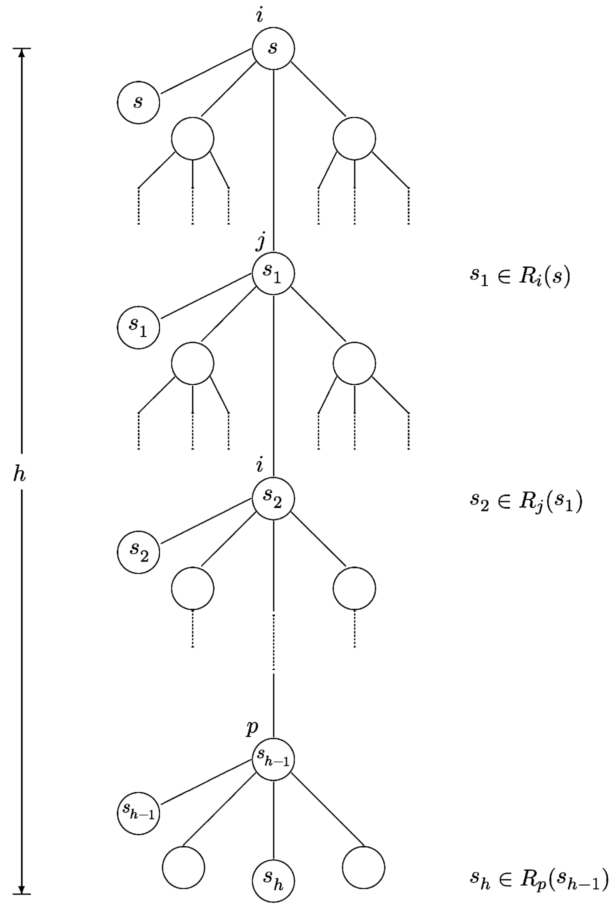

Fig. A.1. L_h stability for DM i in a two-DM conflict (p is the last mover, so that $p = i$ if h is odd and $p = j$ if h is even).

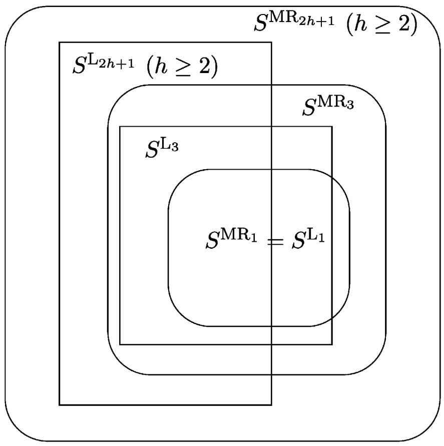

Fig. A.2. Relationship between $S^{\mathrm{MR}_{2h+1}}$ and $S^{\mathrm{L}_{2h+1}}$.

	L	R
U	$(2, 3)$	$(4, 1)$
D	$(1, 2)$	$(3, 4)$

Fig. A.3. A normal-form conflict.

preferred to s by DM i. In other words, s is MR_h stable for DM i. The relationship between $S^{\mathrm{MR}_{2h+1}}$ and $S^{\mathrm{L}_{2h+1}}$ is shown in Fig. A.2.

A non-myopic resolution is a state which is L_h stable for the conflict for all $h \geqslant h^*$, where h is a positive integer [10]. The following theorem was originally given by Zeng et al. [22].

Theorem A.1. *A non-myopic resolution is a PSS.*

Proof. If state s is a non-myopic resolution, then it is L_h stable for all $h \geqslant h^*$ for each DM. Therefore, it is MR_h stable for $h \geqslant h^*$ for each DM, which implies that s is a policy equilibrium by Corollary 1. □

As shown in the appendix of Brams [1], each conflict represented by a 2 × 2 strict ordinal game has at least one non-myopic resolution. Therefore, by Theorem A.1, all such conflicts have at least one PSS.

The converse of Theorem A.1 is not true. For example, in the conflict of Fig. A.3 (which is No. 26 conflict in the appendix of Brams [1]), (U, L) is not a non-myopic resolution but it is a PSS.

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