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Trinification with $\sin^2\theta_W = 3/8$ and see-saw neutrino mass

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Abstract

We realize a supersymmetric trinification model with three families of $(\mathbf{27}_{\text{tri}} + \mathbf{27}_{\text{tri}} + \overline{\mathbf{27}}_{\text{tri}})$ by the Z_3 orbifold compactification with two Wilson lines. It is possible to break the trinification group to the supersymmetric standard model. This model has several interesting features: the *hypercharge quantization*, $\sin^2\theta_W^0 = 3/8$, naturally light neutrino masses, and introduction of R-parity. The *hypercharge quantization* is realized by the choice of the vacuum, naturally leading toward a supersymmetric standard model.

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1. Introduction

Unification of fundamental forces in the last three decades [1] has been a partially successful endeavor. Probably, the most attractive feature of the unification is the quantisation of the electromagnetic charge, $Q_{\text{em}}(\text{proton}) = -Q_{\text{em}}(\text{electron})$. However, the most serious problem in this old grand unification (GUT) is the existence of the fine-tuning problem the so-called gauge hierarchy problem. To understand the gauge hierarchy problem, supersymmetry has been considered, which is then extended to a consistent superstring theory with a big gauge group in ten dimensions (10D). One particularly interesting superstring model is the $\text{E}_8 \times \text{E}_8'$ heterotic string [2], because E_8 contains a chain of symmetry breaking down to $\text{E}_8 \to \text{E}_6$ [3] and then down to $\text{E}_6 \to SO(10) \to SU(5)$. In this $\text{E}_8 \times \text{E}_8'$ heterotic string, the intermediate step E_6 seems to play a crucial role in the classification of standard model (SM) particles. This is because the spinor representation of $SO(10)$ is included in the fundamental representation $\mathbf{27}$ of E_6.

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For the unification of all fundamental forces, the old GUT idea has to be unified with gravity also, which seems to be possible in string theory [4]. Thus, the unification of all forces is better studied in the $E_8 \times E_8'$ heterotic string.[1] In this theory, there must be a reasonable compactification down to four dimensions (4D) so that the SM results as an effective theory at low energy world. The most powerful compactification toward applications in obtaining 4D effective theories seems to be the orbifold compactification [5,6]. However, the adjoint representation needed for breaking the GUT group is not present at the Kac–Moody level $k = 1$.[2] This led to 4D string constructions toward *standard-like* models [8] and flipped $SU(5)$ models [9].

Here, in obtaining an effective 4D model we include all the possibilities of assigning the vacuum expectation values (VEV's) to Higgs fields. For example, if an $SU(5)$ model does (not) include an adjoint representation **24**, then we say that a SM is (not) possible from this model.

However, the standard-like models and the flipped $SU(5)$ models suffer from the $\sin^2\theta_W^0$ ($\equiv$ the value at the GUT scale) problem toward unification [10]. The $SU(5)$, $SO(10)$ or E_6 GUT's with the SM fermions in the spinor(or fundamental) representation gives $\sin^2\theta_W^0 = 3/8$, which will be called the $U(1)_Y$ hypercharge quantization, or simply *hypercharge quantization*. The $\sin^2\theta_W^0$ problem is the hypercharge quantization problem.

The *hypercharge quantization problem* can be understood in the orbifold constructions [10] if the 4D gauge group is the trinification type, $SU(3)^3$ gauge group with 27 chiral fields (let us define this as $\mathbf{27}_{\rm tri}$) in one family, suggested in the middle of eighties [11]. Nevertheless, supersymmetrization of the trinification model does not lead to naturally small neutrino masses. Therefore, it was suggested that in the supersymmetric trinification one must add another vectorlike $\mathbf{27}_{\rm tri} + \overline{\mathbf{27}}_{\rm tri}$ [12].

So far, the trinification with small neutrino masses from the orbifold compactification was possible with the bare value of $\sin^2\theta_W^0 = 1/4$, where one obtains just vectorlike lepton humors in addition to $\mathbf{27}_{\rm tri}$ [13], which however does not satisfy the hypercharge quantization. If the $U(1)_Y$ hypercharge quantization is not satisfied, one must introduce an intermediate scale to fit with data. This is called the optical unification [14], which depends on details of the intermediate scale particles and the magnitudes of the intermediate scales. For the $U(1)_Y$ hypercharge quantization, one needs vectorlike $(\mathbf{27}_{\rm tri} + \overline{\mathbf{27}}_{\rm tri})$'s, not just vectorlike lepton-humor(s) [13].

Therefore, for the $U(1)_Y$ hypercharge quantization it is of utmost importance to obtain vectorlike $(\mathbf{27}_{\rm tri} + \overline{\mathbf{27}}_{\rm tri})$'s. In this Letter, we fulfil such an objective with an orbifold compactification, and hence obtain the bare value of $\sin^2\theta_W^0 = 3/8$ naturally.

2. Trinification with three more $(\mathbf{27} \oplus \overline{\mathbf{27}})$'s

Choosing the hypercharge generator as $Y = -\frac{1}{2}(-2I_1 + Y_1 + Y_2)$, let us denote the trinification spectrum under $SU(3)^3$ as,

$$\mathbf{27}_{\rm tri} = (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}), \tag{1}$$

where

$$\begin{aligned}(\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1}) = \Psi_l \to \Psi_{(\bar{M},I,0)} = {} & \Psi_{(\bar{1},i,0)}(H_1)_{-\frac{1}{2}} + \Psi_{(\bar{2},i,0)}(H_2)_{+\frac{1}{2}} + \Psi_{(\bar{3},i,0)}(l)_{-\frac{1}{2}} \\ & + \Psi_{(\bar{1},3,0)}(N_5)_0 + \Psi_{(\bar{2},3,0)}(e^+)_{+1} + \Psi_{(\bar{3},3,0)}(N_{10})_0,\end{aligned} \tag{2}$$

$$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) = \Psi_q \to \Psi_{(0,\bar{I},\alpha)} = \Psi_{(0,\bar{i},\alpha)}(q)_{+\frac{1}{6}} + \Psi_{(0,\bar{3},\alpha)}(D)_{-\frac{1}{3}}, \tag{3}$$

$$(\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}) = \Psi_a \to \Psi_{(M,0,\bar{\alpha})} = \Psi_{(1,0,\bar{\alpha})}(d^c)_{\frac{1}{3}} + \Psi_{(2,0,\bar{\alpha})}(u^c)_{-\frac{2}{3}} + \Psi_{(3,0,\bar{\alpha})}(\bar{D})_{+\frac{1}{3}}, \tag{4}$$

[1] With duality, one can argue that the perturbative heterotic string can have other realizations. Here, we stick to the perturbative $E_8 \times E_8'$ heterotic string.

[2] At higher k's, it is possible to have an adjoint representation. See, for example, [7].

where the representations will be called carrying three different *humors* as denoted by subscripts: *lepton-*, *quark-*, and *antiquark-humors*. These names are convenient to remember since they contain the designated SM fields. Note that *lepton-humor* field contains also a pair of Higgs doublets which do not carry color charge. With three sets of trinification spectrum, there exist three pairs of Higgs doublets.

We take the following orbifold model with two Wilson lines [6],

$$\begin{aligned}
v &= \left(0\,0\,0\,0\,0\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{2}{3}\right)(0\,0\,0\,0\,0\,0\,0\,0),\\
a_1 &= \left(0\,0\,0\ \ 0\,0\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{2}{3}\right)\left(0\,0\,0\,0\,0\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{2}{3}\right),\\
a_3 &= \left(\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{2}{3}\,0\,0\,0\,0\,0\right)\left(\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{2}{3}\,0\,0\,0\,0\,0\right)
\end{aligned} \tag{5}$$

which results to a gauge group $SU(3)^4 \times [SU(3)']^4$.[3] The massless fields appear only from the twisted sectors, as listed in Table 1.

To obtain all the possible vacuum structure of the compactification, we consider all the possible VEV's also as commented in the Introduction. In this spirit, let there exist the following vacuum expectation values of the scalar components of the fields appearing in Table 1,

$$\big\langle(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3})(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})\big\rangle \neq 0, \qquad \big\langle(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{1},\bar{\mathbf{3}})\big\rangle \neq 0, \tag{6}$$

so that the last factors in $SU(3)^4$'s, i.e., $SU(3)_4 \times SU(3)'_4$, are completely broken, and furthermore, the following link fields for identifications of the primed and unprimed $SU(3)$'s,

$$\begin{aligned}
&\big\langle(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})\big\rangle \neq 0,\\
&\big\langle(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1})(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})\big\rangle \neq 0,\\
&\big\langle(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1})\big\rangle \neq 0.
\end{aligned} \tag{7}$$

Namely, we identify $\mathbf{3}$ and $\bar{\mathbf{3}}$ of $SU(3)'_1$ as $\bar{\mathbf{3}}$ and $\mathbf{3}$ of $SU(3)_2$, $\mathbf{3}$ and $\bar{\mathbf{3}}$ of $SU(3)'_2$ as $\bar{\mathbf{3}}$ and $\mathbf{3}$ of $SU(3)_3$, and $\mathbf{3}$ and $\bar{\mathbf{3}}$ of $SU(3)'_3$ as $\bar{\mathbf{3}}$ and $\mathbf{3}$ of $SU(3)_1$, respectively. Then, the effective theory will be $SU(3)^3$ with the spectrum given in Table 2. Note that there result the needed three families,

$$3\,[\mathbf{27}_{\rm tri} \oplus \mathbf{27}_{\rm tri} \oplus \overline{\mathbf{27}}_{\rm tri}]. \tag{8}$$

Table 1
The massless spectrum of the orbifold (5) with the gauge group $SU(3)^8$

Sector	Twist	Multiplicity	Massless fields
U			None
T0	V	9	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3})(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$
		3	$(\bar{\mathbf{3}},\mathbf{3},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\bar{\mathbf{3}},\mathbf{3},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$
T1	$V + a_1$	3	$(\mathbf{1},\mathbf{1},\mathbf{1},\bar{\mathbf{3}})(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3})$
T2	$V - a_1$	9	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{1},\bar{\mathbf{3}})$
		3	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\bar{\mathbf{3}},\mathbf{1},\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{3},\bar{\mathbf{3}},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{3},\bar{\mathbf{3}},\mathbf{1})$
T3	$V + a_3$	3	$(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1},)(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$
T4	$V - a_3$	3	$(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})(\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$
T5	$V + a_1 + a_3$	3	$(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1})(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})$
T6	$V + a_1 - a_3$	3	$(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$
T7	$V - a_1 + a_3$	3	$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1})$
T8	$V - a_1 - a_3$	3	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$

[3] $SU(3)^8$ was considered before in Ref. [15].

Since the full trinification spectrum is added with the vectorlike combination of 3 $[\mathbf{27}_{\rm tri} \oplus \overline{\mathbf{27}}_{\rm tri}]$, the bare weak mixing angle is $\sin^2\theta_W = 3/8$, fulfilling the hypercharge quantization [10].

3. Phenomenology

There are many indices we deal with: the untwisted and the twisted sector number, the humor (gauge group), and the family indices. So, we use the following convention

$$\Psi_{[\rm family]}(\rm sector)_{(\rm humor)}. \tag{9}$$

For example, $\Psi_{[2]}(\mathrm{T0})_{(\rm a)}$ represents the second (out of the three) antiquark humor $(\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$, appearing in the twisted sector T0. This notation will be generalized to respective fields such as c^c, after the symmetry breaking $SU(3)^3 \to SU(2) \times U(1)_Y \times SU(3)_c$, and by identifying the three remaining light families of fermions. Each twisted sector in Tables 1 and 2 comes in three copies: so it is more accurate to represent, for example, the T0 sectors as T0-1, T0-2, and T0-3. The family indices can be dropped off if unnecessary.

3.1. Neutrino mass

The trinification fields of $(\mathbf{27}_{\rm tri} \oplus \overline{\mathbf{27}}_{\rm tri})$ in Table 2 can be removed at a large mass scale of order M_G by giving VEV's to all singlets in T0

$$\langle \Psi(\mathrm{T0})_{(\mathbf{1},\mathbf{1},\mathbf{1})} \rangle = M_G. \tag{10}$$

The superpotential can be taken as

$$\begin{aligned} g_{ABCDE}\big[\Psi(\mathrm{T0})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\big]\big[&\Psi_{[A]}(\mathrm{T2})_{(l)}\Psi_{[B]}(\mathrm{T6})_{(\bar{l})}\Psi_{[C]}(\mathrm{T3})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\Psi_{[D]}(\mathrm{T0})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\Psi_{[E]}(\mathrm{T0})_{(\mathbf{1},\mathbf{1},\mathbf{1})} \\ &+ \Psi_{[A]}(\mathrm{T2})_{(q)}\Psi_{[B]}(\mathrm{T4})_{(\bar{q})}\Psi_{[C]}(\mathrm{T5})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\Psi_{[D]}(\mathrm{T0})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\Psi_{[E]}(\mathrm{T0})_{(\mathbf{1},\mathbf{1},\mathbf{1})} \\ &+ \Psi_{[A]}(\mathrm{T2})_{(a)}\Psi_{[B]}(\mathrm{T8})_{(\bar{a})}\Psi_{[C]}(\mathrm{T1})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\Psi_{[D]}(\mathrm{T1})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\Psi_{[E]}(\mathrm{T3})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\big] + \text{h.c.}, \end{aligned} \tag{11}$$

where g_{ABCDE} are the couplings and we multiplied three singlet fields to satisfy the point group selection rule [16].

For the case of (11), the three light fermions result from T0. On the other hand, if we change indices in Eq. (11) from $0 \leftrightarrow 2$, then there result light fermions from T2. Also, a more complicated family structure can be obtained

Table 2
The massless spectrum with the identification (7). The gauge group is $SU(3)^3$. The symbol { } in the last column denotes that some entries are Goldstone bosons and some are heavy ones

Sector	Twist	Multiplicity	Massless fields
U		None	
T0	V	27	$\{(\mathbf{1},\mathbf{1},\mathbf{1})\}$
		3	$(\bar{\mathbf{3}},\mathbf{3},\mathbf{1}) + (\mathbf{1},\bar{\mathbf{3}},\mathbf{3}) + (\mathbf{3},\mathbf{1},\bar{\mathbf{3}})$
T1	$V + a_1$	27	$\{(\mathbf{1},\mathbf{1},\mathbf{1})\}$
T2	$V - a_1$	27	$\{(\mathbf{1},\mathbf{1},\mathbf{1})\}$
		3	$(\bar{\mathbf{3}},\mathbf{3},\mathbf{1}) + (\mathbf{1},\bar{\mathbf{3}},\mathbf{3}) + (\mathbf{3},\mathbf{1},\bar{\mathbf{3}})$
T3	$V + a_3$	3	link $SU(3)'_1 \to SU(3)^*_2$: $\{\mathbf{1} \oplus \mathbf{8}\}$
T4	$V - a_3$	3	$(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$
T5	$V + a_1 + a_3$	3	link $SU(3)'_2 \to SU(3)^*_3$: $\{\mathbf{1} \oplus \mathbf{8}\}$
T6	$V + a_1 - a_3$	3	$(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})$
T7	$V - a_1 + a_3$	3	link $SU(3)'_3 \to SU(3)^*_1$: $\{\mathbf{1} \oplus \mathbf{8}\}$
T8	$V - a_1 - a_3$	3	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{3})$

by assigning couplings and VEV's judiciously. One can see that (11) gives only the Dirac neutrino masses. For a see-saw mechanism, we need a huge Majorana neutrino mass at high energy scale. So, we consider the following nonrenormalizable couplings in the superpotential allowed by Z_3 orbifold,

$$\frac{\lambda_{ABCDEF}}{M^3}\big[\Psi_{[A]}(\mathrm{T0})_{(l)}\Psi_{[B]}(\mathrm{T0})_{(l)}\Psi_{[C]}(\mathrm{T6})_{(\bar{l})}\Psi_{[D]}(\mathrm{T6})_{(\bar{l})}\Psi_{[E]}(\mathrm{T7})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\Psi_{[F]}(\mathrm{T7})_{(\mathbf{1},\mathbf{1},\mathbf{1})}\big], \tag{12}$$

where M is of order the string scale. We will assign huge VEV's to $\bar{N}_5$'s in T6 and singlets in T7. Inserting these VEV's to (12), N_5's in T0 obtain huge Majorana masses since the VEV's and M are considered to be of the same order. The N_5's in T0 couple to light lepton doublets by $N_5(\mathrm{T0})l(\mathrm{T0})H_2(\mathrm{T0})$ which render the Dirac neutrino masses of order the electroweak scale. Thus, we have all the ingredients needed for the light see-saw neutrino masses.

3.2. R-parity

The breaking of the trinification gauge group down to the standard model gauge group is achieved by VEV's of N_{10} and N_5 directions [13] (or $\bar{N}_{10}$ and N_5, or $\bar{N}_5$ and N_{10}, or $\bar{N}_{10}$ and $\bar{N}_5$). Note that $\bar{N}_{10}$ and $\bar{N}_5$ appear in the T6 sector.

Let us choose the VEV's of N_{10} and $\bar{N}_5$, where $\bar{N}_5$'s appear only in T6. These VEV's certainly break $SU(3)^3 \to SU(2)_Y \times U(1) \times SU(3)_c$ [10], but the important thing to note is that $\langle\Psi(\mathrm{T6}:\bar{N}_5)_{(\bar{l})}\rangle$ does not couple $(H_2)_{\frac{1}{2}}$ field with the lepton doublet $(l)_{-\frac{1}{2}}$, since with the notation given in Eq. (2), $\bar{N}_5 = \Psi_{(1,\bar{3},0)}$, $H_2 = \Psi_{(\bar{2},i,0)}$, and $l = \Psi_{(\bar{3},i,0)}$. Thus, we obtain a kind of discrete symmetry naturally, forbidding the mixing of $(H_1)_{-\frac{1}{2}}$ and $(l)_{-\frac{1}{2}}$; *the R-parity is introduced* and the proton longevity is understood. Certainly, *the introduction of this discrete symmetry is by not allowing VEV's to N_5 fields* which would have mixed $(H_1)_{-\frac{1}{2}}$ and $(l)_{-\frac{1}{2}}$ if allowed. This is a choice of a specific string vacuum from a multitude of vacua.

The existence of the above discrete symmetry can be understood in the following way. The N_5 in $(\bar{\mathbf{3}},\mathbf{3},\mathbf{1})$ and $\bar{N}_5$ in $(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})$ has the following SM quantum numbers in terms of (2):

$$N_5 : \Psi_{(\bar{1},3,0)}, \qquad \bar{N}_5 : \Psi_{(1,\bar{3},0)}.$$

Thus, N_5 can couple to

$$N_5 H_2 l, \qquad N_5 D d^c, \tag{13}$$

while there is no field which $\bar{N}_5$ can couple to. We assign a huge VEV to $\langle\bar{N}_5\rangle$, but forbid a VEV of N_5. As stressed before, this is chosen by the string vacuum. Note, however, there are two sectors (T0 and T6) where N_5 appear. With our example (11) the N_5's and N_{10}'s in T2 are removed. Of course, N_5's in T0 and T2 do not develop VEV's to forbid $H_1 - l$ mixing. But, all N_{10}'s can develop VEV's. Since $N_5(\mathrm{T2})$'s are removed at a high energy scale, they have the opposite property from $N_5(\mathrm{T0})$'s.[4] Thus, the couplings (13) can be considered as the low energy effective couplings. If we consider the corresponding couplings with $N_5(\mathrm{T2})$, they would give highly suppressed effects at low energy phenomenology. This differentiation through the vacuum allows us to assign an effective low energy R-parity R, to $R(N_5(\mathrm{T0})) = -$ and $R(\bar{N}_5(\mathrm{T6})) = R(N_5(\mathrm{T2})) = +$. On the other hand, N_{10}'s in T0 and T2 can develop huge VEV's, leading to $R(N_{10}) = +$. Since there is the coupling $N_{10}H_1H_2$, H_1 and H_2 have the same R-parity quantum number. The R-parity of the Higgs fields must be positive so that their VEV's do not break the R-parity. From the Yukawa couplings qu^cH_2 and qd^cH_1, u^c and d^c must have the same R-parity. From the second term of (13), D and d^c have the opposite R-parity. Also, H_2 and l have the opposite R-parity, i.e., l has the negative R-parity, implying e^c also has the negative R-parity. Thus, we obtain the standard R-parity quantum number for

[4] Even if more complicated couplings are introduced, three combinations of N_5's remain light.

leptons,

$$R(l) = -1, \qquad R\left(e^c\right) = -1. \tag{14}$$

However, this does not fix the R-parity quantum number of the light quarks, q, u^c, d^c. But it is predicted that the R-parity is opposite for d^c and D^c.

If $R(D^c) = +1$, then we obtain the standard R-parity quantum numbers, $R(q, u^c, d^c) = R(l, e^c) = -1$ and proton lifetime from $qqql$ can be made sufficiently long. On the other hand, if $R(D^c) = -1$, then we obtain $R(q, u^c, d^c) = -R(l, e^c) = +1$. (Thus, the R-parity can be considered as the lepton parity.) In this case, $u^c d^c d'^c$ is forbidden by the gauge symmetry(not by the R-parity) and $u^c d^c D^c$ is forbidden by the R-parity. Also, dimension-5 operators in the superpotential such as $qqql$ are forbidden by R-parity.[5] So, it is hopeless to observe proton decay at the current underground detectors.

3.3. D-flat directions

For the low energy $N = 1$ supersymmetry to be valid, there must exist F-flat and D-flat directions. It is easy to find F-flat directions. For the asymmetric VEV's as we have assigned to $\bar{N}_5$ but not to N_5, search of D-flat directions is nontrivial. Alas, we already have so many VEV's for the consistency of our vacuum. For the D-flatness, we must find at least a direction $\Phi^* F_\alpha \Phi = 0$ for all α, where Φ is the grand column matrix for all the scalar fields and F_α are the gauge group generators. The relevant VEV's for our D-flatness are defined as

$$\begin{aligned}
&\left\langle \Psi(\mathrm{T7})_{(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1})}\right\rangle = \mathrm{diag}(V_{7u}, V_{7d}, V_{7s}), \qquad \langle \bar{N}_5\rangle = \langle \Psi_{(1,\bar{3},0)}\rangle = V_{\overline{N5}},\\
&\langle N_{10}\rangle = \langle \Psi_{(\bar{3},3,0)}\rangle = V_{N10}, \qquad \langle \bar{N}_{10}\rangle = \langle \Psi_{(3,\bar{3},0)}\rangle = V_{\overline{N10}}.
\end{aligned}$$

Thus, the conditions for the D-flatness lead to

$$\begin{aligned}
&\Phi^\dagger (T_3)_1 \Phi = \frac{1}{2}\left(|V_{\overline{N5}}|^2 + |V_{7u}|^2 - |V_{7d}|^2\right) = 0,\\
&\Phi^\dagger (T_3)_2 \Phi = 0,\\
&\Phi^\dagger (Y)_1 \Phi = \frac{1}{3}\left(|V_{\overline{N5}}|^2 + 2|V_{N10}|^2 - 2|V_{\overline{N10}}|^2 + |V_{7u}|^2 + |V_{7d}|^2 - 2|V_{7s}|^2\right) = 0,\\
&\Phi^\dagger (Y)_2 \Phi = \frac{2}{3}\left(|V_{\overline{N5}}|^2 - |V_{N10}|^2 + |V_{\overline{N10}}|^2\right) = 0,
\end{aligned}$$

where the subscripts of the generators represent the $SU(3)$ factors of the trinification group, and $T_3 = \mathrm{diag}(\frac{1}{2}, -\frac{1}{2}, 0)$ and $Y = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$. There are enough independent $(|\mathrm{VEV}|^2)$'s with negative and positive signs to satisfy the above D-flatness conditions. From the above expression, however, our simplified linkage $SU(3)'_3 \to SU(3)^*_1$ in Table 2 has more structures such as $(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})' \to (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})$, but $SU(3)'_3$ and $SU(3)_1$ are completely broken.

Some string orbifold models contain a mechanism for the doublet–triplet splitting by not allowing extra vectorlike quarks but allowing Higgsinos [8]. This has been reconsidered in field theoretic orbifolds by assigning appropriate discrete quantum numbers to the bulk fields so that extra massless vectorlike quarks are forbidden [17]. In essence, the string theory interpretation of the doublet–triplet splitting must arise from the study of the selection rules, summarized in Ref. [16]. In our case, the doublet–triplet splitting must occur after the breaking of the trinification gauge group down to the standard model gauge group by VEV's of N_{10}, $\bar{N}_{10}$, and $\bar{N}_5$. In principle, $\langle N_{10}\rangle$ can remove all the $D - D^c$ fields ($\bar{D} - \bar{D}^c$ also) and $H_1 - H_2$ fields ($\bar{H}_1 - \bar{H}_2$ also). But phenomenologically, we need just one pair of light Higgsinos of the MSSM, surviving this removal process. It is the old μ-problem [18]

[5] Only nonperturbative effects such as by instantons can break this R-parity at low energy. Then, the dimension 5 operator will be sufficiently suppressed.

or the MSSM problem [13]. At the perturbative level, we have not found such a mechanism yet. But, there may be strong dynamics at high energy so that the determinant of the Higgsino mass matrix vanishes [13], which we do not pursue here. In our case, there is no anomalous $U(1)$ symmetry from the string compactification since rank 16 is saturated by $SU(3)^8$. Thus, it is possible to consider the model-independent axion degree which can translate to a Peccei–Quinn symmetry at low energy [19]. This may help to allow a pair of light Higgs doublets [18].

4. Conclusion

We constructed a supersymmetric trinification model with three families of $(\mathbf{27}_{\rm tri} + \mathbf{27}_{\rm tri} + \overline{\mathbf{27}}_{\rm tri})$ by the Z_3 orbifold compactification with two Wilson lines. It is shown that a correct symmetry breaking pattern to the supersymmetric standard model can be achieved. One of the most attractive features is that the *hypercharge quantization*, i.e., the bare value $\sin^2\theta_W^0 = \frac{3}{8}$, is realized by the choice of vacuum. It is an important observation since there is no Z_3 orbifold model with any number of Wilson lines which can directly lead to the needed spectrum $(\mathbf{27}_{\rm tri} + \mathbf{27}_{\rm tri} + \overline{\mathbf{27}}_{\rm tri})$. The model presented in this Letter gives naturally light neutrino masses, and allows an introduction of R-parity. For one choice of the R-charge, the $D = 4$ and $D = 5$ baryon number violating operators are excluded, closing the window to the proton decay experiment. A natural solution of the MSSM problem, however, has to be implemented, which we hope to discuss in a future communication.

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