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Sliding Mode Control for Trajectory Tracking of Intelligent Vehicle

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Abstract

The intelligent vehicle is a complex non-linear system of various machines, sensors and computers, of which the controller design for trajectory tracking is one of the key techniques for intelligent vehicles. For the high qualities of robustness, accuracy and rigid time limitation of high-speed autonomous navigation intelligent vehicle, this paper is about the design of a sliding mode controller, based on the structures and motion model. It realizes that the intelligent vehicle can track random trajectory through controlling the linear velocity and the angular velocity. Through the simulation experiment using Matlab it is showing the effectiveness and reliability of the designed algorithms.

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Keywords: intelligent vehicle; trajectory tracking; sliding mode control

1. Introduction

The intelligent vehicle is a class of robotics, which incorporates the latest research results of automatic control, artificial intelligence, information fusion and other subjects, represents the highest achievements of mechanical and electrical integration and is one of the most active areas of the current technological development. The intelligent vehicle tracking control is a complex controlling problem. Tracking, also known as dynamic tracking, and it requires the mobile robot tracking a trajectory which keeping a functional relationship with time. As the intelligent vehicle system is a typical nonlinear system, there are a variety of unforeseen external disturbances, so sliding mode control becomes a hotspot in recent years. It can work with the range of model parameters without knowing the accurate mathematical model of the controlled object. This controlling method makes the system state slide along the sliding surface by controlling the switching of volume. The system will come with invariance when it is under the parameter perturbation and external disturbances, which attracts the attention of scholars all over the world. This paper is about the design of a sliding mode controller for trajectory tracking, based on the structure and

motion model, with the control input of the linear velocity and the angular velocity of the intelligent vehicle. The effectiveness of this controller was proved out through simulation experiment using Matlab.

2. The structure and motion model of the intelligent vehicle

In this paper, the vehicle platform is with the functional distribution structure. The entire system consists of a set of independent functional modules and the interaction between each functional module is completely transparent and is linked together through a central control module. Its structure is as illustrated in Figure.1. The relative position sensors such as odometers and inertial gyroscopes, combined with the absolute position sensors like GPS and magnetic compasses so that the accuracy of the current intelligent vehicle attitude will be determined, and so are the high positioning precision, good reliability and independence. In unstructured environments like irregular roads or outdoors, setting road signs and using the camera (CCD), radar or others which can detect the external environment and process the visual information accurately in real time is directly related to the operation speed and the ability to avoid obstacles. It plays a decisive role on the real-time character and robustness of the intelligent vehicle. And then deals with the environmental information provided by various sensors, which forms the unified expression of the external environment characteristics. On this basis, the whole and local optimal path of the intelligent vehicle can be planned out. Then according to the requirements of tracking and reaching targets, the controller can be designed to track the desired trajectory.

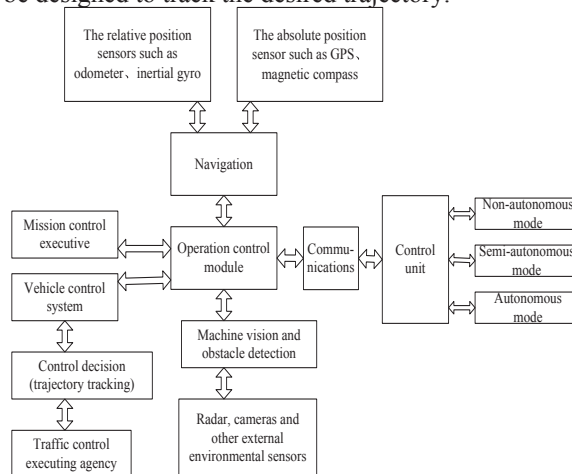


Figure. 1 The sketch map of the intelligent vehicle structure.

For the two-dimensional working space, the intelligent vehicle can be considered approximately that the body, wheels and the ground are rigid. Four wheels contact with the ground, and the wheels make the intelligent vehicle roll and that the wheels meet the condition of no sliding. The two larger rears of the intelligent vehicle are driving wheels, always pointing to the front of the body and with it consistent with the direction of the current speed. The two smaller front wheels are direction wheels and there can be certain deflection angle between them and the direction of the body.

The position and orientation coordinates of the intelligent vehicle are showed in Figure.2. The pose is represented by the coordinate of midpoint M of the two rear axle and the orientation angle θ , which can be described as $p = (x, y, \theta)^T$. The vector $q = (v, w)^T$ represented the control volume of linear velocity and angular velocity of the intelligent vehicle. The motion model is shown in Figure.3. The distance L between the two axles of the front and rear wheels approximately represents the length of the vehicle. ϕ is the front wheels average orientation. If ϕ is fixed to the deflection angle of the front wheels, the robot will do

circular motion around a circle with the radius as ρ , which ρ can be decided by the extension of the front and rear axle lines.

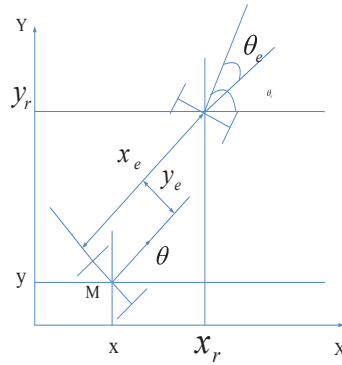


Figure. 2 The pose coordinate of the intelligent vehicle.

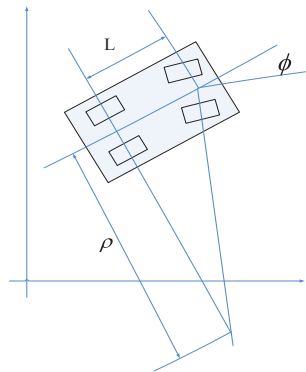


Figure. 3 The motion model of the intelligent vehicle.

For the moving vehicle, the pose P is a differential equation with respect to t . The movement of the intelligent vehicle can be expressed as a set of constraint equations:

$$\begin{cases} \dot{x} = v \cos \theta \\ \dot{y} = v \sin \theta \\ \dot{\theta} = (v / L) \tan \phi = w \end{cases} \quad (1)$$

where \dot{x} , \dot{y} , $\dot{\theta}$ respectively represent the three components of the pose P , assuming the maximum steering angle ϕ_{\max} is $\pi/4$, and then the corresponding minimum turning radius ρ_{\min} is equal to the length of the vehicle L . The angular velocity can be expressed by the minimum turning radius as:

$$\dot{\theta} \leq v / \rho_{\min} \quad (2)$$

Vehicle kinematics equation can be expressed in matrix form as follows:

$$\dot{p} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \tag{3}$$

Considering the pose $p_r = (x_r, y_r, \theta_r)^T$ and the speed $q_r = (v_r, w_r)^T$ as the reference tracking information, and the current pose is $p_c = (x_c, y_c, \theta_c)^T$. The coordinate system $X_c - Y_c$ transforms to the new coordinate system $X_e - Y_e$. The coordinate of the intelligent vehicle in the new coordinate system is $p_e = (x_e, y_e, \theta_e)^T$. According to the transformation formula, the error equation of the intelligent vehicle pose is:

$$\begin{aligned} p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} &= \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_c \\ y_r - y_c \\ \theta_r - \theta_c \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} (p_r - p_c) \end{aligned} \tag{4}$$

The error differential equation of the system is:

$$\dot{p}_e = \begin{bmatrix} y_e w_c - v_c + v_r \cos\theta_e \\ -x_e w_c + v_r \sin\theta_e \\ w_r - w_c \end{bmatrix} \tag{5}$$

The trajectory tracking of the kinematics model is to find the control input $q_c = (v_c, w_c)^T$, under the random initial error the system equation (5) makes P_e bounded and

$$\lim_{t \rightarrow \infty} \|p_e\| = 0 \tag{6}$$

3. The algorithm of trajectory tracking control

3.1. Design of the Switching Function

The equation (5) is a two-input nonlinear system, of which the switching function is difficult to be designed. It can use the method of Backstepping to design the switching function.

Lemma 1 For any $x \in R$ and $|x| < \infty$, there is $\phi(x) = x \sin(\arctan x) \geq 0$, “=” can be set up if and only if $x = 0$.

According to the lemma 1, the switching function can be designed based on the method of Backstepping.

When $x_e = 0$, the Lyapunov candidate is taken as

$$V_y = \frac{1}{2} y_e^2 \tag{7}$$

Suppose $\theta_e = -\arctan(v_r y_e)$, and the derivative of V_y is

$$\begin{aligned} \dot{V}_y &= y_e \dot{y}_e = y_e (-x_e w_c + v_r \sin \theta_e) \\ &= -y_e x_e w_c - v_r y_e \sin(\arctan(v_r y_e)) \end{aligned} \tag{8}$$

Because of

$$v_r y_e \sin(\arctan(v_r y_e)) \geq 0 \tag{9}$$

(if and only if $v_r y_e = 0$, "=" can be set up)

Hence

$$\dot{V}_y \leq 0 \tag{10}$$

It can be seen that the system state y_e converges to zero as long as x_e converges to zero and θ_e converges to $-\arctan(v_r y_e)$.

According to this conclusion, the switching function can be designed as:

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} x_e \\ \theta_e + \arctan(v_r y_e) \end{bmatrix} \tag{11}$$

Through designing sliding mode controller, it can be true that $s_1 \rightarrow 0$ and $s_2 \rightarrow 0$. That means x_e can converge to zero and θ_e can converge to $-\arctan(v_r y_e)$, so it comes with $y_e \rightarrow 0$ and $\theta_e \rightarrow 0$.

3.2. Design of the Sliding Mode Controller

In the real sliding mode control system, chattering must be exist, and if the chattering is eliminated, so will the abilities of anti-proactive and anti-disturbance of the variable structure control. Therefore, it can only weaken the chattering in some degree. The reason why variable structure chatters are mainly that the speed of the system trajectory when reaching the switching surface is so fast and the control of the switch is limited and the inertia of the switch make it difficult to reverse the inertia immediately, which result in switching back and forth across the surface so comes chattering. In this paper, a continuous function is used to weaken the chattering on the sliding mode control itself instead of the sign function.

Following the constant reaching law, let

$$\dot{s} = -k \operatorname{sgn} s \tag{12}$$

The continuous function is

$$\dot{s}_i = -k_i \frac{s_i}{|s_i| + \delta_i}, i = 1, 2 \tag{13}$$

where δ_i is a positive decimal.

Let $\alpha = \arctan(v_r y_e)$, and from (5) and (11) we get:

$$\begin{aligned} \dot{s} = \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} &= \begin{bmatrix} -k_1 \frac{s_1}{|s_1| + \delta_1} \\ -k_2 \frac{s_2}{|s_2| + \delta_2} \end{bmatrix} \\ &= \begin{bmatrix} y_e w_c - v_c + v_r \cos \theta_e \\ w_r - w_c + \frac{\partial \alpha}{\partial v_r} \dot{v}_r + \frac{\partial \alpha}{\partial y_e} (-x_e w_c + v_r \sin \theta_e) \end{bmatrix} \end{aligned} \tag{14}$$

Then the control law is:

$$q_c = \begin{bmatrix} v_c \\ w_c \end{bmatrix} = \begin{bmatrix} y_e w_c + v_r \cos \theta_e + k_1 \frac{s_1}{|s_1| + \delta_1} \\ \frac{w_r + \frac{\partial \alpha}{\partial v_r} \dot{v}_r + \frac{\partial \alpha}{\partial y_e} (v_r \sin \theta_e) + k_2 \frac{s_2}{|s_2| + \delta_2}}{1 + \frac{\partial \alpha}{\partial y_e} x_e} \end{bmatrix} \tag{15}$$

where $\frac{\partial \alpha}{\partial v_r} = \frac{y_e}{1 + (v_r y_e)^2}$, $\frac{\partial \alpha}{\partial y_e} = \frac{v_r}{1 + (v_r y_e)^2}$.

4. Simulation results

The controlled object is the error differential (5). Tracking the circular trajectory of uniform linear velocity and angular velocity, the sliding mode controller is used in this paper. Let $w_r = 1.0, v_r = 1.0$, and we get $\dot{v}_r = 0$ and radius is $r = v_r / w_r = 1.0$. Let $\delta_1 = \delta_2 = 0.01$ and the initial pose error as $[4 \ 0 \ 0]$, using the control law (15), the simulation results are shown in Figure. 4 to Figure. 7. It can be seen that the x error, y error and orientation angle converges rapidly to zero, and the practical trajectory can track the reference trajectory accurately. It uses continuous functional approximation which weakens chattering effectively and improves the dynamic quality of the system.

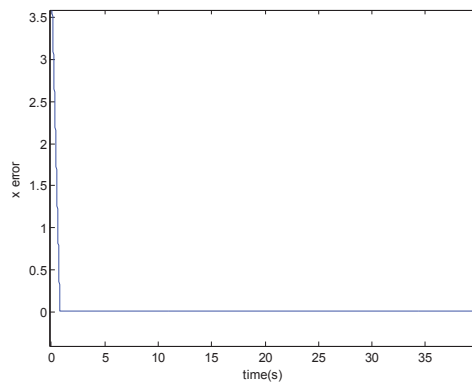


Figure. 4 The error of X-axis.

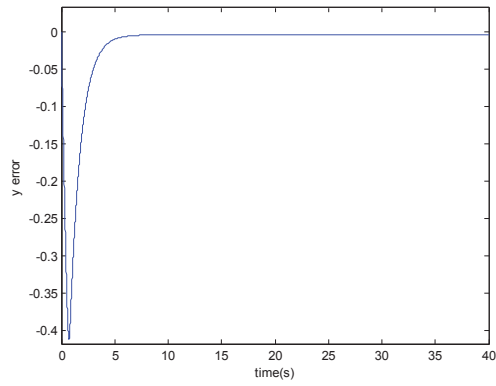


Figure. 5 The error of Y-axis.

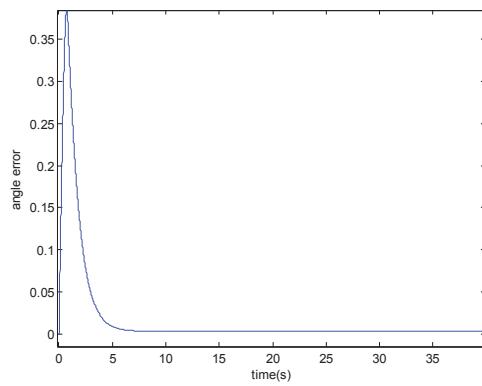


Figure. 6 The error of orientation angle.

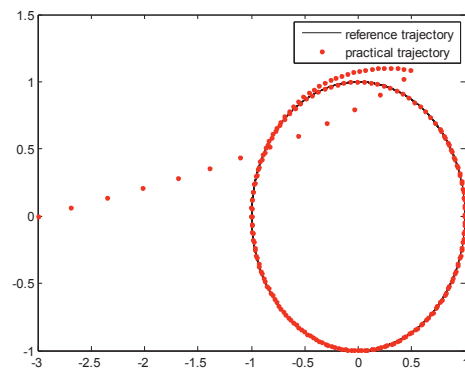


Figure.7 The trajectory tracking of the intelligent vehicle.

5. Conclusion

Trajectory tracking of the intelligent vehicle is a typical time-delay and nonlinear unstable control system. Considering the interferences and the uncertain parameters during the intelligent vehicle movement, this paper proposes a sliding mode control method. Simulation results have demonstrated the control method can not only ensure the high accuracy of the intelligent vehicle's tracking position and direction, but also keep high stability and reliability, which make it possess certain practical value.

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