Numerical simulation of flow past twin near-wall circular cylinders in tandem arrangement at low Reynolds number

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Abstract

Fluid flow past twin circular cylinders in a tandem arrangement placed near a plane wall was investigated by means of numerical simulations. The two-dimensional Navier-Stokes equations were solved with a three-step finite element method at a relatively low Reynolds number of Re = 200 for various dimensionless ratios of 0.25 ≤ G/D ≤ 2.0 and 1.0 ≤ L/D ≤ 4.0, where D is the cylinder diameter, L is the center-to-center distance between the two cylinders, and G is the gap between the lowest surface of the twin cylinders and the plane wall. The influences of G/D and L/D on the hydrodynamic force coefficients, Strouhal numbers, and vortex shedding modes were examined. Three different vortex shedding modes of the near wake were identified according to the numerical results. It was found that the hydrodynamic force coefficients and vortex shedding modes are quite different with respect to various combinations of G/D and L/D. For very small values of G/D, the vortex shedding is completely suppressed, resulting in the root mean square (RMS) values of drag and lift coefficients of both cylinders and the Strouhal number for the downstream cylinder being almost zero. The mean drag coefficient of the upstream cylinder is larger than that of the downstream cylinder for the same combination of G/D and L/D. It is also observed that change in the vortex shedding modes leads to a significant increase in the RMS values of drag and lift coefficients.

Keywords: Navier-Stokes equations; Finite element method; Circular cylinder; Vortex shedding mode; Hydrodynamic force coefficient

1. Introduction

Steady fluid flows past circular cylinders near a plane wall are highly significant to ocean currents over submarine pipelines. Actual pipelines may be close to one another in a tandem arrangement, due to special engineering requirements, leaving a certain center-to-center distance between them. The gap between the pipelines and the plane wall can also be formed by either an uneven seabed or local scour below submarine pipelines, which can be measured using the distance from the seabed to the lowest surface of pipes. These suspended pipelines are subjected to oscillating fluid forces induced by vortex shedding, which may give rise to severe vortex-induced vibration and even strengthen the undesirable local scour. Hence, an understanding of hydrodynamic characteristics is important to practical pipeline design, even if the flow is limited to a rather low Reynolds number (Re).

Investigations of fluid flow past a pair of cylinders in tandem, side-by-side, or staggered arrangements have been carried out in the past, and these investigations have mainly focused on situations in which the cylinders were immersed in an open space and the effect of wall boundaries could be ignored. Zdravkovich (1977, 1987) showed that when more than one body is placed in a fluid flow, the resulting hydrodynamic force coefficient and vortex shedding mode may be completely different from those on a single body at the same Reynolds number. Hence, a variety of vortex shedding modes,
characterized by the different characteristics of near wakes, should be discerned under different arrangements of the circular cylinders or spacings between two circular cylinders.

Flow past two circular cylinders of an identical diameter in a side-by-side configuration was studied by Bearman and Wadcock (1973), Williamson (1985), and Kim and Durbin (1988). Their results showed that only one vortex shedding mode was observed when the distance ratio $L/D \leq 2.0$, where $L$ is the center-to-center distance between the two cylinders, and $D$ is the cylinder diameter. Early experimental studies on the flow past circular cylinders in a tandem arrangement, for example, Ishigai et al. (1972), Kostić and Oka (1972), Tanida et al. (1973), and King and Johns (1976), showed that there were two major flow regimes. For cylinders separated from one another at small values of $L/D$, the flow is separated from the upstream cylinder and reattaches to the downstream one, while when the values of $L/D$ are large, vortices are shed from both the cylinders. Meneghini et al. (2001) and Jester and Kallinderis (2003) studied the flow past two cylinders in tandem and side-by-side arrangements. Meneghini et al. (2001) observed negative drag coefficients of the downstream cylinder for $L/D \leq 4.0$ and $Re = 200$ when the two cylinders were in a tandem arrangement. Mittal et al. (1997) conducted numerical simulations to study fluid flows past two cylinders in tandem and staggered arrangements. They found that, for the two cylinders in a tandem arrangement, the hydrodynamic force coefficient and vortex shedding mode were greatly dependent on the Reynolds number, in comparison to fluid flow past an isolated cylinder.

Flow past a circular cylinder near a plane wall has also been widely studied in the past few decades. Investigations have shown that the vortex shedding can be suppressed with very small gaps between the cylinder and the plane wall. Under the condition of high Reynolds numbers in the sub-critical regime, Bearman and Zdravkovich (1978), Grass et al. (1984), and Lei et al. (1999) confirmed that the vortex shedding can be suppressed when the gap ratio $G/D < 0.3$, although different experimental techniques were employed. Price et al. (2002) experimentally studied the fluid flow past a circular cylinder near a plane wall for Reynolds numbers between 1 200 and 4 960. Their study indicated that, for very small values of $G/D$, the vortex shedding was suppressed or extremely weak, and no regular vortex was shed from the cylinder. Angrilli et al. (1982) investigated the effects of $G/D$ on the Strouhal number at $Re = 2 860, 3 820$, and 7 640. They found that, when $G/D < 0.5$, the gap ratio $G/D$ had a fairly strong effect on the Strouhal number. Bearman and Zdravkovich (1978) investigated the fluid flow over a cylinder close to a plane wall at higher Reynolds numbers of $Re = 2.5 \times 10^4$ and $4.8 \times 10^4$. They found that regular vortex shedding occurred when $G/D > 0.3$, and the Strouhal number was independent of $G/D$.

Cheng et al. (1994) conducted flow visualization measurements to examine the flow past a cylinder close to a plane wall at $Re = 500$. They concluded that the Strouhal number increased with the decrease of $G/D$ when $0.2 < G/D < 0.625$. Lei et al. (2000) found that vortex shedding was suppressed at small gap ratios, and the critical gap ratio, at which the vortex shedding is suppressed, varies with the thickness of the boundary layer that develops on the plane wall.

It is well known that vortex shedding from a circular cylinder becomes three-dimensional when $Re > 200$ (Williamson, 1988, 1989) and turbulent at higher Reynolds numbers. Both the three-dimensional and turbulent effects give rise to a considerable increase in computational requirements. However, two-dimensional simulations at low Reynolds numbers can be used to generate some insights into the vortex dynamics in the wake (Lei et al., 2000; Meneghini et al., 2001). Hence, the numerical investigations of this study were restricted to a limiting Reynolds number of $Re = 200$, which allows us to solve the two-dimensional laminar Navier-Stokes equations with fairly acceptable computational efforts.

The numerical simulations were conducted for $G/D = 0.25, 0.50, 0.75, 1.00, 1.50$, and 2.00 and $L/D$ values ranging from 1.0 to 4.0 with an interval of 0.25. The effects of $L/D$ and $G/D$ on the hydrodynamic force coefficients, Strouhal numbers, and vortex shedding modes were investigated in this study.

2. Governing equations and numerical method

The governing equations are the non-dimensional continuity equation and the non-dimensional time-dependent incompressible Navier-Stokes equations for viscous Newtonian fluid:

$$\frac{\partial u_i}{\partial x_i} = 0$$

(1)

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

(2)

where $u_i$ is the velocity component in the $x_i$ direction ($i = 1, 2$ for the present two-dimensional numerical model with $x_1 = x$ and $x_2 = y$ in this study), $p$ is the pressure, $t$ is time, and $Re$ is defined as $Re = U_0 D / v$, with $U_0$ being the free-stream speed, and $v$ being the kinematic viscosity of the fluid.

The governing equations are solved using a three-step finite element method (Jiang and Kawahara, 1993), which shows high-order accuracy and strong performance for convection-diffusion problems. Using the method, the momentum equation is discretized as follows:

$$u_i^{n+1/3} = u_i^n + \frac{\Delta t}{3} \left( \frac{1}{Re} \frac{\partial^2 u_i^n}{\partial x_j \partial x_j} - u_j \frac{\partial u_i^n}{\partial x_j} - \frac{\partial p^n}{\partial x_j} \right)$$

(3)

$$u_i^{n+1/2} = u_i^n + \frac{\Delta t}{2} \left( \frac{1}{Re} \frac{\partial^2 u_i^{n+1/3}}{\partial x_j \partial x_j} - u_j \frac{\partial u_i^{n+1/3}}{\partial x_j} - \frac{\partial p^{n+1/3}}{\partial x_j} \right)$$

(4)

$$u_i^{n+1} = u_i^n + \Delta t \left( \frac{1}{Re} \frac{\partial^2 u_i^{n+1/2}}{\partial x_j \partial x_j} - u_j \frac{\partial u_i^{n+1/2}}{\partial x_j} - \frac{\partial p^{n+1/2}}{\partial x_j} \right)$$

(5)

where $\Delta t$ denotes the time increment between the $n$th and $(n + 1)$th time levels, and superscripts $n + 1/3$, $n + 1/2$, and $n + 1$ represent the time instants of $(n + 1/3)\Delta t$, $(n + 1/2)\Delta t$, and $(n + 1)\Delta t$, respectively.
and \((n+1)\Delta t\), respectively. Using the divergence operation on both sides of Eq. (5) and considering the continuity equation at the \((n+1)\)th time level, the Poisson-type pressure equation can be obtained:

\[
\frac{\partial^2 p^{n+1}}{\partial x_i^2} = -\frac{1}{\Delta t} \frac{\partial u_i^n}{\partial x_i} - \frac{1}{\Delta t} \frac{\partial}{\partial x_i} \left( u_j^{n+1/2} \frac{\partial u_i^{n+1/2}}{\partial x_j} \right) + \frac{1}{Re} \frac{\partial}{\partial x_i} \left( \frac{\partial u_i^{n+1/2}}{\partial x_j} \right)
\]

(6)

These equations are solved for the unknown velocity \(u_i\) and pressure \(p\) through the finite element method. The numerical procedures can be summarized as follows:

1. \(u_i^{n+1/3}\) is calculated by substituting the known \(u_i^n\) and \(p^n\) into Eq. (3);
2. Eq. (4) is solved with \(u_i^{n+1/3}\) and \(p^n\) to obtain the solution for \(u_i^{n+1/2}\);
3. \(p^{n+1}\) is predicted by substituting \(u_i^{n+1/2}\) and \(u_i^n\) into Eq. (6);
4. Eq. (5) is solved with the available \(u_i^n\), \(u_i^{n+1/2}\), and \(p^{n+1}\) to calculate the velocity \(u_i^{n+1}\) at the \((n+1)\)th time level.

Fig. 1 shows the sketch for the fluid flow past two near-wall circular cylinders with an identical diameter in a tandem arrangement. A rectangular computational domain with a width of 40\(D\) and a height of 10\(D\) was used. The inlet boundary was located 16\(D\) away from the center of the downstream cylinder.

The boundary conditions are as follows: (1) at the inlet, \(u = 1\), \(v = 0\), with \(u\) and \(v\) being the non-dimensional incoming flow velocities in the \(x\) and \(y\) directions, respectively, and \(\partial p/\partial x = 0\); (2) along the outlet boundary, the free outflow boundary condition is \(\partial u/\partial x = 0\), \(\partial v/\partial y = 0\), and \(p = 0\); (3) there is a no-slip boundary condition on the surface of the cylinder: \(u = 0\), \(v = 0\), and \(\partial p/\partial n = 0\), where \(n\) is the outward unit normal vector; (4) at the plane wall, there is a no-slip boundary condition: \(u = 0\), \(v = 0\), and \(\partial p/\partial y = 0\); and (5) along the top boundary, there is a symmetric boundary condition: \(\partial u/\partial y = 0\), \(v = 0\), and \(\partial p/\partial y = 0\).

The time-dependent drag coefficient \(C_D(t)\) and lift coefficient \(C_L(t)\) of each cylinder are obtained by integrating the instantaneous pressure and vorticity over the surface of the cylinder:

\[
C_D(t) = \frac{F_D(t)}{0.5 \rho U_0^2 D} = -\int_0^{2\pi} p(t) \cos \theta d\theta - \int_0^{2\pi} \frac{1}{Re} \omega(t) \sin \theta d\theta
\]

(7)

\[
C_L(t) = \frac{F_L(t)}{0.5 \rho U_0^2 D} = -\int_0^{2\pi} p(t) \sin \theta d\theta + \int_0^{2\pi} \frac{1}{Re} \omega(t) \cos \theta d\theta
\]

(8)

where \(F_D(t)\) and \(F_L(t)\) are the total drag and lift forces, respectively; \(\rho\) is the density of the fluid; \(\theta\) is an angle in the counterclockwise direction, measured from the positive direction of the \(x\)-axis to the line that connects the center of the cylinder and a point on the cylinder surface; and \(\omega(t)\) is the local vorticity, and \(\omega(t) = \partial v/\partial x - \partial u/\partial y\).

The mean drag coefficient \(\overline{C}_D\) is expressed as

\[
\overline{C}_D = \frac{1}{\Delta T} \int_{t_1}^{t_2} C_D(t) dt
\]

(9)

where \(\Delta T = t_2 - t_1\) is the integral time period when the time history of the drag coefficient is stable. Similarly, the mean lift coefficient \(\overline{C}_L\) can be defined as

\[
\overline{C}_L = \frac{1}{\Delta T} \int_{t_1}^{t_2} C_L(t) dt
\]

(10)

The root mean square (RMS) values of drag and lift coefficients, denoted by \(C_D'\) and \(C_L'\), respectively, are defined as

\[
C_D' = \sqrt{\frac{1}{\Delta T} \int_{t_1}^{t_2} (C_D(t) - \overline{C}_D)^2 dt}
\]

(11)

\[
C_L' = \sqrt{\frac{1}{\Delta T} \int_{t_1}^{t_2} (C_L(t) - \overline{C}_L)^2 dt}
\]

(12)

The Strouhal number \(St\) is evaluated according to the variation of the lift coefficient:

\[
St = f U_0 / D
\]

(13)

where \(f\) is the frequency of the lift coefficient obtained with the fast Fourier transform (FFT) method.

3. Spatial and time convergence and numerical validations

In this study, numerical simulations were first performed to confirm the grid independence. The main variables for this study were the mean drag and lift coefficients, RMS values of drag and lift coefficients for the upstream and downstream cylinders, and the Strouhal number. In order to validate the accuracy of this numerical model, fluid flow over two circular
cylinders in a tandem arrangement was simulated and the calculated values of the aforementioned variables were compared with available numerical results.

3.1. Spatial and time convergence

In order to establish grid independence for the numerical model, a typical example of fluid flow over two cylinders in a tandem arrangement when $L/D = 3.0$, $G/D = 1.0$, and $Re = 200$ was considered. Unstructured meshes were used during the numerical simulations, allowing fine meshes around the solid wall and coarse meshes for the flow field far away from the cylinders to be used. The surfaces of the two cylinders were divided into $N_c$ uniform grid cells. Four different cell sizes with $N_c = 80, 160, 240$, and 320 were used to examine grid independence. The numerical results are shown in Table 1, where $C_{D_u}$ and $C_{L_u}$ are the mean drag and lift coefficients of the upstream cylinder, respectively; $C_{D_d}$ and $C_{L_d}$ are the mean drag and lift coefficients of the downstream cylinder, respectively; $C_{l_u}$ and $C_{l_d}$ are the RMS values of the lift coefficient for the upstream and downstream cylinders, respectively; and $St_d$ is the Strouhal number for the downstream cylinder.

The numerical results obtained by mesh 3 and mesh 4 are close to one another. Therefore, mesh 3 was for all subsequent computations for the sake of efficiency and accuracy. In addition, a dynamic time step was used throughout the numerical simulations, and determined by the following equation:

$$\Delta t = C_t \min_k \left( \frac{\sqrt{S_k}}{|u_k|} \right)$$

where $S_k$ is the area of the $k$th computational cell, $|u_k|$ is the absolute velocity at the center of the $k$th cell, and $C_t$ is an empirical coefficient. Considering that the three-step finite element scheme should satisfy the Courant-Friedrichs-Lewy (CFL) condition (Jiang and Kawahara, 1993), we set $C_t = 0.2$ to further guarantee numerical stability.

### 3.2. Numerical validation

The numerical model was then validated by simulating the flow past twin circular cylinders with an identical diameter in a tandem arrangement at $Re = 200$. The numerical results of this study and those obtained by Meneghini et al. (2001) are listed in Table 2, where $St_u$ is the Strouhal number for the upstream cylinder.

It can be seen from Table 2 that strong agreement was achieved between the numerical results from this study and Meneghini et al. (2001), indicating that this numerical model effectively predicts the hydrodynamic force coefficient and Strouhal number.

### 4. Results analysis and discussion

Computations were conducted with the numerical model for the cases of $G/D = 0.25, 0.50, 0.75, 1.00, 1.50$, and 2.00 with different values of $L/D$ ranging from 1.0 to 4.0 and with an interval of 0.25.

#### 4.1. Vortex shedding mode

It is known that vortex shedding can be suppressed for flow over an isolated circular cylinder placed near a plane wall at small values of $G/D$. Lei et al. (2000) reported that the $G/D$ value has a significant effect on vortex shedding. In the case of twin cylinders near a plane wall, it is expected that the vortex shedding will be influenced by the dimensionless parameter $L/D$, in addition to $G/D$.

##### 4.1.1. No-shedding mode

The numerical simulations in this study showed that the vortex shedding can be completely suppressed at low values of $G/D$, even for the total span of $L/D$ considered in this study, meaning that $L/D$ has a limited influence on the vortex shedding at low values of $G/D$. This is referred to as the no-shedding mode in this paper.

Fig. 2 shows the vorticity contours behind two cylinders at $G/D = 0.50$ when $L/D = 1.00$ and 4.00, two typical examples of the no-shedding mode. It can be seen from Fig. 2 that the shear layers are generated from the lateral sides of the cylinders, as they are in the typical situation of laminar flow past an isolated circular cylinder. The asymmetrical distribution of

### Table 1

<table>
<thead>
<tr>
<th>Mesh</th>
<th>$N_c$</th>
<th>$C_{D_u}$</th>
<th>$C_{L_u}$</th>
<th>$C_{l_u}$</th>
<th>$C_{D_d}$</th>
<th>$C_{L_d}$</th>
<th>$C_{l_d}$</th>
<th>$St_d$</th>
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<tr>
<td>1</td>
<td>80</td>
<td>1.34</td>
<td>0.034</td>
<td>0.62</td>
<td>0.74</td>
<td>0.17</td>
<td>1.16</td>
<td>0.180</td>
</tr>
<tr>
<td>2</td>
<td>160</td>
<td>1.40</td>
<td>0.043</td>
<td>0.65</td>
<td>0.77</td>
<td>0.18</td>
<td>1.22</td>
<td>0.185</td>
</tr>
<tr>
<td>3</td>
<td>240</td>
<td>1.43</td>
<td>0.046</td>
<td>0.67</td>
<td>0.78</td>
<td>0.19</td>
<td>1.25</td>
<td>0.186</td>
</tr>
<tr>
<td>4</td>
<td>320</td>
<td>1.44</td>
<td>0.048</td>
<td>0.67</td>
<td>0.79</td>
<td>0.19</td>
<td>1.25</td>
<td>0.186</td>
</tr>
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### Table 2

<table>
<thead>
<tr>
<th>$L/D$</th>
<th>$C_{D_u}$</th>
<th>$C_{D_d}$</th>
<th>$St_u$</th>
<th>$St_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meneghini et al. (2001)</td>
<td>Present study</td>
<td>Meneghini et al. (2001)</td>
<td>Present study</td>
</tr>
<tr>
<td>1.5</td>
<td>1.06</td>
<td>1.059</td>
<td>-0.18</td>
<td>-0.187</td>
</tr>
<tr>
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<td>1.03</td>
<td>1.025</td>
<td>-0.17</td>
<td>-0.191</td>
</tr>
<tr>
<td>3.0</td>
<td>1.00</td>
<td>0.990</td>
<td>-0.08</td>
<td>-0.103</td>
</tr>
<tr>
<td>4.0</td>
<td>1.18</td>
<td>1.186</td>
<td>0.38</td>
<td>0.333</td>
</tr>
<tr>
<td></td>
<td>0.167</td>
<td>0.165</td>
<td>0.167</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>0.130</td>
<td>0.133</td>
<td>0.130</td>
<td>0.133</td>
</tr>
<tr>
<td></td>
<td>0.125</td>
<td>0.127</td>
<td>0.125</td>
<td>0.127</td>
</tr>
<tr>
<td></td>
<td>0.174</td>
<td>0.170</td>
<td>0.174</td>
<td>0.170</td>
</tr>
</tbody>
</table>
vortices can also be observed in the near wakes of the cylinders. For both cases, the negative vortices on the upper side of the cylinders seem to be stronger than the positive vortices along the lower side of the cylinders. Theoretically, this near wake may lead to flow instability. However, one may also observe that the negative vortices occupy the gap between the plane wall and the cylinders. A large vorticity gradient between the negative vortices along the plane wall and the positive vortices attached to the lower sides of cylinders can be observed, which leads to an incline for the positive vortex structures. It is expected that the lower sides of near wakes fall into the boundary layer of the plane wall, causing the near wakes of the two cylinders to remain stable. In other words, the strong viscous effect damps out the disturbance of the flow field and gives rise to a stable flow field.

At a small value of \( G/D \), the positive vortices in the lower shear layer of the cylinder are weakened by the negative vortices in the wall shear layer. Therefore, no vortex shedding occurs behind the cylinders. This is consistent with the observation of Lei et al. (2000).

4.1.2. One-wake mode

The numerical results in this study showed that, although the vortex shedding was observed when \( G/D \geq 0.75 \) for all values of \( L/D \), for small values of \( L/D \), vortex shedding can only be observed from the downstream cylinder. This is referred to as the one-wake mode in this paper.

Fig. 3 shows the evolution of near-wake vortices during one vortex shedding period \( T \) with an interval of \( 0.25T \) when \( G/D = 1.00 \) and \( L/D = 2.75 \). As shown in Fig. 3, the regular vortex shedding seems rather far away from the downstream cylinder. This implies a limited lift oscillation, which will be confirmed later. It was found that the influence of the shear layer along the plane wall on the near wakes of the upstream and downstream cylinders is weak due to the rather large gap ratio, and the near wake between the two cylinders remains stable, showing an almost-constant vorticity field over time. It was also observed that the shear layers that are separated from the upstream cylinder reattach to the front surface of the downstream cylinder. The steady recirculation region, consisting of a pair of stable vortices, develops very slowly in the gap between the cylinders.

4.1.3. Two-wake mode

As the distance ratio \( L/D \) increases to 2.75, the two-wake mode appears at \( G/D = 1.50 \) and 2.00, as shown in Fig. 4. In this study, the two-wake mode was defined as vortex shedding from both the upstream and downstream cylinders.

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**Fig. 2.** Vorticity contours for no-shedding mode for \( G/D = 0.50 \) when \( L/D = 1.00 \) and 4.00.

**Fig. 3.** Evolution of vortex field for one-wake mode when \( G/D = 1.00 \) and \( L/D = 2.75 \).

**Fig. 4.** Evolution of vortex field for two-wake mode when \( G/D = 1.50 \) and \( 2.00 \).
The flow structure of the two-wake mode is completely different from that of the one-wake mode because the vortices are shed from both sides of the two circular cylinders. Compared with the one-wake mode, the vortex dimension in the wake of the downstream cylinder for the two-wake mode is reduced. Strong interference between the two wakes can be observed. The vortex shedding from the upstream cylinder is rolled into the near wake of the downstream cylinder. It seems that the vortices are shed from the upstream and downstream circular cylinders at the same frequency, but with a phase lag. The reason for this is that the vortices are shed from the upstream cylinder and downstream cylinder at the same time but with a positive vortex for the upstream cylinder and a negative vortex for the downstream cylinder, as shown in Figs. 4(c) and (d).

As in the one-wake mode, the shear layer, which develops on the plane wall, has little influence on the near wakes of the two cylinders.

4.1.4. Identification of vortex shedding mode

The vortex shedding modes in the three wake modes were identified according to the computations in this study, and are shown in Table 3. It can be seen from Table 3 that, for $G/D = 0.50$, the no-shedding mode is observed when $L/D \leq 3.25$, while the two-wake mode occurs when $3.50 \leq L/D \leq 4.00$. Hence, there must exist a critical distance ratio $L_{cr}/D$ ranging from 3.25 to 3.5, which determines the transition between the two vortex shedding modes. Table 3 also show that change in the vortex shedding mode from the one-wake mode to the two-wake mode can be observed when $G/D \geq 0.75$, implying the existence of the critical distance ratio. However, no critical distance ratio can be obtained when $G/D = 0.25$ since only the no-shedding mode was observed over the whole span of $L/D$ in this study.

It should be noted that there is a one-wake vortex shedding mode when $G/D = 2.00$ and $L/D = 3.00$, but a two-wake mode when $G/D = 2.00$ and $L/D = 2.75$ and 3.25. It can be inferred that the fluid flows for $2.75 \leq L/D \leq 3.25$ are located at the transition between the one-wake mode and the two-wake mode. In the case of $G/D = 2.00$ and $L/D = 2.75$ to 3.25, the gap between the plane wall and two cylinders is so large that the shear layer generated on the plane wall has limited influence on the near wakes of the two cylinders. However, the center-to-center distance between the two cylinders is not large enough, resulting in a high degree of flow interaction in the gap between the two cylinders. Such strong interaction makes the fluid structure more complex. Thus, a transition

<table>
<thead>
<tr>
<th>$G/D$</th>
<th>Vortex shedding mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L/D=1.00$</td>
<td>$L/D=1.25$</td>
</tr>
<tr>
<td>2.00</td>
<td>S</td>
</tr>
<tr>
<td>1.50</td>
<td>S</td>
</tr>
<tr>
<td>1.00</td>
<td>S</td>
</tr>
<tr>
<td>0.75</td>
<td>S</td>
</tr>
<tr>
<td>0.50</td>
<td>N</td>
</tr>
<tr>
<td>0.25</td>
<td>N</td>
</tr>
</tbody>
</table>

Note: N denotes the no-shedding mode, S denotes the one-wake mode, and P denotes the two-wake mode.
between the one-wake mode and the two-wake mode may occur.

4.2. Hydrodynamic force coefficients of cylinders

The numerical simulations in this study show a close correlation between the hydrodynamic force coefficients and vortex shedding modes. This will be demonstrated by the time history of hydrodynamic force coefficients and the statistics of hydrodynamic force coefficients.

4.2.1. Time series of drag and lift coefficients

Fig. 5 shows the time history of drag and lift coefficients of the upstream and downstream cylinders for the three cases of $G/D = 0.25$ and $L/D = 2.75$, $G/D = 0.75$ and $L/D = 2.75$, and $G/D = 2.00$ and $L/D = 2.75$, which correspond to the previously mentioned no-shedding mode, one-wake mode, and two-wake mode, respectively. In Fig. 5, $C_{Du}$ and $C_{Lu}$ represent the drag and lift coefficients of the upstream cylinder, respectively, and $C_{Dd}$ and $C_{Ld}$ are the drag and lift coefficients of the downstream cylinder, respectively.

It can be seen from Figs. 5(a) and (b) that the drag and lift coefficients of each cylinder attain a constant value over time. The initial oscillations of hydrodynamic fluid coefficients, resulting from the disturbance at the beginning of numerical computations, are quickly damped out. This means that the no-shedding mode is dominated by the absolute flow stability, mainly attributed to the small gap between the cylinders and the plane wall.

Oscillations of drag and lift coefficients of the downstream cylinder can be observed when the one-wake mode appears, as shown in Fig. 5(d), whereas Fig. 5(c) demonstrates that the oscillations of the drag and lift coefficients of the upstream cylinder are rather limited. It should be noted that the drag coefficient of the upstream cylinder is much larger than that of the downstream cylinder. It can also be found that the frequency of the drag coefficient of the downstream cylinder is not twice the frequency of lift coefficient but shares the same value, in contrast to the usual observation for an isolated cylinder in an infinite flow domain. An anti-phase variation with time is identified between the lift and drag coefficients for the downstream cylinder.

For the two-wake mode, as shown in Figs. 5(e) and (f), the amplitudes of both the fluctuating drag and lift coefficients increase drastically, compared with their counterparts in Figs. 5(a) to 5(d). This is mainly due to the appearance of vortex shedding from both cylinders and the interference between them. It is confirmed that $C_{Lu}$ and $C_{Ld}$ have the same oscillating frequency but opposite phases, which is consistent with the observations shown in Fig. 4. However, this is not valid for the drag coefficients of the two cylinders. It seems that the drag coefficient of the upstream cylinder involves multiple frequency components. Generally speaking, the

![Fig. 5. Time history of drag and lift coefficients of upstream and downstream cylinders for different vortex shedding modes at Re = 200.](image-url)
fluctuations of drag and lift coefficients of each cylinder and the relationships between them are much more complex than those of an isolated cylinder in free stream flows.

4.2.2. Mean hydrodynamic force coefficients of cylinders

The mean drag and lift coefficients of two cylinders are presented in Fig. 6. It can be seen that positive mean drag coefficients of the upstream cylinder $C_{Du}$ are observed for all combinations of $G/D$ and $L/D$ considered in this study. At the smallest value of $G/D = 0.25$, $C_{Du}$ increases with the increase of $L/D$, and its growth rate in the range of $1.00 \leq L/D \leq 1.50$ is larger than that in the range of $L/D \geq 1.75$. For $G/D = 0.50$, a gradually growing tendency can be observed for $C_{Du}$ when $L/D \leq 1.50$, and then a slightly decreasing trend for $C_{Du}$ when $1.50 < L/D \leq 3.25$. When the vortex shedding mode changes from the no-shedding mode to the two-wake mode, a minor increase of $C_{Du}$ with $L/D$ can be observed. Then, $C_{Du}$ holds almost a constant when $L/D \geq 3.50$. When the one-wake mode occurs, as shown in Table 3, the values of $C_{Du}$ are almost constant with a constant $L/D$ value when $G/D \leq 1.00$, and $2.00$, and they decrease slightly with the increase of $L/D$. For the two-wake mode, $C_{Du}$ changes little with the increase of $L/D$ for a constant $G/D$ value, indicating that the $L/D$ value has a limited effect on the mean drag coefficient of the upstream cylinder in the two-wake mode.

As far as the mean lift coefficient of the upstream cylinder $C_{Lu}$ is concerned, Fig. 6(b) demonstrates the significant influence of the plane wall on $C_{Lu}$. As shown in Fig. 6(b), the values of $C_{Lu}$ when $0.50 \leq G/D \leq 1.00$ may be either positive or negative, while overall positive values of $C_{Lu}$ are observed when $G/D = 0.25, 1.50$, and $2.00$. The mean lift coefficient of the upstream cylinder $C_{Lu}$ is dependent on the flow speed beneath the upstream cylinder, which is not only directly related to the gap ratio $G/D$, but also partially related to the distance ratio $L/D$. When the gap ratio $G/D$ is small enough and the two cylinders approach one another, the strong blockage effect leads to a low flow speed beneath the upstream cylinder and high pressure along the lower side of the upstream cylinder, which accounts for positive mean lift forces of the upstream cylinder. Otherwise, as the gap ratio $G/D$ increases gradually, the flow speed increases in the gap between the upstream cylinder and plane wall, causing the pressure along the lower side of the upstream cylinder to be smaller than that along the upper side. Therefore, negative mean lift coefficients are observed. Moreover, it can be seen from Fig. 6(b) that the distance ratio has a rather limited effect on the mean lift coefficient when $L/D \geq 3.25$ for all values of $G/D$. It can also be seen in Fig. 6(b) that the mean lift coefficient of the upstream cylinder when $G/D = 0.25$ is much larger than those when $G/D > 0.25$. The reason for this is that the very small gap between the upstream cylinder and the plane wall causes the pressure below the cylinder to be larger than that above it, and the large pressure difference further induces a large positive value of the mean lift coefficient of the upstream cylinder.

Fig. 6(c) displays the mean drag coefficients of the downstream cylinder with respect to $G/D$ and $L/D$. It can be seen from Fig. 6(c) that $C_{Dd}$ is positive for all values of $L/D$ when
$G/D \leq 0.50$. $C_{Dd}$ decreases slowly with the increase of $L/D$ when $G/D = 0.25$, whereas it increases slightly with $L/D$ when $G/D = 0.50$. For the one-wake mode, $C_{Dd}$ increases generally with $L/D$, negative mean drag coefficients can be found, and the bandwidth of $L/D$ for negative mean drag coefficients increases with $G/D$. The appearance of negative mean drag coefficients of the downstream cylinder is mainly induced by the shielding effect from the upstream cylinder. This is confirmed by numerical results showing that an extremely low pressure distribution, together with a well-developed stable vortex structure, is observed between the two cylinders. Comparison of Figs. 6(a) and (c) shows that the switch in the vortex shedding mode, as shown in Table 3, is associated with the jumps of mean drag coefficients of both cylinders. In general, $C_{Dd}$ is smaller than $C_{Du}$ for the same combination of $G/D$ and $L/D$. The variation of $C_{Dd}$ with the gap ratio $G/D$ is not same as that of $C_{Du}$. This is because the hydrodynamic force coefficients of the downstream cylinder are mainly affected by the vortices behind the upstream cylinder.

Fig. 6(d) presents the mean lift coefficient for the downstream cylinder. It can be seen from Fig. 6(d) that, for the no-shedding mode, $C_{Ld}$ decreases with the increase of $L/D$ when $G/D = 0.25$, while it varies slightly with $L/D$ when $G/D = 0.50$. For the one-wake mode, it is observed that $C_{Ld}$ increases with the increase of $L/D$. However, $C_{Ld}$ decreases with the increase of $L/D$ for the two-wake mode. It can also be found that the maximal mean lift coefficients of the downstream cylinder for various $G/D$ values occur when the $L/D$ values are very close to the critical distance ratios $L_{cr}/D$. The positive mean lift coefficient of the downstream cylinder is mainly attributed to the boundary layer developing along the plane wall, which decreases the flow flux in the gap between the cylinder and plane wall and increases the pressure at the lower side of the downstream cylinder. This is evident in the no-shedding mode. In addition, the vortex shedding from the downstream cylinder in the one-wake mode becomes asymmetrical due to the existence of the plane wall boundary layer, which accounts for the positive mean lift coefficient. The vortex shedding from the upstream cylinder in the two-wake mode can further strengthen the asymmetrical vortex shedding, giving rise to the maximal lift coefficient in Fig. 6(d).

4.2.3. Root mean square values of hydrodynamic force coefficients

The RMS values of drag and lift coefficients of both cylinders are shown in Fig. 7. Figs. 7(a) and (b) show that both $C_{Du}$ and $C_{Lu}$ increase abruptly from very small values as the vortex shedding mode changes. For the two-wake mode, it is observed that $C_{Du}$ and $C_{Lu}$ decrease with the increase of the $L/D$ value for a constant $G/D$ value. However, very small values of $C_{Du}$ and $C_{Lu}$ can be seen when the one-wake mode occurs. When $G/D = 0.25$, the values of both $C_{Du}$ and $C_{Lu}$ are zero for all the $L/D$ values. This is attributed to the lack of vortex shedding behind the two cylinders.
Figs. 7(c) and (d) present the RMS values of drag and lift coefficients for the downstream cylinder. The change in the vortex shedding mode again induces dramatic increases of hydrodynamic force coefficient fluctuations. We suggest that the vortex shedding from the upstream cylinder has a more significant influence on the RMS values of hydrodynamic force coefficients of the downstream cylinder than on those of the upstream cylinder. This is further confirmed in Fig. 7(d), in which larger increments of RMS of the lift coefficient from the one-wake mode to the two-wake mode are shown with respect to Fig. 7(b). Comparison between Fig. 7(b) and Fig. 7(d) indicates that the increase of $G/D$ in the two-wake mode has a greater influence on $C_{ld}^r$ than $C_{ld}^r$.

It is also seen in Fig. 7 that the RMS values of the drag and lift coefficients are both very small for the one-wake mode. Though the mean lift coefficients of both cylinders are smaller than the mean drag coefficients for the same combination of $L/D$ and $G/D$ (Fig. 6) in the two-wake mode, the RMS values of the drag and lift coefficients of the downstream cylinder are much greater than the counterparts of the upstream cylinder, respectively, when $L/D$ is larger than $L_{cr}/D$. The main reason for this phenomenon is that the magnitude of the shedding vortices behind the downstream cylinder is larger than that behind the upstream cylinder in the two-wake mode. When the two-wake mode occurs, the RMS values of the lift coefficients of upstream and downstream cylinders increase with $G/D$ when $G/D \leq 1.50$ for a constant value of $L/D$, and the RMS values of the lift coefficient of two cylinders are similar for the same combination of $G/D$ and $L/D$ when $G/D = 1.50$ and 2.00. However, the RMS values of the drag coefficients of both cylinders do not show such tendencies.

Fig. 8 shows the Strouhal number for the downstream cylinder ($S_{ld}$) with different values of $G/D$ and $L/D$. It can be seen from Fig. 8 that the value of $S_{ld}$ is zero in the no-shedding mode. When $G/D = 0.50$, $S_{ld}$ increases with the increase of the $L/D$ value as the two-wake mode occurs. When $G/D = 0.75$, $S_{ld}$ decreases with the increase of $L/D$ for the one-wake mode, and increases with $L/D$ for the two-wake mode. When $G/D > 1.00$, the values of $S_{ld}$ are close to one another for the same value of $L/D$ and different values of $G/D$, implying that the $G/D$ value has a limited effect on $S_{ld}$. However, this is not true for the cases of $G/D = 2.00$ and $L/D = 2.50$, and $G/D = 2.00$ and $L/D = 3.00$, which correspond to the boundaries of different vortex shedding modes, as shown in Table 3.

5. Conclusions

Viscous flow past twin near-wall circular cylinders in a tandem arrangement was investigated numerically at a low Reynolds number. Calculations were carried out for $G/D = 0.25, 0.50, 0.75, 1.00, 1.50$, and 2.00, and $L/D$ ranging from 1.0 to 4.0 with an interval of 0.25. The Reynolds number was kept constant and equal to 200 for all of the computations. The influences of $G/D$ and $L/D$ values on hydrodynamic force coefficients, vortex shedding modes, and Strouhal numbers were examined. The major results can be summarized as follows:

1) There is no vortex shedding from the cylinders for very small values of $G/D$. In this study, vortex shedding was not observed in the cases of $G/D = 0.25$ for all values of $L/D$, and the cases of $G/D = 0.50$ and $L/D \leq 3.25$.

2) When $G/D = 0.25$, vortex shedding can be completely suppressed for all values of $L/D$. When $G/D \geq 0.50$, there is a critical distance ratio $L_{cr}/D$. If $L/D > L_{cr}/D$, it can be found that the vortices are shed from both the upstream and downstream cylinders, a situation defined as the two-wake mode in this study. However, when $L/D \leq L_{cr}/D$, no shedding mode is observed when $G/D = 0.50$, and the vortex is shed only from the downstream cylinder when $G/D \geq 0.75$.

3) The lift coefficients of the downstream and upstream cylinders oscillate at the same frequency in the two-wake mode, implying that the Strouhal number is the same for both cylinders. The Strouhal number for the downstream cylinder is zero in the no-shedding mode, since vortex shedding is totally suppressed. It is also observed that the values of $S_{ld}$ are close to one another for the same value of $L/D$ and different values of $G/D$, implying that the $G/D$ value has a limited effect on $S_{ld}$. However, this is not true for the cases of $G/D = 2.00$ and $L/D = 2.50$, and $G/D = 2.00$ and $L/D = 3.00$.

4) At a very small value of $G/D$, such as $G/D = 0.25$, the vortex shedding is completely suppressed, and the mean drag coefficients for both cylinders change smoothly with the increase of the $L/D$ value. When $G/D \geq 0.50$, the root mean square values of drag and lift coefficients of both cylinders increase abruptly when the vortex shedding mode is changed.

5) When the two-wake mode occurs, the mean drag coefficient of the upstream cylinder increases with $G/D$ when $G/D \leq 1.50$, while the mean drag coefficients are similar to one another when $G/D = 1.50$ and 2.00 for the same values of $L/D$. The mean drag coefficient of the downstream cylinder is negative for small values of $L/D$ and positive for large values of $L/D$ when $G/D > 0.50$, and the bandwidth of $L/D$ for the negative mean drag coefficient of the downstream cylinder increases with $G/D$.

6) The mean drag coefficient of the upstream cylinder is larger than that of the downstream cylinder for the same combination of $G/D$ and $L/D$. However, the RMS values of drag and lift coefficients of the downstream cylinder are much larger than the counterparts of the upstream cylinder in the two-wake mode.
References


