Dynamic Shakedown Analysis of Flexible Pavement under Traffic Moving Loading

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Abstract
Shakedown theory is often used to analyze the plastic behaviors of structures subjected to variable complex loads. It is an effective way to predict the maximum load under which the failure due to plastic collapse or excessive permanent deformation of pavement will not occur. Based on Melan’s lower-bound shakedown theory, this work has proposed a numerical method for estimating the shakedown limit involving the effect of traffic moving speed. The dynamic response of elastic stress to traffic moving speed is computed using combined Finite Element-Infinite Element (FE-IE) method. The shakedown limits for a two-layered pavement system have been investigated at various traffic moving speeds. It shows that the shakedown limit early reduces and subsequently turns to grow as the moving speed increases. The shakedown limit decreases to the minimum when the traffic speed reaches the Rayleigh wave speed of subsoil. In order to provide an optimized design of the pavement system, the dependence of shakedown limits on the material properties of pavement system has also been estimated. Eventually, the characteristic distribution of critical residual stress is discussed.

Keywords: Shakedown, traffic moving load, dynamic response, Finite Element/Infinite Element

1 Introduction
Pavement is subjected to moving traffic load at different stress levels and speeds. The number of repeated loads is extremely large in the designed life period. The pavement may fail due to the excessive permanent deformation of pavement and subsoil. When an elastic–plastic structure is subjected to a cyclic load, the level of load is higher than its static yield limit but lower than its shakedown limit, the increase of the permanent deformation could become so small that a ‘shakedown’ status can be reached. Under shakedown limit, the structure system will adapt itself to the cyclic loads and respond purely elastically to the following load cycles, leading to no further exhibition of plastic strain (Sharp&Booker, 1984).

As early as Sharp&Booker (1984), the lower-Bound shakedown theorem (Melan, 1938) has been widely applied to estimate the maximum traffic load (i.e., shakedown limit) for the pavement system.

Although Melan's lower-bound theorem was widely applied to dynamic problems in the literature, dynamic stresses were always determined using static FEM or static analytical method. However, as traffic load belongs to one type of moving loads, traffic-induced dynamic responses to moving speed are in general very significant, but fail to be considered in previous static shakedown analyses. Hence, the static shakedown analysis method is in essence not suitable for the evaluation of pavement system, particularly under high-speed traffic loading. In order to reasonably determine the dynamic stresses due to moving load, a dynamic finite element analysis is performed, where an artificial boundary is required to efficiently treat the original unbound domain. In addition, an optimization technique for the solution to critical residual stresses is required as well.

The paper is followed by three main parts. In Section 2, the dynamic lower-bound shakedown theorem is briefly introduced and a method is proposed to identify the shakedown limit of cohesive-frictional material. In Section 3, a dynamic FE-IE numerical model is built to compute the dynamic stresses. In Section 4, the shakedown limits of a two-layered system based on Melan’s theory considering speed are calculated, and comparisons between the results in this paper and previous studies based on static approach are made.

2 Shakedown Theorem and Optimization Solution

Melan’s lower-bound shakedown theorem states that a sufficient condition for shakedown to occur under repeated or cyclic loads is that a time-independent, self-equilibrated, residual stress field can be found that, when added to the elastic stress field, produces a combined stress field that nowhere and at no-time violates the yield condition. The Lower-bound shakedown theorem hence can be described as:

\[
\max_\lambda \left\{ f(\lambda \sigma^e_0 + \rho_\lambda) \leq 0, \lambda \geq 0 \right\}
\]

where \( \sigma^e_0 \) is the elastic stress induced by an applied load \( p \), \( \lambda \) is a multiplier and assumed that the elastic stress induced by \( \lambda p \) is \( \lambda \sigma^e_0 \), \( \rho_\lambda \) is the time-independent, self-equilibrated residual stress field, and \( f \) is the yield criterion for the material and here Mohr-Coulomb yield criteria is adopted.

If Mohr-Coulomb criteria is used to define the yield strength of pavement materials, Ineq.(1) can be defined as follows (Yu&Wang, 2012):

\[
f = (\rho_{ss} + M)^2 + N \leq 0
\]

where \( f<0 \) means the material do not violate the yield condition. M and N are defined as below:

\[
M = \lambda \sigma^c_{xx} - \lambda \sigma^c_{zz} + 2 \tan \phi (c - \lambda \sigma^c_{zz} \tan \phi)
\]

\[
N = 4(1 + \tan^2 \phi) \left[ (\lambda \sigma^c_{xx})^2 - (c - \lambda \sigma^c_{zz} \tan \phi)^2 \right]
\]

where \( c \) is the cohesion and \( \phi \) is the internal friction angle.

In order to satisfy Ineq.(2), one condition must be met as follows:

\[
N \leq 0 \Rightarrow \lambda \leq \lambda_{ad} = \frac{c}{\left| \sigma^c_{xx} \right| - \left| \sigma^c_{zz} \right| \tan \phi}
\]

It should be noted that \( M \) and \( N \) differ from point to point. The real solution is believed to be the minimum one among all possible solutions. In order to search for possible real solutions, an efficient optimization technique is proposed. Considering the real critical load multiplier \( \lambda' \) that is less than \( \lambda_{ad} \), in order to obtain \( \lambda' \) that meets Ineq.(2), the solution to determine the real residual stresses is required.
As well known, the residual stresses are constant along the moving direction at the same depth. For a better illustration, the determination of the real residual stress for two representative points (point i and point j) at the same depth has been demonstrated in Fig.1 according to Yu&Wang (2012), where tension is treated as positive. Fig.1 are the solutions of Ineq.(2) to solve $\rho_{xx}$, where points i, j represent the points corresponding to the minimum larger root and maximum smaller root, respectively. According to Ineq.(2), the real residual stresses at point i and point j must meet the following conditions, respectively.

$$-M_i - \sqrt{-N_i} \leq \rho_{xx} \leq -M_i + \sqrt{-N_i} \quad (5)$$
$$-M_j - \sqrt{-N_j} \leq \rho_{xx} \leq -M_j + \sqrt{-N_j} \quad (6)$$

Three distinct cases may take place. (a) the two points share common residual stresses as shown in Fig.1(a), thus the prescribed load multiplier $\lambda'$ is larger than the real solution $\lambda'$, i.e., $\lambda' < \lambda'$. (b) If one unique common solution satisfies Ineqs.(5) and (6), $\lambda'$ can be exactly found, i.e., $\lambda' = \lambda$, as shown in Fig.1(b). (c) No common solution to Ineqs.(5) and (6), or say $\lambda > \lambda'$, which implies the pavement does not shakedown any longer, as shown in Fig.1(c).

![Possible residual stress range](image)

**Fig.1** Shakedown limit analysis based on solutions to critical residual stresses

The above method can be applied to either homogeneous or layered systems. If a layered pavement is considered, the shakedown limit of each layer can be calculated and the minimum one among them is the shakedown limit for the pavement (Wang&Yu, 2013):

$$\lambda' = \min(\lambda_1', \lambda_2', \ldots, \lambda_n') \quad (7)$$

### 3 FE-IE Numerical Model

In this paper, a three-dimensional coupled Infinite Element- Finite Element analysis using ABAQUS is performed to provide realistic estimation of traffic-induced dynamic stresses in the
pavement and subsoil, as shown in Fig.2. The size of the simulated region is 30m×5.5m ×8m in the length (moving direction), width and depth, respectively.

In the numerical model, the simulated wheel load moves from the -10m to 10m in the moving direction on the ground. FEM model has been built using the ABAQUS subroutine VDLOAD, which is capable to simulate the moving wheel load. The simulated region has 967218 elements (C3D8R). Specifically, Infinite Element (CIN3D8) artificial boundaries have been employed to deal with unbounded domains, similar to Connolly et al (2013). The proposed IE-FE coupled method has been successfully applied to many dynamic problems (Khalili et al, 1999).

![Fig.2 Finite element model with infinite element boundaries](image)

### 4 Shakedown Limit Analysis

#### 4.1 Uniform Pavement System

In the three-dimensional IE-FE model (Fig. 2), the wheel contact area is modeled by Hertz distribution load as defined below:

\[
p = \frac{3p}{2\pi a} (a^2 - x^2 - y^2)^{1/2}, \quad (x+vt)^2 + y^2 < a^2
\]

\[
p_0 = \frac{3p}{2\pi a^2}
\]

where \(a\) is the radius of the contact area and \(a=25\) cm is chosen. \(x, y, z\) represent Cartesian coordinates and the \(x\)-axis is the travel direction. \(p\) is the resultant of 3D Hertz distribution load, \(p\) with the peak pressure, \(p_0\).

As shown in Fig.3 (a), comparisons of shakedown limit for the present dynamic method at an extreme speed (\(v=1\)m/s)) and previous static solution (Yu&Wang, 2012, Wang&Yu, 2013) have been presented. Their good consistency has verified the present dynamic shakedown analysis. The shakedown limit is usually represented by a dimensionless factor \(\lambda' p_0/c\).

In order to show the dynamic effect of moving speed on the shakedown limits, the same material parameters given by Yu&Wang (2012) are used to perform a dynamic FE-IE analysis. The predictions have been plotted in Fig.3 (b). The pavement is made of uniform materials with Young’s Modulus \(E=20\)MPa, Poisson’s ratio=0.2 and mass density=1800 kg/m³. As a result, the Rayleigh wave speed of materials is determined to be about 60m/s. It is clear that the shakedown limit decreases gradually to the minimum as the moving speed increases to the Rayleigh wave speed. Subsequently, the shakedown limit tends to increase with increasing moving speed. Specifically, a convenient shakedown limit is found at \(v=1\)m/s, consistent with static solution by Yu&Wang (2012). It is concluded that the strongest dynamic stress response is activated when the moving speed reaches the Rayleigh wave speed, as earlier discussed by Qian et al (2014).
4.2 Two-layered Pavement System

In this section, the shakedown limit analysis for a two-layered pavement system will be discussed. For comparing the results of Wang & Yu (2013), the physical properties of subsoil under pavement are assumed to be constant, but the properties for the top layer are considered to be changeable for an optimal design of the pavement system. With these considerations, the parameters of the two layers in the numerical analysis are listed in Table 1. The thickness of top layer is assumed to be 2a, where a represents the distribution radius of wheel loading as shown in Fig. 4. The same model in Fig. 2 is adopted.

![Image of schematic diagram](image)

Fig. 4 Schematic of the numerical model

<table>
<thead>
<tr>
<th>Layer</th>
<th>Young's modulus (MPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m³)</th>
<th>Cohesion (kPa)</th>
<th>Internal friction angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavement</td>
<td>20–200000</td>
<td>0.2</td>
<td>2100</td>
<td>30–30000</td>
<td>30</td>
</tr>
<tr>
<td>Subsoil</td>
<td>20</td>
<td>0.49</td>
<td>1800</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1 Material parameters used in the model

Fig. 5 shows the influence of modulus ratio ($E_1/E_2$) on shakedown limits for various strength ratios ($c_1/c_2$). Fig. 5 (a) and (b) presents the static results from Wang & Yu (2013) and the present predictions at $v=10$m/s. It shows slight difference between the two predictions. As seen in Fig. 5 (b), for the case that the strength ratio is extremely high, such as $c_1/c_2=1000$, the shakedown limit of the pavement essentially depends on the second layer as presented by the dash line. On the other hand, for an extremely low strength ratio, e.g., $c_1/c_2=1$, the shakedown limit is reached in the first layer. For a medium strength ratio, the shakedown limit may take place either in the first layer or second layer, essentially depending on the modulus ratio $E_1/E_2$. For example, when $c_1/c_2=5$, the shakedown limit takes place in the first layer at a low modulus ratio $E_1/E_2$ and turns to be in the second layer as the modulus ratio $E_1/E_2$ increases. The phenomena agree well with the static prediction by Wang & Yu (2013), particularly at a low speed. However, some findings are quite different from the static results.
as the moving speed increases. First, as the speed increases, all shakedown limits substantially decrease. Second, for both top layer and bottom layer, their growing rates of shakedown limit tend to drop significantly. In addition, the shakedown limits of the bottom layer keep almost constant, differing a lot from static or low-speed cases.

![Image of graphs showing influence of speed on shakedown limits]

**4.3 Influence of Speed to Shakedown Limit**

Fig. 6 shows the relationship between the shakedown limits and the strength ratios at various speed. The modulus ratio \( E_1/E_2 \) equals 10 and the Rayleigh wave speeds of the top and the bottom are about 180 m/s and 60 m/s, respectively. The dash line means that the shakedown failure taking place in the bottom layer, others in the top layer. It can be seen that for some medium values of \( c_1/c_2 \), the failure may take place in the first layer at low speeds, but in the second layer at high speeds, which is in agreement with above analyses. Fig. 6 also indicates that the shakedown limit of second layer drops quickly when the load speed approaches the Rayleigh wave speed, but the shakedown limit of the top layer reduces slightly.

![Image of graph showing influence of speed on shakedown limits]

**Fig. 5 Influence of the modulus ratio to shakedown limits at various strength ratios**

**Fig. 6 Influence of speed on the shakedown limits**
4.4 Critical Residual Stress

Fig. 7 shows the critical residual stress field in a two-layered pavement, in which the level of residual stress is normalized by $c_2$. Note that discontinuous residual stresses are always observed at the interface between two layers. For a low-speed case, two critical residual stress solutions tend to converge at some depth in the top layer, where the shakedown limit is exactly reached. For a high-speed case, two critical residual stress solutions exactly converge at the top of the second layer. The possible explanation is that the dynamic stress caused by the moving speed is stronger at the deep location than that at the shallow location (Qian et al., 2014). As a result, non-shakedown (dynamic failure) will appear in first layer at low speed, but in the second layer at high speed. The other reason may be because the Rayleigh wave speed of first layer is 181.16 m/s and the Rayleigh wave speed of second layer is 58.14 m/s, so the shakedown limits at 10 m/s change slightly in both two layers. Nevertheless, the shakedown limit at 60 m/s drops significantly because the moving speed has approached its Rayleigh wave speed.

Fig. 7 Critical residual stresses in a two-layered pavement ($E_1/E_2=10$, $c_1/c_2=5$)

5 Conclusions

This paper has numerically explored the shakedown behaviors of pavement system subjected to repeated traffic load based on Melan's lower-bound theory. An optimization procedure is also proposed to determine the shakedown limit of pavement. The response of dynamic stress to traffic load.
speed has been computed by Finite Element-Infinite Element method. The dynamic shakedown behaviors are investigated in detail. The main findings can be summarized as follows:

(1) The present FE-IE method is validated by comparison with traditional static method. The proposed optimization technique has provided an effective way to find the exact solution to critical residual stresses in the numerical shakedown analysis.

(2) The shakedown limits strongly depend on the moving speed of the traffic load. It has been found that for most cases, the shakedown limit reduces as the load moving speed when the moving speed is lower than the Rayleigh wave speed of subsoil. Otherwise, the shakedown limit will increases as the moving speed increases if the speed exceeds the Rayleigh wave speed.

(3) For the two-layered pavement, the shakedown limit is quite different from a uniform layer. The shakedown limit varies with the modulus ratio $E_1/E_2$ as well as the strength ratio $c_1/c_2$. The effect of $E_1/E_2$ on the shakedown limit becomes more different from static situation as the moving speed increases. The critical shakedown point at lower speeds tends to initiate in the first layer of pavement, whilst it tends to occur in the second layer at higher speeds. Design of pavements against excessive rutting using the shakedown theory should be carried out by choosing suitable base (or sub-base) materials and layer thickness, while the dynamic effects of moving load speed should be considered.

Acknowledgments

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