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Practical Implementation for the interval type-2 fuzzy PID controller using a low cost microcontroller

Ahmad M. El-Nagar *, Mohammad El-Bardini

Department of Industrial Electronics and Control Engineering, Faculty of Electronic Engineering, Menofia University, Menof 32852, Egypt

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Abstract In this study, we propose an embedded real-time interval type-2 fuzzy proportional – integral – derivative (IT2F-PID) controller which is a parallel combination of the interval type-2 fuzzy proportional – integral (IT2F-PI) controller and the interval type-2 fuzzy proportional – derivative (IT2F-PD) controller. The proposed IT2F-PID controller is able to handle the effect of the system uncertainties due to the structure of the interval type-2 fuzzy logic controller. The proposed IT2F-PID controller is implemented practically using a low cost PIC microcontroller for controlling the uncertain nonlinear inverted pendulum to minimize the effect of the system uncertainties due to the uncertainty in the mass of the pendulum, the measurement error in the rotation angle of the pendulum and the structural uncertainty. The test is carried out using the hardware-in-the-loop (HIL) simulation. The experimental results show that the performance of the IT2F-PID controller improves significantly the performance over a wide range of system uncertainties.

1. Introduction

Most of the industrial processes are still the conventional PID controllers due to their simple control structures, affordable price, and effectiveness for linear systems [1]. However, when the process to be controlled has a high level of complexity, such as, time delay, high order, modeling nonlinearities, vague systems without precise mathematical models, and structural uncertainties, the performance of a PID control system becomes unsatisfactory and fails to guarantee the requirements in most of the practical situations [2]. For these reasons, many researchers have attempted to combine a conventional PID controller with a fuzzy logic controller (FLC) in order to achieve better system performance over a conventional PID controller. The fuzzy PI controller [3] and the fuzzy PD controller [4] are developed to improve the system performance rather than the conventional PID controllers. The fuzzy PI and the fuzzy PD controllers are combined to get a fuzzy PID controller in order to improve the system performance

* Corresponding author. Tel.: +20 1006469369.
E-mail addresses: Ahmed_elnagar@menofia.edu.eg (A.M. El-Nagar), dralbardini@ieee.org (M. El-Bardini).
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[5]. It's knowledge base consists of two dimensional rule base for the PI and PD controllers to obtain overall improved performance. There are different control structures for the fuzzy PID controller mentioned in [6,7], in order to improve the closed loop systems. Despite the significant improvement of system performance with fuzzy PID controllers over their conventional counterparts, it should be noted that they are usually not effective if cases where the system to be controlled has system uncertainties as ordinary fuzzy controllers (type-1 fuzzy logic controllers; T1-FLCs) have limited capabilities to directly handle data uncertainties [8].

The concept of type-2 fuzzy set (T2-FS) is an extension and generalization of the type-1 fuzzy sets (T1-FSs) and it was first introduced by Zadeh [9]. FLCs that are described with at least one T2-FS are called type-2 fuzzy logic controllers (T2-FLCs) [10]. It has also been shown that T2-FSs are much more powerful to handle uncertainties and nonlinearities directly [11]. The major problem with T2-FLC is that the computations are much more complicated compared to T1-FLCs. Therefore, Mendel [10] has been proposed a special type of T2-FS called an interval type-2 fuzzy set (IT2-FS) in which the output of interval type-2 fuzzy is uncertain with an interval. The IT2-FLCs have been successfully implemented in controller design [12–25]. The structure of the IT2-FLC has four components, which are a fuzzifier, a rule base, a fuzzy inference engine, and an output processor. In the IT2-FLC, the output sets are interval type-2, so we have to use an extended version of type-1 defuzzification methods. The extended defuzzification operation with the type-2 case gives a T1-FS at the output. Since this operation takes us from the T2-FSs of the IT2-FLC to a T1-FS, this operation is called type-reduction and calls the type-1 set so obtained a type-reduced set [26]. The type-reduced set is a collection of the outputs of all of the embedded T1-FLCs [27]. The type-reduction is usually performed by iterative Karnik–Mendel (KM) algorithms [28], which are computationally intensive. However, the IT2-FLCs have a computational overhead associated with the computation of the type-reduced fuzzy sets using the KM algorithm [10]. This computational overhead reduces the real-time performance of the IT2-FLC, especially when operating on industrial embedded controllers which have limited computational and memory capabilities. So, the type-2 computational overhead can limit the application of the IT2-FLCs in industrial embedded controllers. Wu and Mendel [29] proposed a method to approximate the type-reduced set, thus avoiding the use of the iterative KM algorithm. This method calculates the inner- and outer-bound sets for the type-reduced set. These two sets can not only be used to estimate the uncertainty contained in the output of the IT2-FLC, but can also be used to directly derive the defuzzified output under certain conditions. Thus, the inner- and outer-bound sets have the potential to eliminate the computational bottleneck of an IT2-FLC.

As reported in [5,30], it is convenient to combine the fuzzy PI and the fuzzy PD controllers in a parallel structure to form a T1F-PID controller. Such a fuzzy PID structure is simple and can be theoretically analyzed. However, the main drawback of the T1F-PID controller is its limited capability to directly handle data uncertainties [8]. Thus, the main objective of this paper is developing and implementing an IT2F-PID controller taking into its consideration the advantages of using a parallel structure consisting of fuzzy PI and fuzzy PD controllers and the advantages of the type-2 fuzzy controllers to overcome the uncertainty problems. The proposed IT2F-PID controller using the Wu-Mendel type-reduction method and the Mamdani interval type-2 fuzzy rule base is implemented practically using a low cost PIC microcontroller (P18F4685) and its performance is tested using HIL simulation for controlling an uncertain nonlinear inverted pendulum. The main problem in an inverted pendulum system is the uncertainty which is divided into three groups; the uncertainty in the mass of the pendulum, the structural uncertainty which caused by parameters are difficult or impossible to get a precise measure or that the parameters tend to vary as a function of time, temperature, etc. and the uncertainty due to the measurement error in the rotation angle of the pendulum. The proposed IT2F-PID controller is designed and implemented to overcome the uncertainties in the inverted pendulum system. The rest of the paper is organized as follows. In Section 2, the IT2F-PID controller is included. The description of the mathematical model of the nonlinear inverted pendulum is presented in Section 3. Section 4, presents the hardware implementation of the IT2F-PID controller and practical results followed by the conclusions and the relevant references.

2. Interval type-2 fuzzy PID controller

Generally, an IT2-FLC is either a PD or a PI depending on the output of fuzzy control rules; if the output is the control signal it is said to be an IT2F-PD controller and if the output is the change of control signal it is said to be an IT2F-PI controller. An IT2F-PID controller may be constructed by introducing the third information besides error and derivative of the error, which is sum of error, with a 3-D rule base. Such an IT2F-PID controller with a 3-D rule base is difficult to construct because a 3-D rule base can be very complex when the number of quantizations of each rule dimension increases; in this case, the control rule number increases cubically with the number of quantizations. In this paper, we propose a parallel combination of an IT2F-PD controller and an IT2F-PI controller to achieve an IT2F-PID controller. The proposed IT2F-PID controller uses the Mamdani interval type-2 fuzzy rule and the Wu-Mendel type-reduction method for the IT2F-PI controller and the IT2F-PD controller. The proposed controller structure using the Wu-Mendel type-reduction method is suitable for real-time applications and implement in industrial embedded controllers, thus avoiding the use of the iterative KM algorithm which not suitable for real-time applications. The structure of the controller is first presented along with the interaction rules, followed by the description of the fuzzification, rule base and finally defuzzification involving type reduction.

2.1. The structure of the proposed controller

The overall structure of the proposed IT2F-PID controller is shown in Fig. 1. In Fig. 1, $e(k)$ and $\Delta e(k)$ represent the input variables and $u_{PID}(k)$ represents the output variable. For an IT2F-PI controller, two scaling factors G_{e1} and $G_{\Delta e1}$ are employed to scale $e(k)$ and $\Delta e(k)$, the error and the derivative error respectively, as follows:

$$E_{PI}(k) = G_{e1}e(k) = G_{e1}(y_r(k) - y(k))$$
$$\Delta E_{PI}(k) = G_{\Delta e1}\Delta e(k) = G_{\Delta e1}(e(k) - e(k-1)) \quad (1)$$

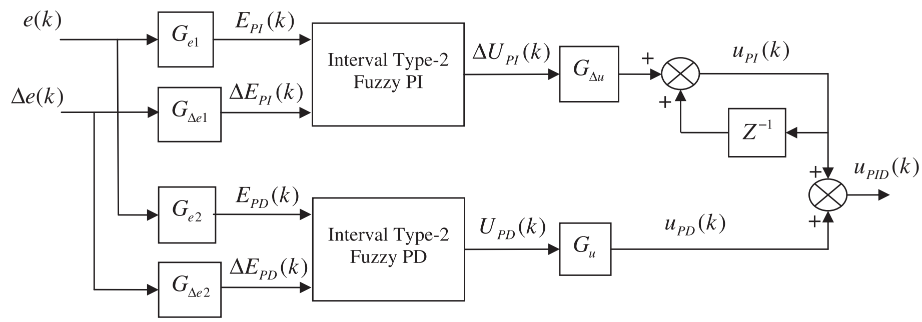

Figure 1 Overall structure of the interval type-2 fuzzy PID controller.

where $y_r(k)$ is the system output reference signal, $y(k)$ is the output of the system under control, and k is the sampling instance. The output variable of an IT2F-PI controller $u_{PI}(k)$ is obtained from the incremental output $\Delta u_{PI}(k)$ and the previous output $u_{PI}(k-1)$ as follows:

$$u_{PI}(k) = u_{PI}(k-1) + \Delta u_{PI}(k) \quad (2)$$

Now, internal variable $\Delta U_{PI}(k)$ and its scaling factor $G_{\Delta u}$ are introduced such that

$$\Delta u_{PI}(k) = G_{\Delta u} \Delta U_{PI}(k) \quad (3)$$

Also, there are two scaling factors G_{e2} and $G_{\Delta e2}$ are employed to scale $e(k)$ and $\Delta e(k)$, respectively, for an IT2F-PD controller as follows:

$$E_{PD}(k) = G_{e2} e(k) = G_{e2}(y_r(k) - y(k))$$
$$\Delta E_{PD}(k) = G_{\Delta e2} \Delta e(k) = G_{\Delta e2}(e(k) - e(k-1)) \quad (4)$$

The output variable $u_{PD}(k)$ of an IT2F-PD controller and its scaling factor G_u are given by the following equation:

$$u_{PD}(k) = G_u U_{PD}(k) \quad (5)$$

Therefore, the output of an IT2F-PID controller $u_{PID}(k)$ is obtained by the following equation:

$$u_{PID}(k) = u_{PI}(k) + u_{PD}(k) \quad (6)$$

The structure of the IT2F-PI controller and the IT2F-PD controller is elaborated in Fig. 2. These are works as follows [8]: The crisp input from the input variables is first fuzzified into input IT2-FSs. The input IT2-FSs then activate the inference engine and the rule base to produce output IT2-FSs. The IT2-FLC rules remain same as those for a T1-FLC, but the antecedents and/or the consequences are represented by IT2-FSs. The inference engine combines the fired rules and gives a mapping from input IT2-FSs to output IT2-FSs. The IT2 fuzzy outputs of the inference engine are then processed by the type reducer, which combines the output sets and performs a centroid calculation that leads to T1-FSs called the type-reduced sets. In this paper, the Wu-Mendel uncertainty bounds method [29] has been used to approximate the type-reduced set. After the type reduction process, the approximate type-reduced sets are defuzzified (by taking the average of the approximated type-reduced set) to obtain crisp outputs that are sent to the actuators.

2.2. Interval type-2 fuzzification

The fuzzification for an IT2F-PI controller is essentially same as that for an IT2F-PD controller. Fig. 3, shows an example for fuzzification of an IT2-FS. For each point of the universe of discourse, two membership levels are computed, one called *upper membership value* which is obtained from the *upper membership function*, and the other called *lower membership value* which is obtained from the *lower membership function*. For the IT2F-PI controller as shown in Fig. 3, when $E_{PI}(k) = x'_1$, the vertical line at x'_1 intersects footprint of uncertainty FOU($\widetilde{A}$) everywhere in the interval $[\underline{\mu}_{\widetilde{A}}(x'_1), \bar{\mu}_{\widetilde{A}}(x'_1)]$; and, when $\Delta E_{PI}(k) = x'_2$, the vertical line at x'_2 intersects FOU($\widetilde{B}$) everywhere in the interval $[\underline{\mu}_{\widetilde{B}}(x'_2), \bar{\mu}_{\widetilde{B}}(x'_2)]$.

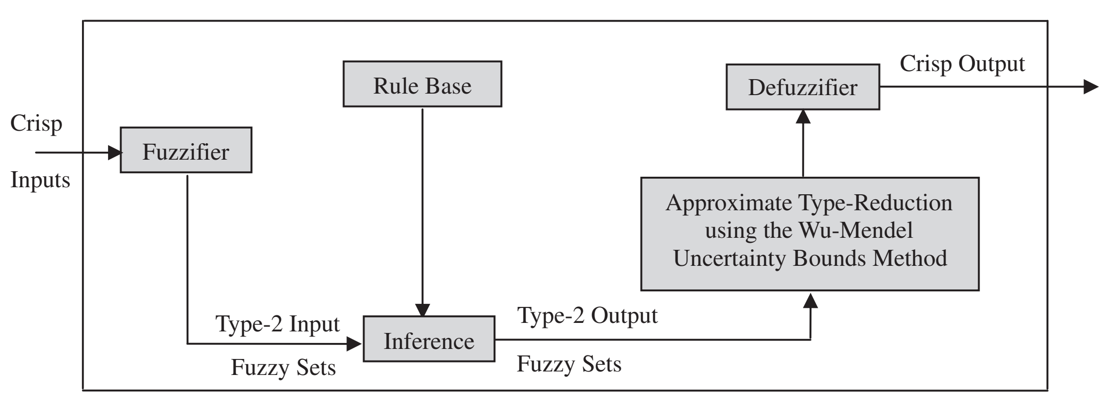

Figure 2 Structure of the interval type-2 fuzzy controller.

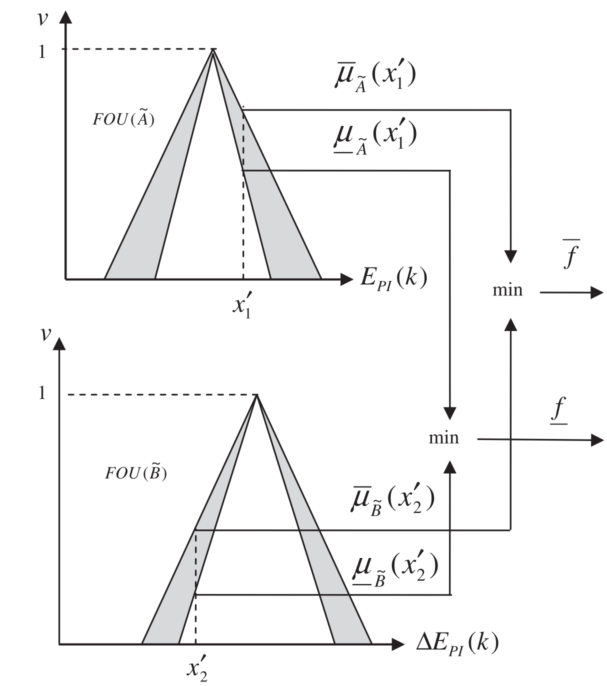

Figure 3 Fuzzification and inference of the IT2F-PI controller [8].

2.3. Rule base and inference engine

The structure of the Mamdani interval type-2 fuzzy rule for the IT2F-PI controller and the IT2F-PD controller is written as described in Eqs. (7) and (8), respectively.

$$\text{IF } E_{PI}(k) \text{ is } \tilde{A} \text{ AND } \Delta E_{PI}(k) \text{ is } \tilde{B} \text{ THEN } \Delta U_{PI}(k) \text{ is } \tilde{C} \quad (7)$$

$$\text{IF } E_{PD}(k) \text{ is } \tilde{A} \text{ AND } \Delta E_{PD}(k) \text{ is } \tilde{B} \text{ THEN } U_{PD}(k) \text{ is } \tilde{C} \quad (8)$$

where $\tilde{A}, \tilde{B}$, and $\tilde{C}$ are IT2-FSs. The inference engine combines the fired rules and gives a mapping from input IT2-FSs to output IT2-FSs. The inference process for a single rule is shown in Fig. 3. Two firing levels are then computed, a lower firing level, $\underline{f}$, and an upper firing level $\bar{f}$, where $\underline{f} = \min[\underline{\mu}_{\tilde{A}}(x_1'), \underline{\mu}_{\tilde{B}}(x_2')]$ and $\bar{f} = \min[\bar{\mu}_{\tilde{A}}(x_1'), \bar{\mu}_{\tilde{B}}(x_2')]$. The main observed comment from the result of input and antecedent operations is the firing interval F, where $F = [\underline{f}, \bar{f}]$. Note that other structures of fuzzy rules and the product t-norm also be used in Eqs. (7) and (8). However, this paper focuses on the Mamdani fuzzy rule and minimum t-norm.

2.4. Type-reduction and defuzzification

In this paper, the Wu-Mendel uncertainty bounds method is used to calculate the output of the IT2F-PI controller and the IT2F-PD controller. The steps of this method are calculated as follows [8]:

A. Computation of centroids of M consequent IT2-FSs:

U_l^i and U_r^i ($i = 1, \ldots, M$), the end points of the centroids of the M consequent IT2-FSs, are computed using KM algorithms [10,31]. These computations can be performed after the design of the IT2-FLC has been completed and the only have to be done once.

B. Computation of boundary T1-FLC centroids:

An IT2-FLS can be interpreted as a collection of embedded T1-FLCs. The following embedded T1-FLCs only use the LMFs (or UMFs) of the input and antecedent fuzzy sets, together with the left (or right) end point of the centroids of the consequents:

$$U_l^{(0)}(x) = \sum_{i=1}^{M} \underline{f}^i U_l^i \Big/ \sum_{i=1}^{M} \underline{f}^i \quad U_l^{(M)}(x) = \sum_{i=1}^{M} \bar{f}^i U_l^i \Big/ \sum_{i=1}^{M} \bar{f}^i$$
$$U_r^{(0)}(x) = \sum_{i=1}^{M} \bar{f}^i U_r^i \Big/ \sum_{i=1}^{M} \bar{f}^i \quad U_r^{(M)}(x) = \sum_{i=1}^{M} \underline{f}^i U_r^i \Big/ \sum_{i=1}^{M} \underline{f}^i \quad (9)$$

These equations refer to as boundary T1-FLCs for an IT2-FLC. These boundary T1-FLCs are very important in deriving the inner- and outer-bound sets of a type-reduced set.

C. Computation of uncertainty bounds:

The inner-bound set $[\overline{U}_l(x), \underline{U}_r(x)]$ and the outer-bound set $[\underline{U}_l(x), \overline{U}_r(x)]$ for the type-reduced set are calculated as follows:

$$\underline{U}_l(x) \leqslant U_l(x) \leqslant \overline{U}_l(x), \quad \underline{U}_r(x) \leqslant U_r(x) \leqslant \overline{U}_r(x) \quad (10)$$

$$\overline{U}_l(x) = \min\{U_l^{(0)}(x), U_l^M(x)\},$$
$$\underline{U}_r(x) = \max\{U_r^{(0)}(x), U_r^M(x)\} \quad (11)$$

$$\underline{U}_l(x) = \overline{U}_l(x) - \left[\frac{\sum_{i=1}^{M}(\bar{f}^i - \underline{f}^i)}{\sum_{i=1}^{M}\bar{f}^i \sum_{i=1}^{M}\underline{f}^i} \frac{\sum_{i=1}^{M}\underline{f}^i(U_l^i - U_l^1)\sum_{i=1}^{M}\bar{f}^i(U_l^M - U_l^i)}{\sum_{i=1}^{M}\underline{f}^i(U_l^i - U_l^1) + \sum_{i=1}^{M}\bar{f}^i(U_l^M - U_l^i)} \right] \quad (12)$$

$$\overline{U}_r(x) = \underline{U}_r(x) + \left[\frac{\sum_{i=1}^{M}(\bar{f}^i - \underline{f}^i)}{\sum_{i=1}^{M}\bar{f}^i \sum_{i=1}^{M}\underline{f}^i} \frac{\sum_{i=1}^{M}\bar{f}^i(U_r^i - U_r^1)\sum_{i=1}^{M}\underline{f}^i(U_r^M - U_r^i)}{\sum_{i=1}^{M}\bar{f}^i(U_r^i - U_r^1) + \sum_{i=1}^{M}\underline{f}^i(U_r^M - U_r^i)} \right] \quad (13)$$

D. Computation of the approximate type-reduction sets:

The two end points $[U_l(x), U_r(x)]$ for the type-reduced set are calculated as follows:

$$[U_l(x), U_r(x)] \approx [\hat{U}_l(x), \hat{U}_r(x)]$$
$$= [(\underline{U}_l(x) + \overline{U}_l(x))/2, (\underline{U}_r(x) + \overline{U}_r(x))/2] \quad (14)$$

E. Computation of approximate defuzzified output:

Finally, the crisp output of an IT2-FLC is calculated as:

$$U(x) \approx \hat{U}(x) = \frac{1}{2}[\hat{U}_l(x) + \hat{U}_r(x)] \quad (15)$$

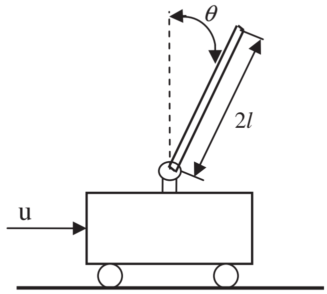

Figure 4 Inverted Pendulum on a cart.

3. Uncertain nonlinear inverted pendulum and its controller

3.1. Mathematical model

The inverted pendulum system defined here is shown in Fig. 4, which is formed from a cart, a pendulum and a rail for defining the position of the cart. The pendulum is hinged in the center of the top surface of the cart and can rotate around the pivot in the same vertical plane with the rail. The cart can move right or left on the rail freely. It is given that no friction exists in the system between the cart and the rail or between the cart and the pendulum [32].

The dynamic equations of the uncertain inverted pendulum system can be expressed as [32–34]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ f_1(x_1, x_2) \end{bmatrix} + \Delta A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ f_2(x_1) \end{bmatrix} u \tag{16}$$

$$a = m_p + \Delta m_p + m_c,$$
$$b = \frac{4l}{3} - \frac{(m_p + \Delta m_p) l \cos^2(x_1 + \Delta x_1)}{a} \tag{17}$$

$$f_1(x_1, x_2) = \left[g\sin(x_1 + \Delta x_1) - \frac{(m_p + \Delta m_p) l x_2^2 \sin(x_1 + \Delta x_1)\cos(x_1 + \Delta x_1)}{a} \right] \Big/ b \tag{18}$$

$$f_2(x_1) = \cos(x_1 + \Delta x_1)/ab \tag{19}$$

where x_1 is the angle of the pendulum, $x_2 = \dot{x}_1$ is the rate change of the pendulum angle, and u is the control force in the unit (Newton) applied horizontally to the cart. The parameters, m_c and m_p are, respectively, the mass of the cart and the mass of the pendulum in the unit (kg), and $g = 9.8\ \text{m/s}^2$ is the gravity acceleration. The parameter l is the half length of the pendulum in meters. ΔA is the structural uncertainty of the inverted pendulum which indicate the parameter uncertainties which can be caused by parameters are difficult or impossible to get a precise measure or that the parameters tend to vary as a function of time, temperature, etc. For this pendulum system, the mean uncertain parameters are the Coulomb friction constants. Also, this uncertainty indicates to unmodelled dynamic uncertainty. Δm_p is the uncertainty in the mass of the pendulum. Δx_1 is a disturbance element which is caused by measurement error.

3.2. The proposed controller for the inverted pendulum

Fig. 5, shows the block diagram of the IT2F-PID controller for balancing an inverted pendulum on a cart. y_r denotes the desired angular position of the pendulum. The goal is to balance the pendulum in the upright position (i.e., $y_r = 0$) when it initially starts with nonzero angle of the vertical (i.e., $x_1 \neq 0$). u_{PID} is the control signal (force). The proposed IT2F-PID controller for the inverted pendulum uses triangular membership functions. Figs. 6 and 7 show the membership function for the error signal ($e(k)$) and the derivative of the error signal ($\Delta e(k)$). Figs. 8 and 9 show the output signal of the IT2F-PI controller ($\Delta U_{PI}(k)$) and the output of the IT2F-PD controller ($U_{PD}(k)$). The universe of discourse is divided into five overlapping interval type-2 fuzzy set values labeled NL (Negative Large), NS (Negative Small), Z (Zero), PS (Positive Small) and PL (Positive Large). The rule base for the IT2F-PI and the IT2F-PD controllers is described in Tables 1 and 2 respectively. The scaling factors of the IT2F-PID controllers are $G_{e1} = -1$, $G_{\Delta e1} = -12$, $G_{e2} = 0.5$, $G_{\Delta e2} = 30$, $G_u = 2$ and $G_{\Delta u} = 1$.

4. Implementation of the interval type-2 fuzzy PID controller

4.1. Hardware-in-the-loop simulation

Hardware-in-the-loop simulation is used increasingly required for the design, implementation and testing of control systems, where some of the control loop components are real hardware and some are simulated. Usually, the controlled system is simulated in real-time and the controllers are in hardware [35]. HIL simulation has been developed in industrial control for testing of systems comprising some physical and some simulated components [36–38]. Simulation is used to represent processes that are physically unavailable, or whose use would be too costly, dangerous, or time-consuming. Proven benefits of HIL simulation include reproducibility of experiments and the ability to perform tests which would otherwise be impossible, impractical or unsafe. The proposed IT2F-PID controller which described in Sections 2 and 3 is implemented using a low cost PIC microcontroller (P18F4685). The architecture of the P18F4685 is Harvard architecture which the program memory (14 bit) and the data memory (8 bit) as separate memories and are accessed from separate buses. With P18F4685, the instruction is fetched in a single instruction cycle (execution time

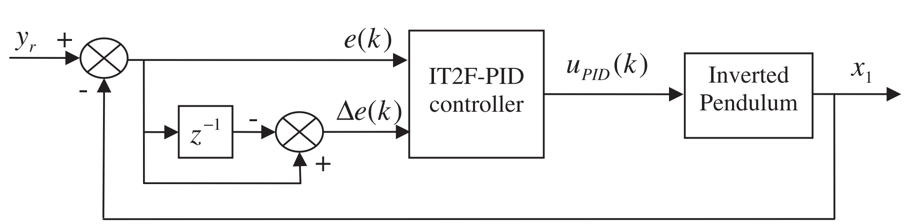

Figure 5 Interval type-2 fuzzy PID controller for an inverted pendulum.

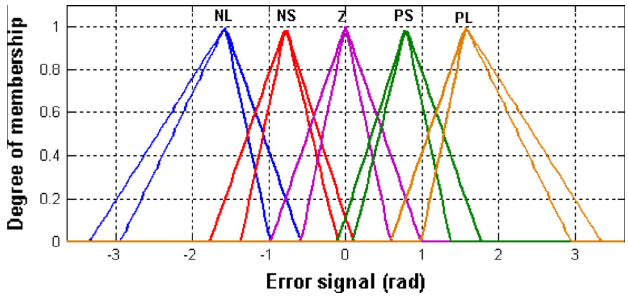

Figure 6 Membership functions for the error signal.

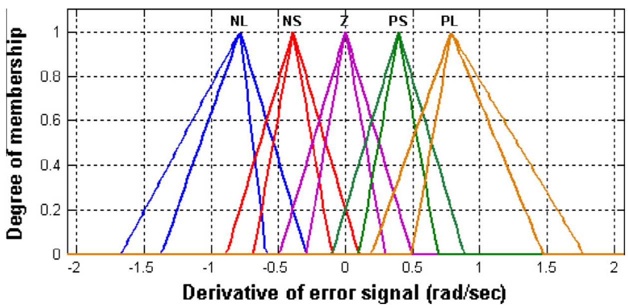

Figure 7 Membership functions for the derivative of error signal.

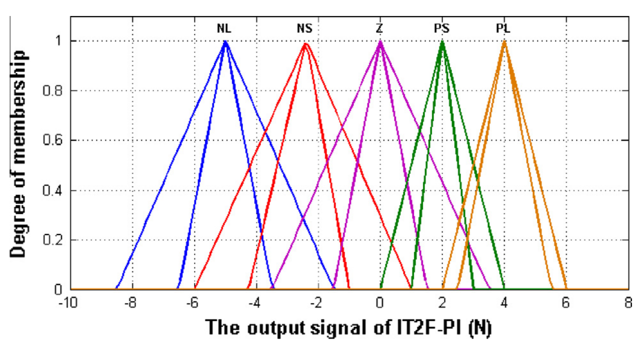

Figure 8 Membership functions for the output signal of the IT2F-PI controller.

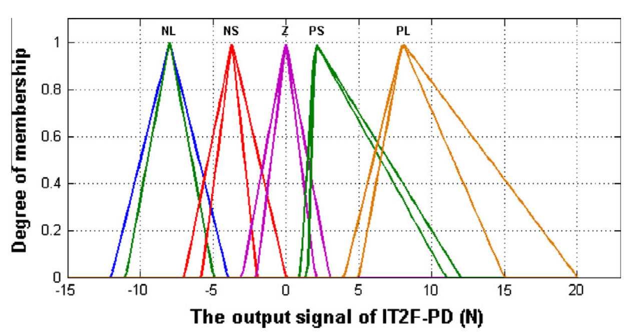

Figure 9 Membership functions for the output signal of the IT2F-PD controller.

equal 0.1 μs) which reduces the computation time and improves the performance. The P18F4685 has high computational performance at an economical price with the addition

Table 1 The rule base for the IT2F-PI controller.

Derivative of error signal	Error signal				
	N	NS	Z	PS	PL
NL	NL	NL	NL	NS	Z
NS	NL	NL	NS	Z	PS
Z	NL	NS	Z	PS	PL
PS	NS	Z	PS	PL	PL
PL	Z	PS	PL	PL	PL

Table 2 The rule base for the IT2F-PD controller.

Derivative of error signal	Error signal				
	N	NS	Z	PS	PL
NL	PL	PL	PL	PS	Z
NS	PL	PL	PS	Z	NS
Z	PL	PS	Z	NS	NL
PS	PS	Z	NS	NL	NL
PL	Z	NS	NL	NL	NL

of high-endurance, 98 kB of internal read only memory (ROM), 32 kB of internal random access memory (RAM), 40 MHz crystal frequency and addressable universal synchronous asynchronous receiver transmitter (USART). All programming is done in the C language using the mikroC environment. The nonlinear inverted pendulum is simulated using MATLAB via real-time toolboxes where the sampling time for the system is 0.01 s. The communication between the MATLAB and PIC microcontroller is performed by serial communication via RS232. Fig. 10, shows the schematic diagram of the HIL simulation for the system. Fig. 11, shows the real implementation of HIL simulation setup for the nonlinear inverted pendulum.

4.2. Practical results

In this section, the real-time simulation results for a system representing an inverted pendulum-cart system controlled using the proposed IT2F-PID controller are presented. In order to clarify the improvements of the proposed controller, the simulation results with a T1F-PID controller also are implemented for comparison purposes using the same number of membership functions, a number of rules, the same universe of discourse and the gains that shown in Fig. 1. The parameters of an inverted pendulum-cart system are given in Table 3. There are three experimental tasks performed for the inverted pendulum. For all the experimental tasks the initial conditions are $x_1 = 0.3$ rad and $x_2 = 0$ rad/s.

4.2.1. Task 1: Uncertainty in the mass of the pendulum

Fig. 12, shows the response of an inverted pendulum when adding $\Delta m_p = 2.5$ kg after 2 s from starting the experiment. The proposed IT2F-PID controller can realize tracking of an inverted pendulum with good robustness with small overshoot and settling time rather than the T1F-PID controller. Fig. 13, shows the response of the inverted pendulum when adding $\Delta m_p = 2.7$ kg after 2 s. It's clear that, the response of the in-

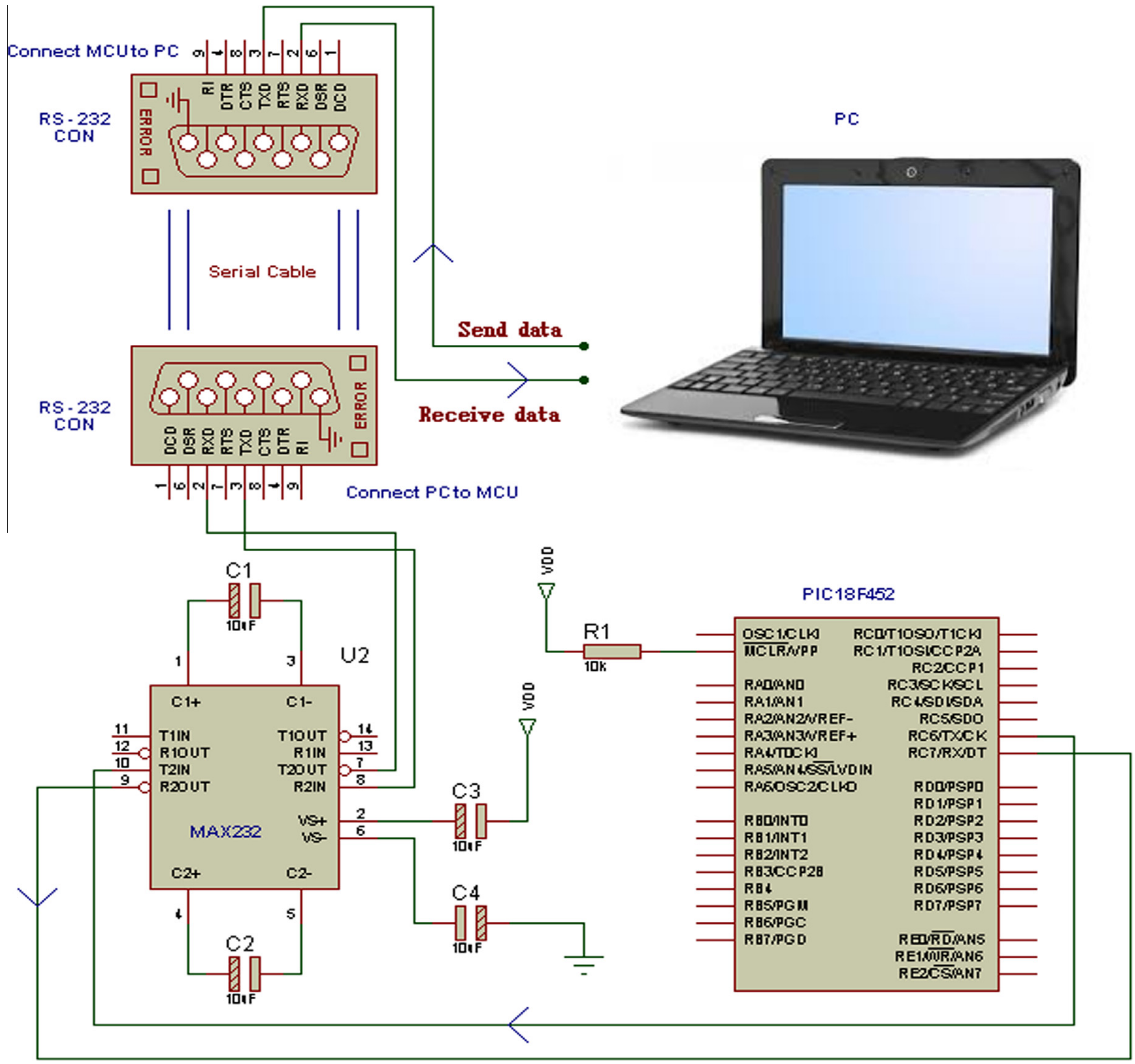

Figure 10 Schematic diagram of the HIL simulation of the system.

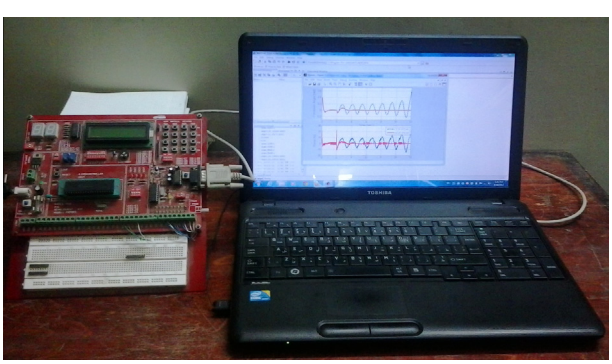

Figure 11 Experimental setup used in the study.

verted pendulum system remains stable for the proposed IT2F-PID controller and unstable for the T1F-PID controller. To show the improvements of the proposed controller, the mean absolute error (MAE) which is defined in Eq. (20) for the proposed IT2F-PID controller and the T1F-PID controller is shown in Figs. 14 and 15. It's clear that the MAE for the proposed IT2F-PID controller is lower than that obtained for the T1F-PID controller. So, the response of the proposed IT2F-PID controller is made significantly better than the T1F-PID controller to handle the uncertainty due to the mass of the pendulum.

Table 3 The parameters of the inverted pendulum system.

Symbol	Parameter name	Values
m_c	The mass of the cart	0.5 kg
m_p	The mass of the pendulum	0.2 kg
I	The half length of the pendulum	0.5 m
g	The gravity acceleration	9.8 m/s^2

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|e(t)| \tag{20}$$

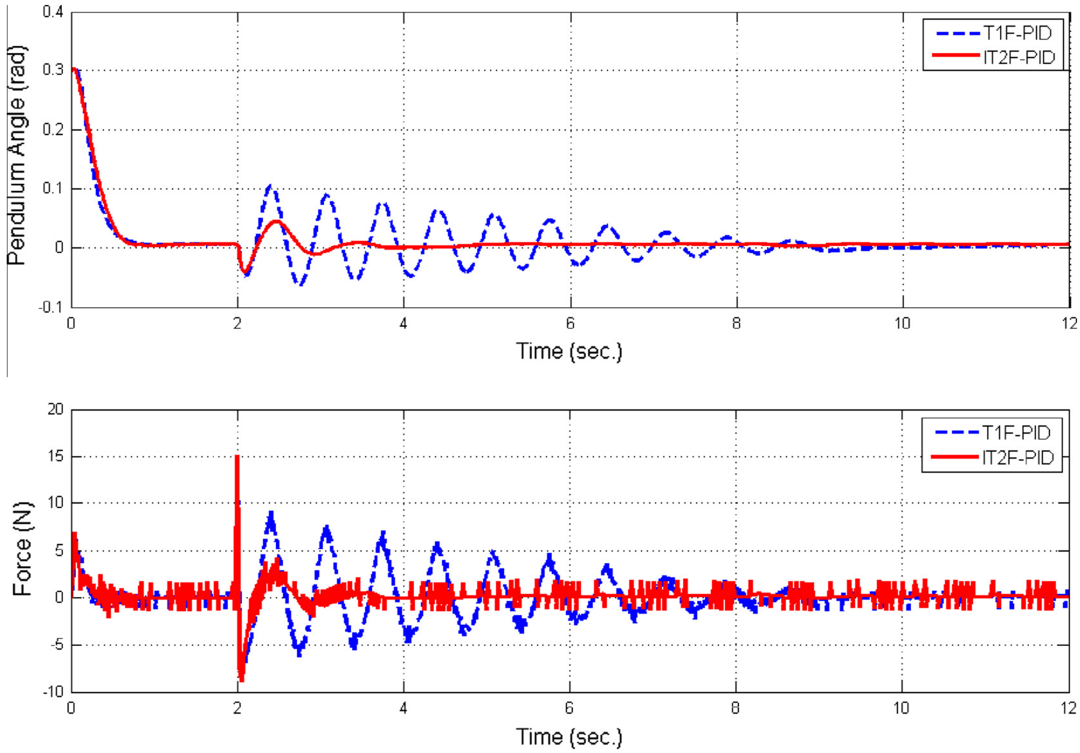

Figure 12 Response of the inverted pendulum system for uncertainty in the mass of the pendulum ($\Delta m_p = 2.5$ kg).

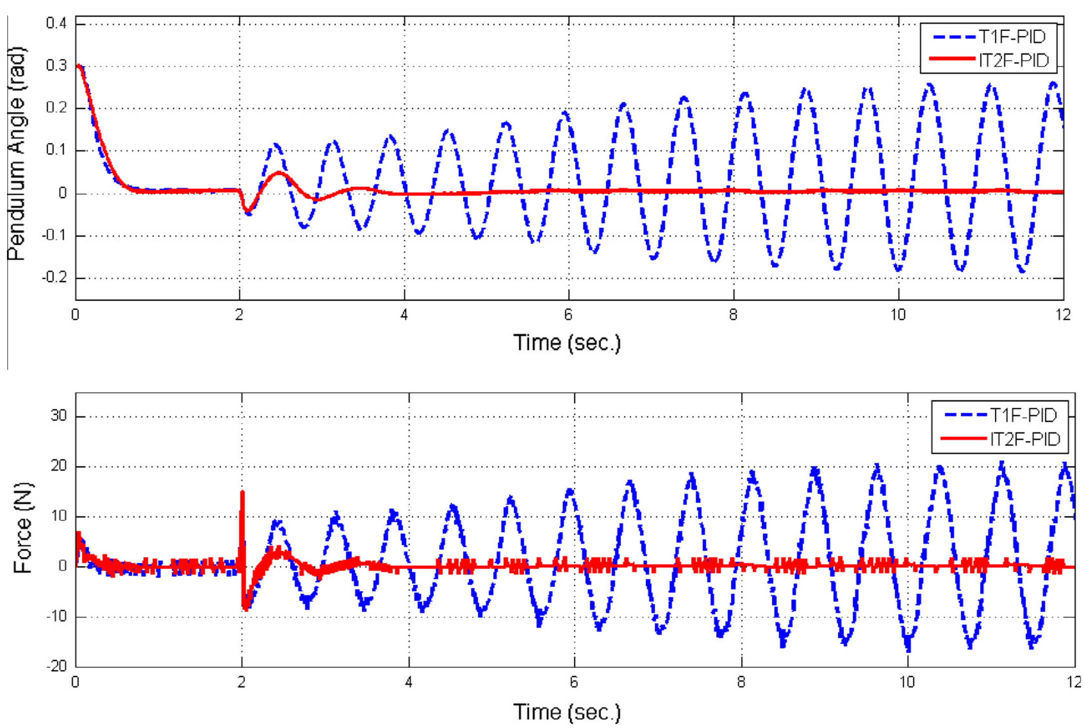

Figure 13 Response of the inverted pendulum system for uncertainty in the mass of the pendulum ($\Delta m_p = 2.7$ kg).

4.2.2. Task 2: Uncertainty due to the measurement error in the angle of the pendulum

This task shows the effect of the uncertainty due to the measurement error in the angle of the pendulum. The response of an inverted pendulum for this task using $\Delta x_1 = 0.052$ rad after 2 s from starting the simulation is shown in Fig. 16. It is clear that, the response of the proposed IT2F-PID controller after adding uncertainty value is better than the T1F-PID

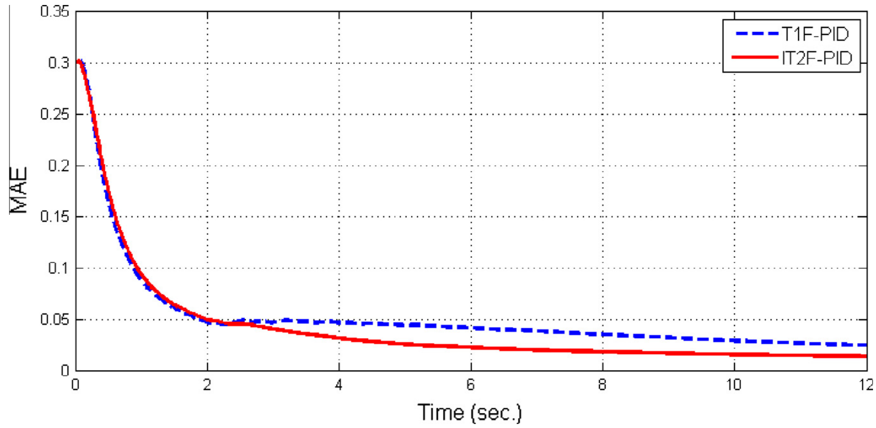

Figure 14 MAE for uncertainty in the mass of the pendulum (Δm_p = 2.5 kg).

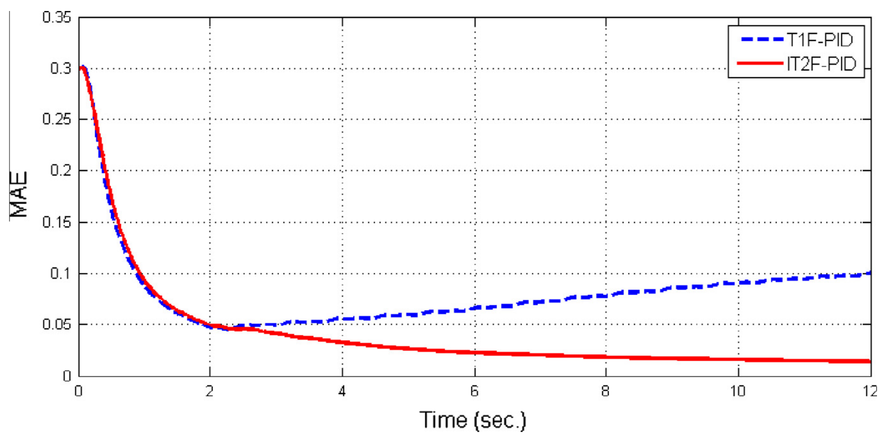

Figure 15 MAE for uncertainty in the mass of the pendulum (Δm_p = 2.7 kg).

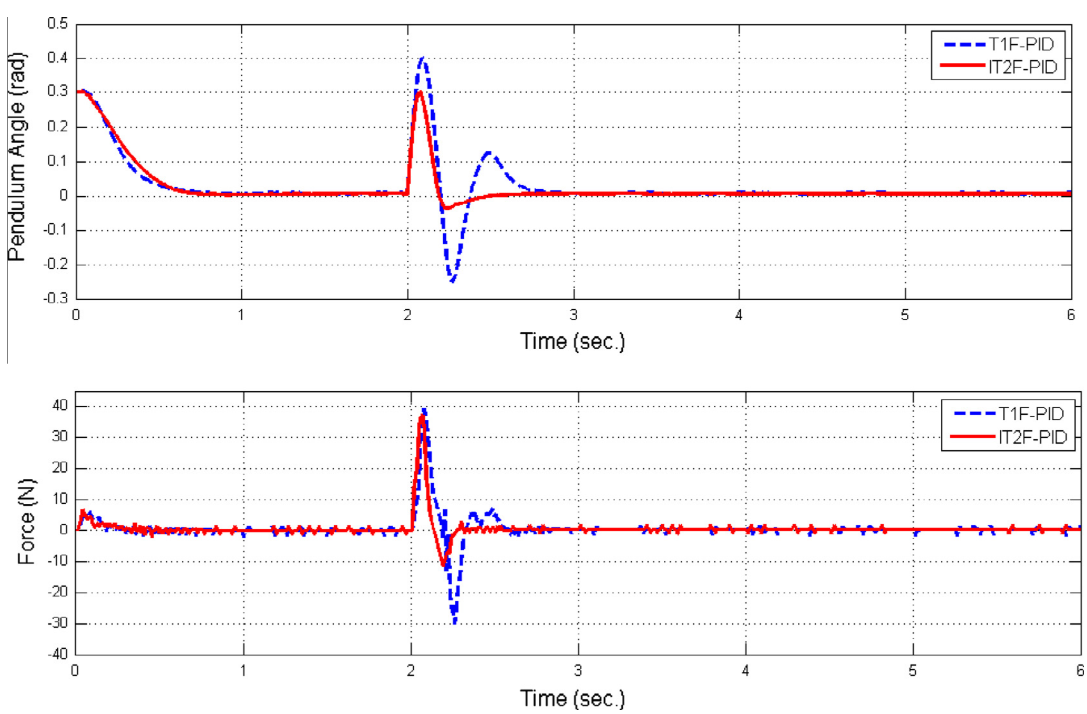

Figure 16 Response of the inverted pendulum system when the value Δx_1 = 0.052 rad.

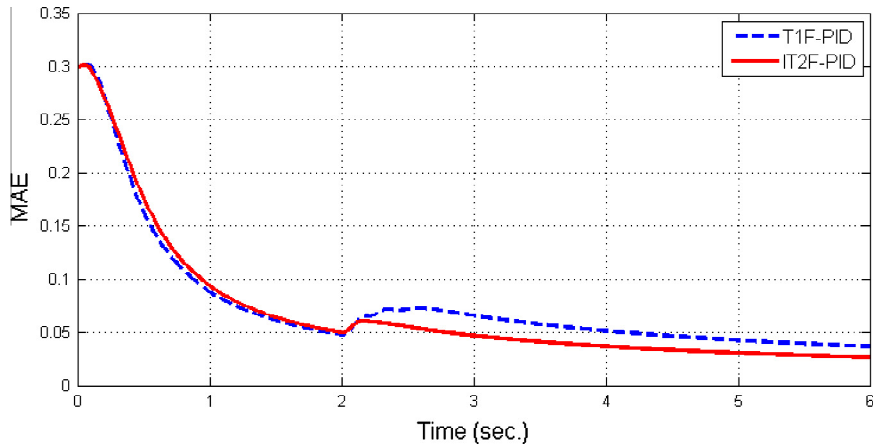

Figure 17 MAE for the uncertainty due to the measurement error in the angle of the pendulum.

controller where the settling time for the proposed IT2F-PID controller after adding the uncertainty value is less than the settling time for the T1F-PID controller. Fig. 17, shows the MAE for both the proposed and the T1F-PID controller. The MAE for the proposed IT2F-PID controller is lower than the T1F-PID controller. So, the proposed IT2F-PID controller is able to respond the uncertainty due to the measurement error of the pendulum angle rather than the T1F-PID controller.

4.2.3. Task 3: Structural uncertainty

To simulate this type of uncertainty, we add the value of ΔA to the system states (x_1 and x_2) as shown above in Eq (16). When this value is added, this mean that the parameters of the inverted pendulum are changed. Fig. 18, shows the response of an inverted pendulum system using the IT2F-PID and the T1F-PID controllers for this task. The uncertainty value, ΔA is defined as:

$$\Delta A = \begin{bmatrix} 0.03 & 0.03 \\ 0.03 & 0.03 \end{bmatrix}$$

This value is added at time equal 2 s. As shown in Fig. 18, the response of the pendulum angle which is seemed to oscillate far from the origin after adding the uncertainty value for the T1F-PID controller but, the response of the pendulum angle for the proposed IT2F-PID controller is good without an oscillation after adding the uncertainty value. The MAE for the proposed

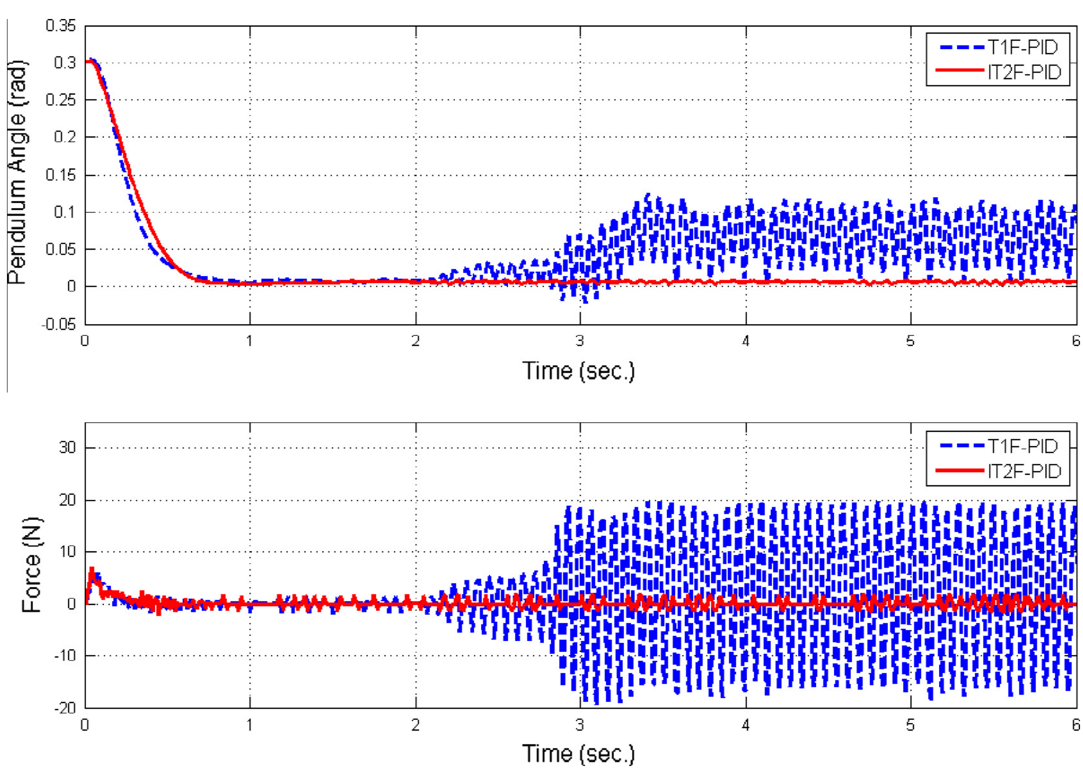

Figure 18 Response of the inverted pendulum system for the structural uncertainty.

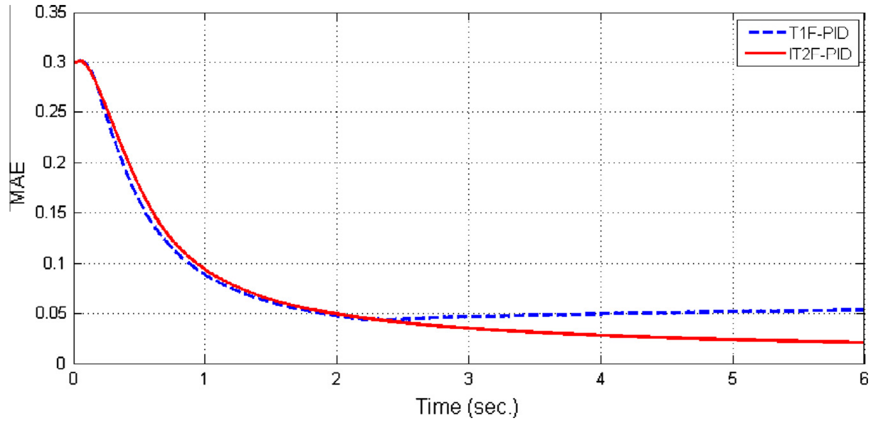

Figure 19 MAE for the structural uncertainty.

IT2F-PID controller and the T1F-PID controller is shown in Fig. 19. It's clear that, the MAE for the proposed controller is lower than the T1F-PID controller. So, the proposed IT2F-PID controller is superior to respond the uncertainty rather than the T1F-PID controller.

To show the visual indications of control performance, an objective measure of error performance was made using the integral of square of errors (ISE), the root mean square error (RMSE) and the integral of the absolute error (IAE) criteria. The ISE, the RMSE and the IAE are defined in Eqs. (21)–(23) respectively.

$$\mathrm{ISE} = \int_0^\infty [e(t)]^2 dt \tag{21}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(e(t))^2} \tag{22}$$

$$\mathrm{IAE} = \int_0^\infty |e(t)| dt \tag{23}$$

Tables 4–6 list the ISE, the RMSE and the IAE values respectively, for the T1F-PID controller and the proposed IT2F-PID controller for all the above experimental tasks. As shown in Tables 4–6 the values of the ISE, the RMSE and the IAE for the proposed IT2F-PID controller are lower than that the values obtained for the T1F-PID controller. So, the proposed IT2F-PID controller is superior to respond the uncertainties in the inverted pendulum system rather than the T1F-PID controller.

The used memory for the proposed IT2F-PID controller is 32% of RAM and 43% of ROM and for the T1F-PID controller is 14% of RAM and 23% of ROM. The computation time for the proposed IT2F-PID controller is 4.5 ms and 2.4 ms for the T1F-PID controller. It's mean that, the computation time for the proposed IT2F-PID controller is less than the sampling time for the system (0.01 s). Although the computation time for the proposed controller is larger than the time for the T1F-PID controller, the proposed IT2F-PID controller improves the performance and respond to the uncertainties rather than the T1F-PID controller.

Table 4 ISE values.

	Task 1 (Δm_p = 2.5 kg)	Task1 (Δm_p = 2.7 kg)	Task 2	Task 3
TIF-PID	2.733	18.62	4.0432	3.434
IT2F-PID	2.047	2.055	2.713	1.965

Table 5 RMSE values.

	Task 1 (Δm_p = 2.5 kg)	Task1 (Δm_p = 2.7 kg)	Task 2	Task 3
TIF-PID	0.0477	0.1246	0.0821	0.0756
IT2F-PID	0.0413	0.0414	0.0672	0.0572

Table 6 IAE values.

	Task 1 (Δm_p = 2.5 kg)	Task1 (Δm_p = 2.7 kg)	Task 2	Task 3
T1F-PID	29.39	120.04	21.71	30.23
IT2F-PID	16.55	16.6	15.68	12.25

5. Conclusions

In this paper, the IT2F-PID controller is proposed for controlling the uncertain inverted pendulum on a cart. The proposed IT2F-PID controller is a parallel combination of IT2F-PI controller and IT2F-PD controller. The IT2F-PID controller is implemented using a low cost PIC microcontroller (P18F4685). The test is carried out using HIL simulation. The inverted pendulum system is simulated using a MATLAB. The communication between the controller and the simulated system is performed using serial communication. The proposed IT2F-PID controller has been tested by using three experimental tasks including the mass uncertainties of the pendulum, the structural uncertainties, and the uncertainty due to the measurement error in the angle of the pendulum. The experimental results of the proposed controller are compared with the results of the T1F-PID controller. For the mass uncertainty case, the proposed IT2F-PID controller can realize tracking of an inverted pendulum with good robustness, small overshoot and small settling time rather than the T1F-PID controller. When the uncertainty value of the mass of the pendulum increased, the proposed controller remains the stability of the system. For the structured uncertainty case, the inverted pendulum system remains stable for the proposed controller. But, it becomes unstable for the T1F-PID controller. For the uncertainty due to the measurement error case, the proposed controller has the ability to respond the effect of this type of uncertainties rather than the T1F-PID controller. Four performance indices; MAE, ISE, RMSE and IAE are measured for the proposed IT2F-PID controller and the T1F-PID controller for all cases and these indices establish the superiority of the proposed controller to respond to system uncertainties rather than the T1F-PID controller. Thus, the methodology proposed in this study can be used to realize a robust, practically realizable, fuzzy controller capable of controlling a real-life plant with system uncertainties with acceptable closed-loop response. For the calculations of the proposed IT2F-PID controller, the Wu-Mendel method is used to perform the type-reduction which decreases the time of computation rather than KM algorithms. So, in the future work we can propose another type-reduction methods which decrease the time of computations rather than Wu-Mendel method.

The major contributions of this study are: (1) the successful development of the fuzzy PID controller to the IT2F-PID controller. (2) The successful implementation of the proposed IT2F-PID controller using a low cost PIC microcontroller. (3) The successful application of an embedded IT2F-PID controller for controlling the uncertain inverted pendulum on a cart system using the HIL simulation. (4) The success of the proposed controller to minimize the effect of the system uncertainties where the MAE, ISE, RMSE and IAE for all HIL simulation results obtained for the proposed controller are lower than that obtained for the T1F-PID controller.

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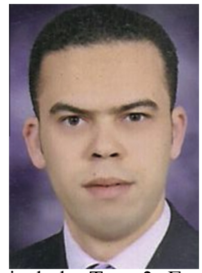

Ahmad M. El-Nagar received the B.Sc. and M.Sc. degrees in electronic engineering from Faculty of Electronic Engineering, Menoufia University, Menouf, Egypt. He is currently working toward the Ph.D. degree in electronic engineering. He is currently Assistant Lecture with the department of Industrial Electronics and Control Engineering, Faculty of Electronic Engineering, Menoufia University, Menouf Egypt. His current research interests include Type-2 Fuzzy Logic Systems and Type-2 Fuzzy Neural Networks.

Mohammad El Bardini is currently an Associate Professor with the Department of Industrial Electronic and Control Engineering, Faculty of Electronic Engineering; Menoufia University. His research interests include Robotics, Computer Controlled Systems and the Embedded System design for Control Systems. He has coauthored many journal and conference papers and has supervised many Ph.D. and M.Sc. for students working in the field of Intelligent Control Systems, Embedded Control Systems, Advanced Techniques for Systems Modeling, and Vision based Control of Robotic Systems. He is the recipient of the best students project Award in Egyptian Engineering Day.