# A proposal for M2-brane-anti-M2-brane action 

Mohammad R. Garousi<br>Department of Physics, Ferdowsi University of Mashhad, PO Box 1436, Mashhad, Iran

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#### Abstract

We propose a manifestly $S O(8)$ invariant BF type Lagrangian for describing the dynamics of M2-brane-anti-M2-brane system in flat spacetime. When one of the scalars which satisfies a free-scalar equation takes a large expectation value, the M2-brane-anti-M2-brane action reduces to the tachyon DBI action of D2-brane-anti-D2-brane system in flat spacetime.


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## 1. Introduction

Following the idea that the Chern-Simons gauge theory may be used to describe the dynamics of coincident M2-branes [1], Bagger and Lambert [2] as well as Gustavsson [3] have constructed three-dimensional $\mathcal{N}=8$ superconformal SO(4) Chern-Simons gauge theory based on 3-algebra. It is believed that the BLG world volume theory at level one describes two M2-branes on $R^{8} / Z_{2}$ orbifold [4]. The world volume theory of $N$ M2-branes on $R^{8} / Z_{k}$ orbifold has been constructed in [5] which is given by $\mathcal{N}=6$ superconformal $U(N)_{k} \times U(N)_{-k}$ Chern-Simons gauge theory.

The signature of the metric on 3-algebra in the BLG model is positive definite. This assumption has been relaxed in [6] to study $N$ coincident M2-branes in flat spacetime. The so called BF membrane theory with arbitrary semi-simple Lie group has been proposed in [6]. This theory has ghost fields, however, there are different arguments that model may be unitary due to the particular form of the interactions [6,7]. The bosonic part of the Lagrangian for gauge group $U(N)$ is given by

$$
\begin{equation*}
L=\operatorname{Tr}\left(\frac{1}{2} \epsilon^{a b c} B_{a} F_{b c}-\frac{1}{2} \hat{D}_{a} X^{I} \hat{D}^{a} X^{I}+\frac{1}{12} M^{I J K} M^{I J K}\right)+\left(\partial_{a} X_{-}^{I}-\operatorname{Tr}\left(B_{a} X^{I}\right)\right) \partial^{a} X_{+}^{I} \tag{1}
\end{equation*}
$$

where $A_{a}, B_{a}, X^{I}$ are in adjoint representation of $U(N)$ and $X_{-}^{I}, X_{+}^{I}$ are singlets under $U(N)$, and

Obviously the above Lagrangian is invariant under global $S O(8)$ transformation and under $U(N)$ gauge transformation associated with the $A_{a}$ gauge field. It is also invariant under gauge transformation associated with the $B_{a}$ gauge field

$$
\begin{equation*}
\delta_{B} X^{I}=X_{+}^{I} \Lambda, \quad \delta_{B} B_{a}=D_{a} \Lambda, \quad \delta_{B} X_{+}^{I}=0, \quad \delta_{B} X_{-}^{I}=\operatorname{Tr}\left(X^{I} \Lambda\right) \tag{3}
\end{equation*}
$$

The Lagrangian (1) is a candidate to describe the dynamics of $N$ stable M2-branes in flat supergravity background. A nonlinear extension of this Lagrangian in non-abelian case is proposed in [8,9] (see also [10,11]). ${ }^{1}$

In this Letter, we would like to study the dynamics of unstable M2-brane-anti-M2-brane system. The instability of this system can be either unperturbative effect or it can be the result of having tachyon fields in the spectrum of M2-brane-anti-M2-brane system, as in the D2-brane-anti-D2-brane system. Assuming the latter case, one may then use the Higgs mechanism [13] to find the effective action

[^0]by including appropriately the tachyons in the nonlinear action [8,9]. That is, when one of the scalars $X_{+}^{I}$ takes a large expectation value, M2-brane-anti-M2-brane action should be reduced to the D2-brane-anti-D2-brane action. However, this mechanism does not work for the tachyon potential because the M2-brane-anti-M2-brane action should describe the D2-brane-anti-D2-brane system at strong coupling. One expects the tachyon potential at the strong coupling to be totally different than the tachyon potential at the weak coupling. So the Higgs mechanism cannot fix the tachyon potential in terms of the tachyon potential of D2-brane-anti-D2-brane system. To find the M2-brane-anti-M2-brane action we do as follows: Near the unstable point, one can set the tachyon potential to one, and find the other parts of the M2-brane-anti-M2-brane action by the Higgs mechanism. Then one multiplies the result by the unknown M2-brane-anti-M2-brane tachyon potential.

In the next section we review the construction of the effective action of $D_{2} \bar{D}_{2}$ system proposed in [15] which is a non-abelian extension of the tachyon DBI action. Then we use de Wit-Herger-Samtleben duality transformation to write the $D_{2} \bar{D}_{2}$ action in a BF theory. In Section 3, we propose an $S O$ (8) invariant BF type action for $M_{2} \bar{M}_{2}$ system which reduces to the above theory when one of the scalars $X_{+}^{I}$ takes a large expectation value.

## 2. D2-brane-anti-D2-brane effective action

An effective action for $D_{9} \bar{D}_{9}$ system has been proposed in [22] whose vortex solution satisfies some consistency conditions. This action has been written as a non-abelian extension of the tachyon DBI action in [15]. However, the ordering of the matrices in the action is not consistent with the $S$-matrix elements. Hence, another effective action has been proposed in [15] which is consistent with the $S$-matrix elements. This second action may be related to the action proposed in [22] by some field redefinition. In the following we are going to review this second construction of the effective action for $D_{2} \bar{D}_{2}$ system.

The effective action for describing the dynamics of one non-BPS $D_{p}$-brane in flat background in static gauge is given by [16-19]

$$
\begin{equation*}
S=-T_{p} \int d^{p+1} \sigma V\left(T^{2}\right) \sqrt{-\operatorname{det}\left(\eta_{a b}+\partial_{a} X^{i} \partial_{b} X^{i}+\lambda F_{a b}+\lambda \partial_{a} T \partial_{b} T\right)} \tag{4}
\end{equation*}
$$

where $\lambda \equiv 2 \pi \alpha^{\prime}$ and $V=1-\frac{\pi}{2} T^{2}+O\left(T^{4}\right)$ is the tachyon potential. ${ }^{2}$ The action for $N$ non-BPS $D_{p}$-branes may be given by some nonabelian extension of the above action. To study the non-abelian extension of the above action for arbitrary $p$, one may first consider the non-abelian action for $p=9$ case which has no transverse scalar field, and then use the $T$-duality transformations to find the non-abelian action for any $p$.

The following non-abelian action has been proposed in [17] for describing the dynamics of $N$ non-PBS $D_{9}$-branes:

$$
\begin{equation*}
\left.S=-T_{9} \operatorname{STr} \int d^{10} \sigma V\left(T^{2}\right) \sqrt{-\operatorname{det}\left(\eta_{\mu \nu}+\lambda F_{\mu \nu}+\lambda D_{\mu} T D_{\nu} T\right.}\right) \tag{5}
\end{equation*}
$$

where the symmetric trace make the integrand to be a Hermitian matrix. In the above, the gauge field strength and covariant derivative of the tachyon are

$$
F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu}-i\left[A_{\mu}, A_{\nu}\right], \quad D_{\mu} T=\partial_{\mu} T-i\left[A_{\mu}, T\right]
$$

Obviously the action (5) has $U(N)$ gauge symmetry and reduce to (4) for $N=1$.
The trace in the non-abelian action (5) is the symmetric trace. That is, if one expands the square root and the tachyon potential, then the non-abelian expressions of the form $F_{\mu \nu}, D_{\mu} T$ and the individual $T$ of the tachyon potential must appear in each term of the expansion as symmetric. This property make it possible to treat the non-abelian expressions $F_{\mu \nu}, D_{\mu} T$ and $T$ as ordinary number when manipulating them. Various couplings in the action (5) are consistent with the appropriate disk level $S$-matrix elements in string theory [17,20,21]. In particular, the calculation in [21] shows that the consistency is hold only if one uses the symmetric trace prescription.

Using the effective action of $N$ non-BPS $D_{9}$-branes (5), one finds the effective action of $N$ non-BPS $D_{2}$-branes by using $T$-duality [17]. The proposal for the effective action of $D_{2} \bar{D}_{2}[14,15]$ is then to project the effective action of $N=2$ non-BPS $D_{2}$-branes with ( -1$)^{F_{L}}$, i.e., the matrices $A_{a}, X^{i}$ and $T$ take the following form:

$$
A_{a}=\left(\begin{array}{cc}
A_{a}^{(1)} & 0  \tag{6}\\
0 & A_{a}^{(2)}
\end{array}\right), \quad X^{i}=\left(\begin{array}{cc}
X^{i(1)} & 0 \\
0 & X^{i(2)}
\end{array}\right), \quad T=\left(\begin{array}{cc}
0 & \tau \\
\tau^{*} & 0
\end{array}\right)
$$

which reduces the $U(2)$ gauge symmetry to $U(1) \times U(1)$ gauge symmetry.
Replacing the above matrices in the effective action of $N=2$ non-BPS $D_{2}$-branes [17], one finds that the effective action of $D_{2} \bar{D}_{2}$ takes the following form:

$$
\begin{equation*}
S^{D \bar{D}}=-T_{2} \int d^{3} \sigma \operatorname{STr}\left(V \sqrt{\operatorname{det}(Q)} \sqrt{-\operatorname{det}\left(\eta_{a b}+\lambda^{2} g_{Y M}^{2} \partial_{a} X^{i}\left(Q^{-1}\right)_{i j} \partial_{b} X^{j}+\lambda\left(F_{a b}+\frac{1}{T_{2}} T_{a b}^{A}\right)+\frac{1}{T_{2}} T_{a b}^{S}\right)}\right) \tag{7}
\end{equation*}
$$

The matrices $Q^{i j}, T_{a b}^{S}, T_{a b}^{A}$ are

$$
\begin{aligned}
& Q^{i j}=I \delta^{i j}-\frac{g_{Y M}^{2}}{T_{2}}\left[X^{i}, T\right]\left[X^{j}, T\right], \quad \operatorname{det}(Q)=1+\frac{g_{Y M}^{2}}{T_{2}}\left[X^{i}, T\right]\left[T, X^{i}\right] \\
& T_{a b}^{S}=D_{a} T D_{b} T+\frac{g_{Y M}^{2}}{T_{2}} D_{a} T\left[X^{i}, T\right]\left(Q^{-1}\right)_{i j}\left[X^{j}, T\right] D_{b} T
\end{aligned}
$$

[^1]\[

$$
\begin{equation*}
T_{a b}^{A}=i g_{Y M}^{2} \partial_{a} X^{i}\left(Q^{-1}\right)_{i j}\left[X^{j}, T\right] D_{b} T-i g_{Y M}^{2} D_{a} T\left[X^{i}, T\right]\left(Q^{-1}\right)_{i j} \partial_{b} X^{j} \tag{8}
\end{equation*}
$$

\]

Here the transverse scalars in [17] are normalized as $\Phi^{i}=g_{Y M} \lambda X^{i}$ where $g_{Y M}$ is the three-dimensional Yang-Mills coupling constant, i.e., $\lambda^{2} T_{2}=1 / g_{Y M}^{2}$, and a factor of $\sqrt{\lambda T_{2}}$ has been absorbed into the tachyon field. The tachyon potential is then a function of $T^{2} /\left(\lambda T_{2}\right)$. The trace in the action is completely symmetric between all matrices $F_{a b}, \partial X^{i}, D_{a} T,\left[X^{i}, T\right]$ and individual $T$ of the tachyon potential. Hence, $\left(Q^{-1}\right)_{i j}$ appears in symmetric form. Moreover, the symmetric trace makes the matrix $\eta_{a b}+\frac{1}{T_{2}} \partial_{a} X^{i}\left(Q^{-1}\right)_{i j} \partial_{b} X^{j}+\frac{1}{T_{2}} T_{a b}^{S}$ in the action to be symmetric and matrix $T_{a b}^{A}$ to be antisymmetric.

Now we use the following de Wit-Herger-Samtleben duality transformation [8]:

$$
\begin{equation*}
-T_{2} \sqrt{-\phi \operatorname{det}\left(g_{a b}+\lambda F_{a b}\right)} \rightarrow-T_{2} \sqrt{-\phi \operatorname{det}\left(g_{a b}+\frac{g_{Y M}^{2}}{T_{2}} \frac{B_{a}^{\prime} B_{b}^{\prime}}{\phi}\right)}+\frac{1}{2} \epsilon^{a b c} B_{a}^{\prime} F_{b c} \tag{9}
\end{equation*}
$$

for any scalar $\phi$, any symmetric matrix $g_{a b}$ and any antisymmetric matrix $F_{a b}$. Using this duality in which $\phi=V \sqrt{\operatorname{det}(Q)}$ and $B^{\prime}=V B$, the action (7) can be written in the following form:

$$
\begin{align*}
S^{D \bar{D}}= & -T_{2} \int d^{3} \sigma \operatorname{STr}\left(V \sqrt{\operatorname{det}(Q)} \sqrt{-\operatorname{det}\left(\eta_{a b}+\frac{1}{T_{2}} \partial_{a} X^{i}\left(Q^{-1}\right)_{i j} \partial_{b} X^{j}+\frac{g_{Y M}^{2}}{T_{2}} \frac{B_{a} B_{b}}{\operatorname{det}(Q)}+\frac{1}{T_{2}} T_{a b}^{S}\right)}\right) \\
& +\frac{1}{2} \int d^{3} \sigma \operatorname{STr}\left(V \epsilon^{a b c} B_{a}\left(F_{b c}+\frac{1}{T_{2}} T_{b c}^{A}\right)\right) \tag{10}
\end{align*}
$$

where

$$
B_{a}=\left(\begin{array}{cc}
B_{a}^{(1)} & 0 \\
0 & B_{a}^{(2)}
\end{array}\right)
$$

Near the unstable point of the tachyon potential one cat set $V \sim 1$. In the next section we are going to write an action for M 2 -brane-anti-M2-brane system around its unstable point that reduces to the above action around its unstable point under the Higgs mechanism [13].

## 3. M2-brane-anti-M2-brane effective action

Using the prescription given in [13], one may expect that effective action of the $M_{2} \bar{M}_{2}$ system to be reduced to the effective action of $D_{2} \bar{D}_{2}$ system when $X_{+}^{I}$ takes a large expectation value. However, the tachyon potential in the $M_{2} \bar{M}_{2}$ system may not be related to the tachyon potential in the $D_{2} \bar{D}_{2}$ system in this way since the $M_{2} \bar{M}_{2}$ action should describe the $D_{2} \bar{D}_{2}$ system at the strong coupling limit. Moreover, it is expected that the tachyon potential at the strong coupling to be totally different than the tachyon potential at the weak coupling. However around their unstable point both potential are one. In this Letter we are going to fix the effective action of $M_{2} \bar{M}_{2}$ around its unstable point by using the Higgs mechanism [13].

The prescription given in [13] has been used in [8,9] to find a nonlinear action for multiple M2-branes. Following [8], the $M_{2} \bar{M}_{2}$ extension of $S^{D \bar{D}}$ in (10) should have $S O(8)$ invariant terms $\tilde{\partial}_{a} X^{I}\left(\tilde{Q}^{-1}\right)_{I J} \tilde{\partial}_{b} X^{J}$ where $\tilde{\partial}_{a} X^{I}$ and $\tilde{Q}_{I J}$ should be defined to be invariant under the $B_{a}$ gauge transformation and when $X_{+}^{I}=v \delta^{I 10}$ where $v=g_{Y M}$, they satisfy the boundary condition:

$$
\begin{equation*}
\tilde{\partial}_{a} X^{I}\left(\tilde{Q}^{-1}\right)_{I J} \tilde{\partial}_{b} X^{J} \rightarrow \partial_{a} X^{i}\left(Q^{-1}\right)_{i j} \partial_{b} X^{j}+v^{2} \frac{B_{a} B_{b}}{\operatorname{det}(Q)} \tag{11}
\end{equation*}
$$

This fixes $\tilde{\partial}_{a} X^{I}$ to be [8]

$$
\begin{equation*}
\tilde{\partial}_{a} X^{I}=\partial_{a} X^{I}-X_{+}^{I} B_{a}-\left(\frac{X_{+} \cdot X}{X_{+}^{2}}\right) \partial_{a} X_{+}^{I} \tag{12}
\end{equation*}
$$

where $X_{+}^{2}=X_{+}^{I} X_{+}^{I}$. This is invariant under the gauge transformation (3). The boundary value of $\tilde{Q}_{I J}$ is [8]

$$
\begin{equation*}
\tilde{Q}^{i j}=Q^{i j}, \quad \tilde{Q}^{i 10}=\tilde{Q}^{10 i}=0, \quad \tilde{Q}^{1010}=\operatorname{det}(Q) \tag{13}
\end{equation*}
$$

At the boundary, one has $\operatorname{det}(\tilde{Q})=(\operatorname{det}(Q))^{2}$.
An ansatz for $\tilde{Q}^{I J}$ which is consistent with the above boundary condition may be

$$
\tilde{Q}^{I J}=a \delta^{I J}+b M^{I K} M^{K J}
$$

where $a, b$ are some $S O(8)$ invariants which can be found from the above boundary condition, and

$$
\begin{equation*}
M^{I J} \equiv X_{+}^{I}\left[X^{J}, T\right]+X_{+}^{J}\left[T, X^{I}\right] \tag{14}
\end{equation*}
$$

in which $X_{+}^{I}$ is a singlet under $U(1) \times U(1)$ and

$$
X^{I}=\left(\begin{array}{cc}
X^{I(1)} & 0  \tag{15}\\
0 & X^{I(2)}
\end{array}\right), \quad T=\left(\begin{array}{cc}
0 & \tau \\
\tau^{*} & 0
\end{array}\right)
$$

Note that $\delta_{B}\left(M^{I J}\right)=0$ and consequently $\delta_{B}\left(\tilde{Q}^{I J}\right)=0$ if one assumes the tachyon to be invariant under the $B_{a}$ gauge transformation. Imposing the boundary condition $\tilde{Q}^{i j}=Q^{i j}$ on the above ansatz, one finds

$$
\begin{equation*}
\tilde{Q}^{I J}=\delta^{I J}+\frac{1}{T_{2}} M^{I K} M^{K J} \tag{16}
\end{equation*}
$$

It also satisfies the boundary condition $\tilde{Q}^{1010}=\operatorname{det}(Q)$. Using the relation between type IIA theory and $M$-theory, i.e., $\ell_{p}=g_{s}^{1 / 3} \ell_{s}, T_{2}$ can be written in terms of 11-dimensional Plank length $\ell_{p}$ as $T_{2}=1 /(2 \pi)^{2} \ell_{p}^{3}$.

The matrices $\tilde{T}_{a b}^{S}$ and $\tilde{T}_{a b}^{A}$ should be determined by forcing them to be invariant under global $S O$ (8) and under gauge transformation associated with $B_{a}$, and by imposing the boundary condition that at the boundary $X_{+}^{I}=v \delta^{I 10}$ they should be reduced to those in (8). The result is

$$
\begin{align*}
& \tilde{T}_{a b}^{S}=D_{a} T\left(\frac{1}{\sqrt{\operatorname{det}(\tilde{Q})}}+\frac{1}{T_{2}} M^{I K}\left(\tilde{Q}^{-1}\right)_{I J} M^{K J}\right) D_{b} T, \\
& \tilde{T}_{a b}^{A}=i \tilde{\partial}_{a} X^{I}\left(\tilde{Q}^{-1}\right)_{I J} M^{J K} D_{b} T X_{+}^{K}-i D_{a} T M^{I K}\left(\tilde{Q}^{-1}\right)_{I J} \tilde{\partial}_{b} X^{J} X_{+}^{K} . \tag{17}
\end{align*}
$$

Note that the tachyon is invariant under the $B_{a}$ gauge transformation.
Taking the above points, one finds that the extension of the $D_{2} \bar{D}_{2}$ action (10) around its unstable point to $M_{2} \bar{M}_{2}$ is then given by the following action:

$$
\begin{aligned}
& \int d^{3} \sigma \operatorname{STr}\left(-T_{2}(\operatorname{det}(\tilde{Q}))^{1 / 4} \sqrt{-\operatorname{det}\left(\eta_{a b}+\frac{1}{T_{2}} \tilde{\partial}_{a} X^{I}\left(\tilde{Q}^{-1}\right)_{I J} \tilde{\partial}_{b} X^{J}+\frac{1}{T_{2}} \tilde{T}_{a b}^{S}\right)}\right. \\
& \left.\quad+\frac{1}{2} \epsilon^{a b c}\left(B_{a} F_{b c}-\frac{2 i}{T_{2}} \tilde{\partial}_{a} X^{K} \tilde{\partial}_{b} X^{I}\left(\tilde{Q}^{-1}\right)_{I J} M^{J K} D_{c} T\right)\right)
\end{aligned}
$$

where we have replaced $B_{a} X_{+}^{K}$ in the last line by the covariant expression - $\tilde{\partial}_{a} X^{K}$. This action is manifestly invariant under global SO(8), satisfies the Higgs mechanism and is also invariant under gauge transformations associated with gauge fields $A_{a}$ and $B_{a}$. The symmetric trace is between the gauge invariants $\tilde{\partial}_{a} X^{I}, D_{a} T, M^{I J}$.

The above action is not complete yet. Its tachyon potential is at its unstable point, i.e., $\mathcal{V} \sim 1$, and it dose not reduce to the action (1) at low energy and for $T=0$. We assume that the tachyon potential appears in the action as an overall function as in tachyon DBI action. The tachyon potential should be a function of only tachyon. The tachyon is a dimensionfull field, so the tachyon potential should be a function of $\mathcal{V}\left(\frac{1}{T_{2}^{1 / 3}} T^{2}\right)$. As we discussed before, this potential is not expected to be reduced to the tachyon potential of the $D_{2} \bar{D}_{2}$ system under the Higgs mechanism. To have consistency with action (1), one should add some extra terms to the above action [8,9]. Hence, our proposal for the effective action of $M_{2} \bar{M}_{2}$ is the following:

$$
\begin{align*}
& \int d^{3} \sigma \operatorname{STr}\left(\mathcal { V } \left[-T_{2}(\operatorname{det}(\tilde{Q}))^{1 / 4} \sqrt{-\operatorname{det}\left(\eta_{a b}+\frac{1}{T_{2}} \tilde{\partial}_{a} X^{I}\left(\tilde{Q}^{-1}\right)_{I J} \tilde{\partial}_{b} X^{J}+\frac{1}{T_{2}} \tilde{T}_{a b}^{S}\right)}\right.\right. \\
& \quad+\frac{1}{2} \epsilon^{a b c}\left(B_{a} F_{b c}-\frac{2 i}{T_{2}} \tilde{a}_{a} X^{K} \tilde{\partial}_{b} X^{I}\left(\tilde{Q}^{-1}\right)_{I J} M^{J K} D_{c} T\right) \\
& \left.\left.\quad+\left(\partial_{a} X_{-}^{I}-\operatorname{Tr}\left(B_{a} X^{I}\right)\right) \partial^{a} X_{+}^{I}-\operatorname{Tr}\left(\frac{X_{+} \cdot X}{X_{+}^{2}} \hat{D}_{a} X^{I} \partial^{a} X_{+}^{I}-\frac{1}{2}\left(\frac{X_{+} \cdot X}{X_{+}^{2}}\right)^{2} \partial_{a} X_{+}^{I} \partial^{a} X_{+}^{I}\right)\right]\right) \tag{18}
\end{align*}
$$

The last line in the above action has been added to have consistency at low energy and for $T=0$ with the action (1) for gauge group $U(1) \times U(1)[8,9]$. This action is manifestly invariant under global SO(8) and is also invariant under gauge transformations associated with gauge fields $A_{a}$ and $B_{a}$. The symmetric trace in the first two lines is between the gauge invariants $\tilde{\partial}_{a} X^{I}, D_{a} T, M^{I J}$ and individual $T$ of the tachyon potential, and in the last line it is only over the tachyon potential.

Let us now compare the two actions (18) and (10) around their unstable points where $\mathcal{V} \sim 1 \sim V$. Action (18) gives the equation of motion for $X_{-}^{I}$ to be $\partial_{a} \partial^{a} X_{+}^{I}=0$. If one of the scalars $X_{+}^{I}$ takes large expectation value, i.e., $X_{+}^{I}=v \delta^{I 10}$, then $\tilde{\partial}_{a} X^{i}=\partial_{a} X^{i}$, $\tilde{\partial}_{a} X^{10}=$ $\partial_{a} X^{10}-v B_{a}$ and $X_{+} \cdot \partial_{a} X=v \partial_{a} X^{10}$. Fixing the gauge symmetry (3) by setting $X^{10}=0$, one then recovers the $D_{2} \bar{D}_{2}$ action (10). On the other hand, if the shift symmetry $X_{-}^{I} \rightarrow X_{-}^{I}+c^{I}$ is gauged as in $[23,24]$ by introducing a new field $C_{a}^{I}$ and writing $\partial_{a} X_{-}^{I}$ as $\partial_{a} X_{-}^{I}-C_{a}^{I}$, then equation of motion for the new field gives $\partial_{a} X_{+}^{I}=0$ which has only constant solution $X_{+}^{I}=v^{I}$. Using the $S O(8)$ symmetry, one can write it as $X_{+}^{I}=v \delta^{I 10}$. Then the $M_{2} \bar{M}_{2}$ theory (18) would be classically equivalent to the $D_{2} \bar{D}_{2}$ theory (10).

The $M_{2} \bar{M}_{2}$ action (18), for constant $X_{+}^{I}$, are written almost entirely in terms of covariant derivative of the scalars/tachyon and 3bracket $M^{I J}$. Similar observation has been made in [11] for the higher derivative corrections to the Lorentzian-signature $M_{2}$-brane action. As pointed out in [11], one expects this part of the action which has no dependency on $X_{+}^{I}$ to be relevant to the theories beyond the Lorentzian signature that we have considered here.

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[^0]:    E-mail address: garousi@mail.ipm.ir.
    ${ }^{1}$ Nonlinear action of M2-brane in the presence of background fields for abelian case has been discussed in [12].

[^1]:    ${ }^{2}$ Our index convention is that $\mu, v, \ldots=0,1, \ldots, 9 ; a, b, \ldots=0,1, \ldots, p ; i, j, \ldots=p+1, \ldots, 9$ and $I, J, \ldots=3,4, \ldots, 10$.

