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Deformed two-photon squeezed states in noncommutative space

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Abstract

Recent studies on nonperturbation aspects of noncommutative quantum mechanics explored a new type of boson commutation relations at the deformed level, described by deformed annihilation–creation operators in noncommutative space. This correlated boson commutator correlates different degrees of freedom, and shows an essential influence on dynamics. This Letter devotes to the development of formalism of deformed two-photon squeezed states in noncommutative space. General representations of deformed annihilation–creation operators and the consistency condition for the electromagnetic wave with a single mode of frequency in noncommunicative space are obtained. Two-photon squeezed states are studied. One finds that variances of the dimensionless Hermitian quadratures of the annihilation operator in one degree of freedom include variances in the other degree of freedom. Such correlations show the new feature of spatial noncommutativity and allow a deeper understanding of the correlated boson commutator.

In the investigation of physics in noncommutative (NC) space [1–7] motivated by arguments of string theory, apart from studies of field theory, noncommutative quantum mechanics (NCQM) has recently attracted some attention [8–21]. As the one-particle sector of NC quantum field theory, studies of NCQM show special meaning. It may clarify some phenomenological consequences in solvable models. NC effects will only become apparent as the NC high energy scale is approached. But it is expected that, because of the incomplete decoupling mechanism between the high energy sector and the low energy sector, there should be some relics of NC effects appear in the low energy sector. Studies of NCQM may explore such low energy relics of effects of spatial noncommutativity. In literature perturbation aspects of NCQM have been studied in detail. The perturbation approach is based on the Weyl–Moyal correspondence [22–24], according to which the usual product of functions should be replaced by the star-product. Because of the exponential differential factor in the Weyl–Moyal product the nonperturbation treatment is difficulty.

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Recent studies on nonperturbation aspects of NCQM clarified [20] that in order to maintain Bose–Einstein statistics at the nonperturbation level described by deformed annihilation–creation operators in NC space, when the state vector space of identical bosons is constructed by generalizing one-particle quantum mechanics, the consistent ansatz of commutation relations of the phase space variables should simultaneously include space–space noncommutativity and momentum–momentum noncommutativity. A new type of boson commutation relations at the deformed level, called the correlated boson commutator, was explored. The correlated boson commutator correlates different degrees of freedom, and shows an essential influence on dynamics. For example, it explored that the spectrum of the angular momentum possesses, because of such correlating effects, fractional eigenvalues [20].

In literature many interesting topics of NC quantum theories have been extensively investigated, from the Aharonov–Bohm effect to the quantum Hall effect [25–31]. In order to further explore the influence of the correlated boson commutator on dynamics, in this Letter we study deformed two-photon squeezed states.

In commutative space the boson algebra shows that annihilation–creation operators in one degree of freedom are independent of ones in the other degree of freedom. In NC space new future appears, the correlated boson commutator shows that there is a correlation between different degrees of freedom. In order to explore the correlated effects this Letter devotes to the development of formalism of deformed two-photon squeezed states in noncommutative space. It turns out that the variances of the dimensionless Hermitian quadratures of the annihilation operator in one degree of freedom include the ones in the other degree of freedom. These results show the new future of spatial noncommutativity and allow a deeper understanding of the correlated boson commutator.

In the following we first review the background [10,20,32].

In order to develop the NCQM formulation we need to specify the phase space and the Hilbert space on which operators act. The Hilbert space can consistently be taken to be exactly the same as the Hilbert space of the corresponding commutative system [8].

As for the phase space we consider both space–space noncommutativity (space–time noncommutativity is not considered) and momentum–momentum noncommutativity. There are different types of NC theories, for example, see a review paper [32].

In the case of simultaneously space–space noncommutativity and momentum–momentum noncommutativity the consistent NCQM algebra are

$$[\hat{x}_i, \hat{x}_j] = i\xi^2\theta\epsilon_{ij}, \qquad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij},$$
$$[\hat{p}_i, \hat{p}_j] = i\xi^2\eta\epsilon_{ij} \quad (i, j = 1, 2), \tag{1}$$

where θ and η are the constant parameters, independent of position and momentum; their dimensions are, respectively, L^2 and $M^2L^2T^{-2}$, where M, L, T are, respectively, dimensions of mass, length and time. ϵ_{ij} is an antisymmetric unit tensor, $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$. $\xi = (1 + \theta\eta/4\hbar^2)^{-1/2}$. When $\eta = 0$, we have $\xi = 1$, the NCQM algebra (1) reduces to the one which is extensively discussed in literature for the case that only space–space are noncommuting.

There are different ways to construct the creation–annihilation operators. In order to explore NC effects at the nonperturbation level for our purpose we consider a system with a single mode of frequency ω, for example, the plain electromagnetic wave of single frequency. For the dimension i we find the general representations of the deformed annihilation–creation operators $\hat{a}_i$, $\hat{a}_i^\dagger$ $(i = 1, 2)$

$$\hat{a}_i = \frac{\omega}{\sqrt{2}c}\left(\hat{x}_i + i\frac{c^2}{\hbar\omega^2}\hat{p}_i\right),$$
$$\hat{a}_i^\dagger = \frac{\omega}{\sqrt{2}c}\left(\hat{x}_i - i\frac{c^2}{\hbar\omega^2}\hat{p}_i\right). \tag{2}$$

(c is speed of light in vacuum.) We notice that Eqs. (2) are ω dependent. Their NC parameter dependent structures are different from the ones of two-dimensional harmonic oscillator in Ref. [20].

When the state vector space of identical bosons is constructed by generalizing one-particle quantum mechanics, in order to maintain Bose–Einstein statistics at the level described by $\hat{a}_i^\dagger$, the basic assumption is that operators $\hat{a}_i^\dagger$ and $\hat{a}_j^\dagger$ should be commuting. This requirement leads to a consistency condition of NCQM algebra [20]

$$\eta = \hbar^2\omega^4c^{-4}\theta. \tag{3}$$

1. Deformed boson algebra

From Eqs. (1)–(3) it follows that the deformed boson algebra of $\hat{a}_i$ and $\hat{a}_j^\dagger$ read [20]

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij} + i\xi^2\omega^2 c^{-2}\theta,$$
$$[\hat{a}_i, \hat{a}_j] = 0 \quad (i, j = 1, 2). \tag{4}$$

For the case $i = j$, Eqs. (4) is the same commutation relations as the ones in commutative space. This confirms that for the same degree of freedom i the operators $\hat{a}_i, \hat{a}_i^\dagger$ are the correct deformed annihilation–creation operators. For the case $i \neq j$, a new type of deformed commutation relations between $\hat{a}_i$ and $\hat{a}_j^\dagger$, called the correlated boson commutator, emerges,

$$[\hat{a}_1, \hat{a}_2^\dagger] = i\xi^2\omega^2 c^{-2}\theta, \tag{5}$$

which correlates different degrees of freedom. At the level of deformed operators the effect of spatial noncommutativity are coded in Eq. (5).

It is worth noting that Eq. (5) is consistent with *all* principles of quantum mechanics and Bose–Einstein statistics.

The NCQM algebra (1) has different possible perturbation realizations [10]. To the linear terms of phase space variables in commutative space, the ansatz of the perturbation expansions of $\hat{x}_i$ and $\hat{p}_i$, consistent with NCQM algebra (1), is

$$\hat{x}_i = \xi\left[x_i - \frac{1}{2\hbar}\theta\epsilon_{ij}p_j\right],$$
$$\hat{p}_i = \xi\left[p_i + \frac{1}{2}\hbar\omega^4 c^{-4}\theta\epsilon_{ij}x_j\right], \tag{6}$$

where (x_i, p_i) are the phase space variables in commutative space, and $[x_i, x_j] = [p_i, p_j] = 0$, $[x_i, p_j] = i\hbar\delta_{ij}$.

The ansatz of the perturbation expansions of $\hat{a}_i$ and $\hat{a}_i^\dagger$ is

$$\hat{a}_i = \xi\left[a_i + \frac{i}{2}\omega^2 c^{-2}\theta\epsilon_{ij}a_j\right],$$
$$\hat{a}_i^\dagger = \xi\left[a_i - \frac{i}{2}\omega^2 c^{-2}\theta\epsilon_{ij}a_j\right], \tag{7}$$

where $(a_i, a_i^\dagger)$ are annihilation–creation operators in commutative space. The relations between $(a_i, a_i^\dagger)$ and (x_i, p_i) are $x_i = \sqrt{\hbar/(2\mu\omega)}(a_i + a_i^\dagger)$, $p_i = -i\sqrt{\mu\omega\hbar/2}(a_i - a_i^\dagger)$. We have $[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$, $[a_i, a_j^\dagger] = i\delta_{ij}$. Eq. (7) is consistent with the deformed boson algebra (4), specially including correlating effects of Eq. (5).

In summary, structures of the operators $(\hat{a}_i, \hat{a}_i^\dagger)$, the consistency condition and the perturbation expansions of $(\hat{x}_i, \hat{p}_i)$ and $(\hat{a}_i, \hat{a}_i^\dagger)$ are determined by the characteristic parameters of the system. The NC parameter dependent structures of Eqs. (2), (3), (5)–(7) are different from the ones of two-dimensional harmonic oscillator in Ref. [20].

Now we investigate the influence of the correlated boson commutator Eq. (5) on deformed two-photon squeezed states in NC space.

In discussions of the Heisenberg minimal uncertainty relation special attention has been focused on coherent states and squeezed states of the light field [33]. The idea of squeezing has both fundamental and practical interests. Here we consider a special squeezed state suggested in Ref. [34]. Such kind of squeezed states is easy to generalize to the case in NC space.

In commutative space a coherent state is defined as an eigenstate of the annihilation operator a, $a|\alpha\rangle = \alpha|\alpha\rangle$ with a complex eigenvalue α. The coherent state $|\alpha\rangle$ is represented as

$$|\alpha\rangle = N_\alpha\left[\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle\right] = N_\alpha \exp\bigl(\alpha a^\dagger\bigr)|0\rangle, \tag{8}$$

where $|n\rangle$ is the number states, $N|n\rangle = n|n\rangle$, $N = a^\dagger a$ is the number operator. From Eq. (8) $\langle\alpha| - \alpha\rangle = N_{-\alpha}^* N_\alpha \exp(-\alpha^2)$. For simplicity we may choose the phase factor so that α is real, thus in (8) the normalization constant is $N_\alpha = \exp(-\alpha^2/2)$.

The special squeezed state considered in Ref. [34] is as follows. An effective squeezing can be achieved by superposition of coherent states along a straight line on the α plane. This mechanism opens new possibility for squeezing, e.g., of the molecular vibrations during a Franck–Condon transition induced by a short coherent light pulse. For a single mode of frequency ω the electric field operator $E(t)$ is represented as $E(t) = E_0[a\exp(-i\omega t) + a^\dagger \exp(i\omega t)]$, where a and $a^\dagger$ are the annihilation and creation operators of photon field. This squeezed state is defined as

$$|\alpha, \pm\rangle = c_\pm\bigl(|\alpha\rangle \pm |-\alpha\rangle\bigr), \tag{9}$$

which satisfy $a|\alpha,\pm\rangle = \alpha c_\pm c_\mp^{-1}|\alpha,\mp\rangle$, $a^2|\alpha,\pm\rangle = \alpha^2|\alpha,\pm\rangle$, and $\langle\alpha,\pm|\alpha,\mp\rangle = 0$. The normalization constants are $c_\pm^2 = \exp(\alpha^2)/\{2[\exp(\alpha^2)\pm\exp(-\alpha^2)]\}$.

In the state $|\alpha,+\rangle$ squeezing appears. Let X and Y be the dimensionless Hermitian quadratures of the annihilation operator: $a = X + iY$. The variances of X and Y in this state are [34]

$$(\Delta X)^2 = \frac{1}{4} + \frac{\alpha^2\exp(\alpha^2)}{\exp(\alpha^2)+\exp(-\alpha^2)},$$
$$(\Delta Y)^2 = \frac{1}{4} - \frac{\alpha^2\exp(-\alpha^2)}{\exp(\alpha^2)+\exp(-\alpha^2)}. \tag{10}$$

Here for any normalized state $|\psi\rangle$, the variances of an operator F is defined as $\Delta F \equiv [(\psi,(F-\bar F)^2\psi)]^{1/2}$, $\bar F \equiv (\psi, F\psi)$. It is noticed that the state $|\alpha,+\rangle$ is squeezed, i.e., the variance $(\Delta Y)^2$ is less than Heisenberg's minimal uncertainty 0.25. The maximum squeezing appears at $\alpha_0^2 = 0.64$, where $(\Delta Y)_0^2 = 0.111$. Beyond α_0^2, as α^2 increases, $(\Delta Y)^2$ monotonically increases to Heisenberg's minimal uncertainty 0.25.

2. Two-photon squeezed states

The correlated boson commutator (5) shows that in NC space there is a correlation between different degrees of freedom. Now we clarify the influence of (5) on two-photon squeezed states. For this purpose we first clarify the situation of two-photon squeezed states in commutative space.

From two-photon coherent state $|\alpha_1\rangle|\alpha_2\rangle$, $a_i|\alpha_1\rangle|\alpha_2\rangle = \alpha_i|\alpha_1\rangle|\alpha_2\rangle$ $(i = 1, 2)$ we can construct three types of two-photon squeezed states:

(i) $|\mathrm{I},\pm\rangle \equiv |\alpha_1,\pm\rangle|\alpha_2\rangle$,

(ii) $|\mathrm{II},\pm\rangle \equiv |\alpha_1,\pm\rangle|\alpha_2,\pm\rangle$,

(iii) $|\mathrm{III},\pm\rangle = c_{3\pm}\big(|\alpha_1,\alpha_2\rangle \pm |-\alpha_1,-\alpha_2\rangle\big)$. (11)

From the definitions the states $|\mathrm{I},\pm\rangle$ and $|\mathrm{II},\pm\rangle$ are normalized. For the states $|\mathrm{III},\pm\rangle$ the normalization constants $c_{3\pm}^2 = \exp(\alpha_1^2+\alpha_2^2)/\{2[\exp(\alpha_1^2+\alpha_2^2)\pm\exp(-\alpha_1^2-\alpha_2^2)]\}$.

For the degree of freedom i let $a_i = X_i + iY_i$. Where the variables X_i and Y_i satisfy $[X_i, Y_j] = i\frac{1}{2}\delta_{ij}$. In the state $|\mathrm{I},+\rangle$ the variances of X_1 and Y_1 of the photon 1 are the same as for a single squeezed state represented by Eq. (10): $(\Delta X_1)^2_{\mathrm{I}+} = \frac{1}{4} + \alpha_1^2\exp(\alpha_1^2)/[\exp(\alpha_1^2)+\exp(-\alpha_1^2)]$, $(\Delta Y_1)^2_{\mathrm{I}+} = \frac{1}{4} - \alpha_1^2\exp(-\alpha_1^2)/[\exp(\alpha_1^2)+\exp(-\alpha_1^2)]$; the photon 2 is a ‘spectator’ in a coherent state, the variances of X_2 and Y_2 are: $(\Delta X_2)^2_{\mathrm{I}+} = (\Delta Y_2)^2_{\mathrm{I}+} = \frac{1}{4}$. In the states $|\mathrm{II},+\rangle$ the photons 1 and 2 are independently squeezed: $(\Delta X_1)^2_{\mathrm{II}+} = (\Delta X_1)^2_{\mathrm{I}+}$, $(\Delta Y_1)^2_{\mathrm{II}+} = (\Delta Y_1)^2_{\mathrm{I}+}$; changing the subindex $1\to 2$, we obtain $(\Delta X_2)^2_{\mathrm{II}+}$ and $(\Delta Y_2)^2_{\mathrm{II}+}$. In the state $|\mathrm{III},+\rangle$ the variances of X_1 and Y_1 of the photon 1 are

$$(\Delta X_1)^2_{\mathrm{III}+} = \frac{1}{4} + \frac{\alpha_1^2\exp(\alpha_1^2+\alpha_2^2)}{\exp(\alpha_1^2+\alpha_2^2)+\exp[-(\alpha_1^2+\alpha_2^2)]}, \tag{12a}$$

$$(\Delta Y_1)^2_{\mathrm{III}+} = \frac{1}{4} - \frac{\alpha_1^2\exp[-(\alpha_1^2+\alpha_2^2)]}{\exp(\alpha_1^2+\alpha_2^2)+\exp[-(\alpha_1^2+\alpha_2^2)]}. \tag{12b}$$

Changing the subindex $1\to 2$, we obtain $(\Delta X_2)^2_{\mathrm{III}+}$ and $(\Delta Y_2)^2_{\mathrm{III}+}$. In the following we only consider the states $|\mathrm{I},+\rangle$ and $|\mathrm{III},+\rangle$.

3. Deformed two-photon squeezed states in NC space

From Eqs. (6) and (7) the dimensionless Hermitian quadratures of the deformed annihilation operator $\hat a_i = \hat X_i + i\hat Y_i$ are

$$\hat X_i = \xi\left[X_i - \frac{1}{2}\omega^2 c^{-2}\theta\epsilon_{ij}Y_j\right],$$
$$\hat Y_i = \xi\left[Y_i + \frac{1}{2}\omega^2 c^{-2}\theta\epsilon_{ij}X_j\right]. \tag{13}$$

The variances of $\hat X_i$ and $\hat Y_i$ in the state $|\mathrm{I},+\rangle$ are

$$(\Delta\hat X_1)^2_{\mathrm{I}+} = \xi^2\left[(\Delta X_1)^2_{\mathrm{I}+} + \frac{1}{16}\omega^4c^{-4}\theta^2\right],$$
$$(\Delta\hat Y_1)^2_{\mathrm{I}+} = \xi^2\left[(\Delta Y_1)^2_{\mathrm{I}+} + \frac{1}{16}\omega^4c^{-4}\theta^2\right], \tag{14}$$

$$(\Delta\hat X_2)^2_{\mathrm{I}+} = \frac{1}{4}\xi^2\left[1+\omega^4c^{-4}\theta^2(\Delta Y_1)^2_{\mathrm{I}+}\right],$$
$$(\Delta\hat Y_2)^2_{\mathrm{I}+} = \frac{1}{4}\xi^2\left[1+\omega^4c^{-4}\theta^2(\Delta X_1)^2_{\mathrm{I}+}\right]. \tag{15}$$

The variances of $\hat{X}_1$ and $\hat{Y}_1$ in the state $|\mathrm{III},+\rangle$ are

$$(\Delta\hat{X}_1)^2_{\mathrm{III}+} = \xi^2\left[(\Delta X_1)^2_{\mathrm{III}+} + \frac{1}{4}\omega^4 c^{-4}\theta^2(\Delta Y_2)^2_{\mathrm{III}+}\right],$$
$$(\Delta\hat{Y}_1)^2_{\mathrm{III}+} = \xi^2\left[(\Delta Y_1)^2_{\mathrm{III}+} + \frac{1}{4}\omega^4 c^{-4}\theta^2(\Delta X_2)^2_{\mathrm{III}+}\right]. \qquad (16)$$

Changing the subindex $1 \to 2$, we obtain $(\Delta\hat{X}_2)^2_{\mathrm{III}+}$ and $(\Delta\hat{Y}_2)^2_{\mathrm{III}+}$.

We notice that Eqs. (15) and (16) show correlating effects. For example, the variance $(\Delta\hat{X}_1)^2_{\mathrm{III}+}$ in the degree of freedom 1 includes a θ dependent variance $(\Delta Y_2)^2_{\mathrm{III}+}$ in the degree of freedom 2, etc.

Now we estimate the contribution of the correlating effects. There are different bounds of the scale of the parameter θ set by experiments: the estimation from the Lorentz symmetry violation [35], measurements of the Lamb shift [8] and clock-comparison experiments [36]. If we adopt the bound of the parameter $\theta/(\hbar c)^2 \leqslant (10^4\ \mathrm{GeV})^{-2}$ [8], and want the coefficient of the θ dependent term $\frac{1}{4}\omega^4 c^{-4}\theta^2 = \frac{1}{4}(\hbar\omega)^4(\theta/\hbar^2 c^2)^2$ in Eqs. (15) and (16) to reach a value, say $\sim 10^{-4}$, the corresponding frequency is $\nu \sim 10^{26}$ Hz.

The correlated boson commutator, exposed in the maintenance of Bose–Einstein statistics at the non-perturbation level described by deformed annihilation–creation operators, is a clear indication of NC effects. Any physical observable can be formulated by a bilinear representation of $\hat{a}_i$ and $\hat{a}_i^\dagger$, thus the ω^4 factor in the θ dependent term in Eqs. (15) and (16) is a general feature for processes with a single mode of frequency in NC space. We conclude that for exploring NC effects a process with a high frequency is favorable.

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