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ORIGINAL ARTICLE

Analytical investigation of the one dimensional heat transfer in logarithmic various surfaces

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Abstract The purpose of the present study was to investigate of the effect of temperature variation on the logarithmic surface. By using the appropriate similarity transformation for the generation components and temperature, the basic equations governing flow and heat transfer are reduced to a set of ordinary differential equations. These equations have been solved approximately subject to the relevant boundary conditions with numerical and analytical techniques. The reliability and performance of the present method have been compared with the numerical method (Runge–Kutta fourth-rate) to solve this problem. Then, LSM is used to solve nonlinear equation in heat transfer. This method is useful and practical for solving the nonlinear equation in heat transfer. It is observed that the obtained results by present analytical method are very close to result of the numerical method. Furthermore, the results show that the temperature profiles decreased by increasing the α number, and, temperature profiles increased by increasing the β number.

1. Introduction

In the heart of all the different engineering sciences, everything explains itself in some kinds of mathematical relations and most of these problems and phenomena are modeled by linear and nonlinear equations. Therefore, many different methods have been recently introduced to solve these equations. Analytical methods have made a comeback in research methodology after taking a backseat to the numerical techniques for the latter half of the preceding century.

Most scientific problems and phenomena such as heat transfer occur nonlinearly. Except a limited number of these problems, it is difficult to find the exact analytical solutions for them. Therefore, approximate analytical solutions are searched and were introduced, among which Adomian Decomposition Method (ADM) [1–3], Variational Iteration Method (VIM) [4–7], and Homotopy Perturbation Method (HPM) [8] are the most effective and convenient ones for both weakly and strongly nonlinear equations. Perturbation method [9] provides the most versatile tools available in nonlinear analysis of engineering problem, but its limitations restrict its application [10,11]. Perturbation method [12] is based on

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assuming a small parameter. The majority of nonlinear problems, especially those having strong nonlinearity, have no small parameters at all. Hajmohammadi and Nourazar have studied the solution of characteristic value problems arising in linear stability analysis. The results indicate that the present algorithm based on DTM could be used as a promising method for solving characteristic value problems [13]. There are some simple and accurate approximation techniques for solving nonlinear differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square Method (LSM) are examples of the WRMs which are introduced by Ozisik [14] for using in heat transfer problem. Stern and Rasmussen [15] used collocation method for solving a third order linear differential equation. Vaferi et al. [16] have studied the feasibility of applying Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Recently Hatami et al. [17] used LSM for heat transfer study through porous fins and also the problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field is investigated. Hatami and Ganji [18] found that LSM is more appropriate than other analytical methods for solving the nonlinear heat transfer equations. Recently several authors investigated about this subject and heat transfer [19–25].

In this paper, the least square method, as simple, accurate and computationally efficient analytical tools, is used to solve the nonlinear heat transfer equation. The accuracy of this method is demonstrated by comparing its results with this generated by numerical method.

2. Applications

2.1. Heat transfer problem description with various logarithmic surfaces

The one-dimensional heat transfer in a logarithmic various surface $A(x)$ and logarithmic various heat generation $G(x)$ was studied (Fig. 1). It is also assumed that the conduction coefficient, k, can be variable as a function of temperature.

The energy equation and the boundary conditions for this geometry are as follows:

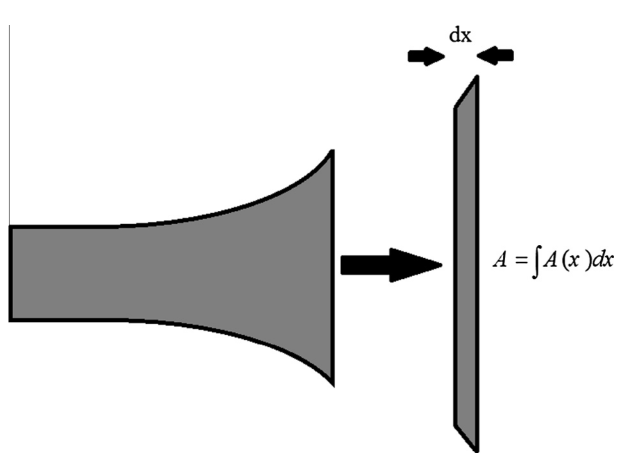

Figure 1 Geometry of the problem.

$$\frac{d}{dx}\left(k(T)\cdot A(x)\cdot\frac{dT}{dx}\right)+G(x)=0, \tag{1}$$

$$\{x=0\rightarrow T=T_0,\quad x=L\rightarrow T=T_L. \tag{2}$$

where

$$\{A(x)=A_0e^{\alpha x},\quad G(x)=G_0e^{-\alpha x}. \tag{3}$$

Assuming k as a linear function of temperature, we have

$$k(T)=k_0(1+\beta T). \tag{4}$$

Here, β shows the rate of effectiveness of temperature variation on thermal conductivity coefficient and k_0 is the thermal conductivity of the fin at the ambient.

After simplification, we have

$$\alpha\left(\frac{d}{dx}\theta(x)\right)+\alpha\cdot\beta\cdot T_0\cdot\theta(x)\cdot\left(\frac{d}{dx}\theta(x)\right)+\beta\cdot T_0\left(\frac{d}{dx}\theta(x)\right)^2 + \left(\frac{\partial^2}{\partial x^2}\theta(x)\right)+\beta\cdot T_0\cdot\theta(x)\left(\frac{\partial^2}{\partial x^2}\theta(x)\right)+c\cdot e^{-\alpha x}=0. \tag{5}$$

where

$$\theta=\frac{T}{T_0},\quad c=\frac{G}{k_0\cdot A_0\cdot\beta\cdot T_0}. \tag{6}$$

With making the boundary conditions dimensionless we have

$$x=0\ \rightarrow\ \theta=1,\quad x=L\ \rightarrow\ \theta=\frac{T_L}{T_0}=z. \tag{7}$$

And α, z, and L are constants to be determined through the initial conditions.

3. Describe least square method and applied to the problem

3.1. Describe least square method

As Fakour et al. [26] defined, least square method is one of the weighted residual methods which are constructed on minimizing the residuals of the trial function introduced to the nonlinear differential equation. For perception of the principle of LSM, consider a differential operator D is acted on a function u to produce a function p:

$$D(u(x))=p(x). \tag{8}$$

It is considered that u is estimated by a function, $\tilde{u}$ which is a linear combination of fundamental functions chosen from a linearly independent set. This is,

$$u\cong\tilde{u}=\sum_{i=1}^{n}c_i\varphi_i. \tag{9}$$

by substituting Eq. (9) into the differential operator, D, the result of the operations generally is not $p(x)$ and a difference will be appeared. Hence an error or residual will exist as follows:

$$R(x)=D(\tilde{u}(x))-p(x)\neq 0. \tag{10}$$

The main concept of LSM is to force the residual to zero in some average sense over the domain. So,

$$\int_x R(x)W_i(x)=0\quad i=1,2,\ldots,n. \tag{11}$$

where the number of weight functions W_i, is accurately equal the number of unknown coefficients c_i in $\tilde{u}$. The result is a set of n algebraic equations for the undefined coefficients c_i. If the continuous summation of all the squared residuals is minimized, the rationale behind the LSM's name can be seen, in other words, a minimum of

$$S = \int_x R(x)R(x)dx = \int_x R^2(x)dx. \tag{12}$$

In order to achieve a minimum of this function Eq. (12), the derivatives of S with respect to all the each unknown parameter should be zero, i.e.

$$\frac{\partial S}{\partial c_i} = 2\int_x R(x)\frac{\partial R}{\partial c_i}dx = 0. \tag{13}$$

Comparing with Eq. (13), the weighted functions for LSM will be,

$$W_i = 2\frac{\partial R}{\partial c_i}. \tag{14}$$

Because the "2" coefficient can be eliminated, it can be negligible in the equation. So the weighted functions, W_i, for the least square method are the derivatives of the residuals with respect to the unknown constants

$$W_i = \frac{\partial R}{\partial c_i}. \tag{15}$$

3.2. The LSM applied to the problem

It should be noted that the trial solution must satisfy the boundary conditions [14], so the trial solution can be written as

$$\theta(x) = \frac{e^{-\alpha L} - z}{-1 + e^{-\alpha L}} + \frac{(z-1)e^{-\alpha x}}{-1 + e^{-\alpha L}} + C_1(x - x^2) + C_2(x - x^3) + C_3(x - x^4). \tag{16}$$

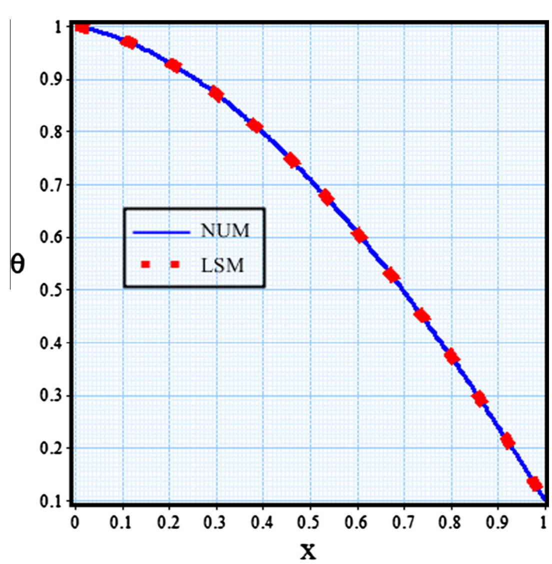

Figure 2 Comparison between results obtained via numerical solution and LSM at $\beta = 0$, $\alpha = 4$, $T_0 = 10$, $c = 2$, $z = 0.1$, $L = 1$.

Table 1 Comparison between the numerical results and LSM solution for $\theta(x)$ when $\beta = 0$, $\alpha = 2$, $T_0 = 10$, $c = 2$, $z = 0.1$, $L = 1$.

x	LSM	NUM
0.0	1.00000000000000	1.00000000000000
0.1	0.86482427174124	0.86482427193265
0.2	0.73931067033517	0.73931067054619
0.3	0.62439798224122	0.62439798377139
0.4	0.52036716645904	0.52036716651016
0.5	0.42704898501483	0.42704898401945
0.6	0.34397799491287	0.34397799521372
0.7	0.27050549573856	0.27050549714003
0.8	0.20588125772145	0.20588125814171
0.9	0.14931164297411	0.14931164310081
1.0	0.10000000000000	0.10000000000000

By introducing this equation into Eq. (4) residual function will be found and by substituting the residual function into Eq. (16) a set of equations with seven equations and seven unknown coefficients will be appeared and by solving this system of equations, coefficients C_1–C_3 will be determined. By using LSM, when $\alpha = 4$, $T_0 = 10$, $c = 2$, $z = 0.1$, $L = 1$ and $\beta = 0$ following equations will be determined for temperature distribution on logarithmic surface.

$$\theta(x) = -3.96499 + 4.96499e^{(-0.2x)} - 0.4251181388x^2 + 0.3097707x + 0.2755875x^3 - 0.1602401x^4. \tag{17}$$

4. Results and discussion

The objective of the present study was to apply least square method to obtain an explicit analytical solution for heat transfer equation of logarithmic surface profiles (Fig 1).

For showing the efficiency of applied analytical method a special case is considered and results are compared with numerical method as shown in Fig 2.

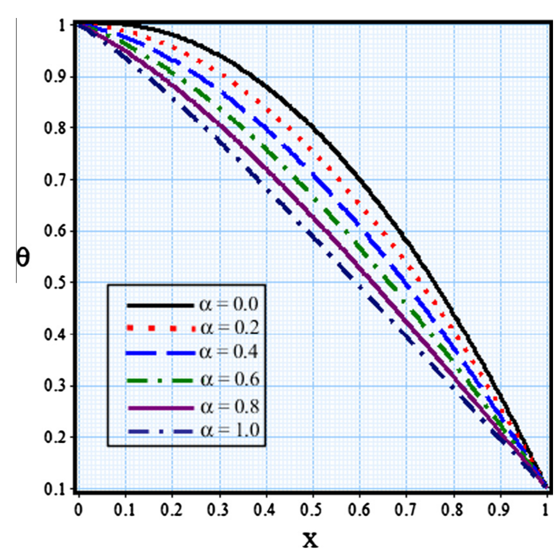

Figure 3 Effect of α on θ when $\beta = 0$, $T_0 = 10$, $c = 2$, $z = 0.1$, $L = 1$.

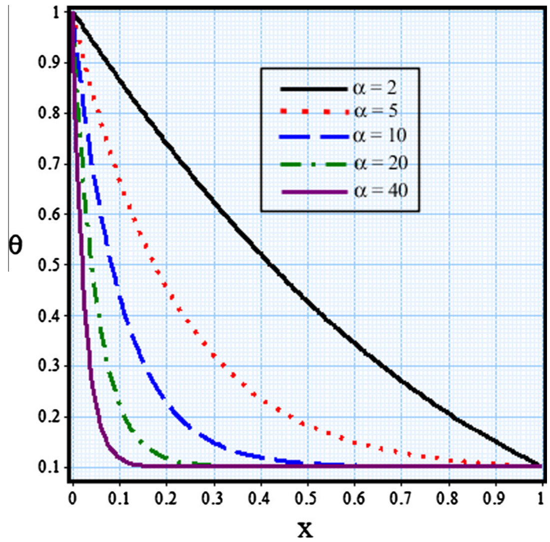

Figure 4 Effect of α on θ when $\beta = 0$, $T_0 = 10$, $c = 2$, $z = 0.1$, $L = 1$.

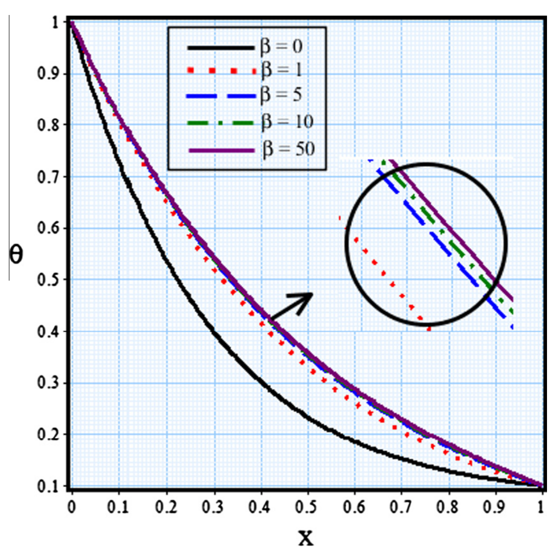

Figure 5 Effect of β on θ when $\alpha = 4$, $T_0 = 10$, $c = 2$, $z = 0.1$, $L = 1$.

The numerical solution is performed using the algebra package Maple 18.0. The package uses a fourth order Runge–Kutta procedure to solve nonlinear boundary value (B–V) problem. The validity of LSM is shown in Table 1. The graphical representation of obtained data shows that the results are precise and accurate in solving a wide range of mathematical and engineering problems especially fluid mechanic cases. This accuracy gives high confidence to us about validity of this problem and reveals an excellent agreement of engineering accuracy.

Moreover, Figs 3–6 present the effects of α and β on the temperature profile. Figs 3 and 4 show the effect of α on temperature profile. As seen in these figures by increasing α, temperature profiles decreased for the values of α in the range of $0 < \alpha < 1$, also by increasing α number, temperature profiles decreased for the values of α beyond 1.0. In addition, the

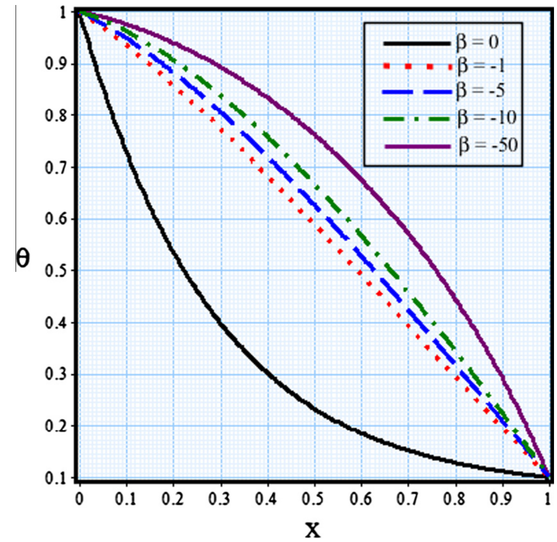

Figure 6 Effect of β on θ when $\alpha = 4$, $T_0 = 10$, $c = 2$, $z = 0.1$, $L = 1$.

dimensionless temperature distributions along the fin surface are depicted in Figs. 5 and 6, respectively. In the case of $\beta > 0$, the temperature distribution increases as x increases. For the case of $\beta < 0$, which is illustrated in Fig. 6, the temperature increases as x increases.

5. Conclusion

In this paper, the basic idea of the least square method is introduced and this method has been successfully applied to the governing differential equations of a selected geometry with a logarithmic various surface. The obtained results here were compared with the exact solutions. The results show that these methods enable to convert a difficult problem into a simple problem which can be solved easily. The important objective of our research is the examination of the convergence of LSM. The comparisons of the obtained results here provide more realistic solutions, reinforcing the conclusions about the efficiency of these methods. Therefore the LSM is powerful mathematical tools and can be applied to a large class of linear and nonlinear equations arising in heat transfer problems. Also by increasing α number, temperature profiles decreased and by increasing β number, temperature profiles increased.

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