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Maximum entropy principle for Kaniadakis statistics and networks

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ABSTRACT

In this Letter we investigate a connection between Kaniadakis power-law statistics and networks. By following the maximum entropy principle, we maximize the Kaniadakis entropy and derive the optimal degree distribution of complex networks. We show that the degree distribution follows $P(k) = P_0 \exp_\kappa(-k/\eta_\kappa)$ with $\exp_\kappa(x) = (\sqrt{1+\kappa^2 x^2} + \kappa x)^{1/\kappa}$, and $|\kappa| < 1$. In order to check our approach we study a preferential attachment growth model introduced by Soares et al. [Europhys. Lett. 70 (2005) 70] and a growing random network (GRN) model investigated by Krapivsky et al. [Phys. Rev. Lett. 85 (2000) 4629]. Our results are compared with the ones calculated through the Tsallis statistics.

1. Introduction

Over the last years, a lot of effort has been dedicated to studies of *networks*. In this concern, a network can be defined as a mathematical abstraction created to represent a relationship between objects. Usually, the objects are called *nodes* and the relationships are called *edges*. The number of edges owned by a node is referred as the node's degree and the networks are classified according to a distribution of that degree (also called connectivity). Considering its definition, the network can be used to model a great quantity of natural and artificial systems [1].

Primordially, networks without an evident organization were described with the random graph theory introduced by Erdös and Rényi (ER) [2]. The associated model, ER model, gives rise to random networks whose connectivity distributions $P(k)$ are Poisson distributions.

The technological advances allowed to study larger amounts of data and new conclusions were found about the apparently disordered networks. In 1999, Albert et al. reported that the WWW links connectivity distribution obeys a power law [3] which could indicate a subjacent organization in that network. In order to try to understand the mechanisms that could lead to a non-evident order, Barabási and Albert (BA) [4] introduced a model that presents two ingredients: growth and preferential attachment. By considering this model, BA have shown that the $P(k)$ decays as a power law for large k, independently of the nature of the system [5].

On the other hand, as it is well known, some restrictions to the applicability of the standard statistical mechanics have motivated investigations of non-standard statistics, both from theoretical and experimental viewpoints. In fact, the Tsallis nonextensive statistical mechanics [6] and the generalized power-law statistics developed by Kaniadakis [7] are the most investigated frameworks. Several consequences (in different branches) of the former framework have been investigated in the literature [8], which include the study of Tsallis statistics in the context of complex networks [9–11]. In this concern, the Thurner–Tsallis model [9] shows that growth is not necessary for having scale-free degree distributions. The Kaniadakis statistics in turn is characterized by a κ-entropy that emerges naturally in the framework of the so-called kinetic interaction principle [7]. Several physical features of a κ-distribution have also been theoretically investigated [12].

In this Letter, by following the maximum q-entropy method in the context of complex networks [10,11], we derive an optimal degree distribution which maximizes the κ-entropy based on the Kaniadakis statistics [7]. As an application, we analyze the κ- and q-degree distributions in two scale-free network model, e.g. the preferential attachment growth [11] and the growing random network (GRN) model [13,14].

This Letter is organized as follows. A brief summary of Kaniadakis statistics is presented in Section 2. In Section 3, we present the maximum entropy method for the calculation of optimal degree distribution in the context of Kaniadakis framework.

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In Section 4, by using the preferential attachment growth model and growing random network (GRN) model, we investigate the κ- and q-degree distributions. We summarize our main conclusions in Section 5.

2. Kaniadakis framework

Recent studies on the kinetic foundations of the so-called κ-statistics led to the power-law distribution function and the κ-entropy which emerge naturally in the framework of the kinetic interaction principle (see, e.g., Ref. [7]). Formally, the κ-framework is based on the κ-exponential and the κ-logarithm functions defined as [7]

$$\exp_\kappa(x) = \left(\sqrt{1+\kappa^2 x^2} + \kappa x\right)^{1/\kappa}, \tag{1}$$

$$\ln_\kappa(x) = \frac{x^\kappa - x^{-\kappa}}{2\kappa}, \tag{2}$$

with

$$\ln_\kappa\left(\exp_\kappa(x)\right) = \exp_\kappa\left(\ln_\kappa(x)\right) = x. \tag{3}$$

The κ-parameter belongs to the mathematical interval $|\kappa| < 1$ and in the case $\kappa = 0$ these expressions reduce to the usual exponential and logarithmic functions. The κ-entropy associated with the κ-framework is given by

$$S_\kappa = -\int d^3p f \ln_\kappa f \tag{4}$$

which fully recovers standard Boltzmann–Gibbs entropy, $S_{\kappa=0}(f) = -\int f \ln f d^3p$. As a matter of fact, the Kaniadakis entropy also can be a particular case of the Borges–Roditi entropy [15].

3. Maximum entropy method

3.1. Tsallis degree distributions

We recall the main aspects of the connection between the Tsallis statistics and complex networks. Specifically, the main result is the q-optimal degree distribution that maximizes the Tsallis entropy given by [6]

$$S_q = -\frac{1}{1-q}\left(1 - \sum_i p_i^q\right), \tag{5}$$

where q represents the entropic index and p_i the probability distribution of the state i. Such entropy reduces to the Boltzmann–Gibbs–Shannon in the limit $q \to 1$. Here, the q-degree distribution reads [10,11]

$$P(k) = P_0 \exp_q\left(-\frac{k}{\eta_q}\right), \tag{6}$$

where $\eta_q > 0$ defines the characteristics number of links, k is the connectivity and the q-exponential function is defined as

$$\exp_q(x) \equiv \left[1 + (1-q)x\right]^{\frac{1}{1-q}} \tag{7}$$

if $1 + (1-q)x > 0$ and zero otherwise.

3.2. New approach

Now, let us discuss the standard method of maximization of the Kaniadakis entropy. Here and hereafter, the Boltzmann constant is set equal to unity for the sake of simplicity. Thus, the functional entropy to be maximized is

$$\delta S_\kappa^* = \delta\left(S_\kappa + \alpha \sum_k P(k) + \beta \sum_k k P(k)\right) \tag{8}$$

where α and β are the Lagrange multipliers. The Kaniadakis entropy is given by [7]

$$S_\kappa = -\frac{1}{2\kappa}\sum_k \left[\frac{1}{1+\kappa} P(k)^{1+\kappa} - \frac{1}{1-\kappa} P(k)^{1-\kappa}\right], \tag{9}$$

and the above constraints used are the normalization of the degree distribution and the averaged coordination number

$$\sum_k P(k) = 1 \quad \text{and} \quad \sum_k k P(k) = \langle k \rangle. \tag{10}$$

By considering the same arguments of Ref. [10], we derive, after some algebra, the following expression for the κ-degree distribution

$$P(k) = P_0 \exp_\kappa\left(-\frac{k}{\eta_\kappa}\right), \tag{11}$$

with

$$\exp_\kappa\left(-\frac{k}{\eta_\kappa}\right) = \left[\sqrt{1 + \kappa^2\left(\frac{k}{\eta_\kappa}\right)^2} - \kappa\left(\frac{k}{\eta_\kappa}\right)\right]^{\frac{1}{\kappa}}. \tag{12}$$

Therefore, this new degree distribution, based on the Kaniadakis framework, is the power law that generalizes the exponential distribution. In particular, $\kappa \sim 0$ it behaves like the Tsallis degree distribution. Indeed, by using the asymptotic analytical behaviors of the q-exponential and κ-exponential functions, we obtain the following relation between the entropic parameters

$$\kappa = 1 - q, \tag{13}$$

where the Gaussian limits $\kappa = 0$ and $q = 1$ are satisfied simultaneously in (13).

4. Applications

4.1. The preferential attachment growth model

4.1.1. Numerical model

In order to test the viability of the new degree distribution [Eq. (11)], let us consider the preferential attachment growth model. In this regard, we use the same model proposed in Ref. [11] that considered the following rules for the growing of lattice:

1. First, one site is fixed ($i = 1$) at some arbitrary origin of the plane.
2. The second site ($i = 2$) is randomly and isotropically chosen at a distance r distributed according to the probability law

   $$P_G(r) \propto 1/r^{2+\alpha_G} \tag{14}$$

   with $\alpha_G \geqslant 0$ (G stands for *growth*). This second site is then linked to the first one.
3. To locate the next sites ($i = 3, 4, 5, \ldots, N$), the origin is moved to the barycenter of the existing sites and the distribution $P_G(r)$ is applied again from this new origin. The new site is now going to be linked to only one of the pre-existing sites in the lattice. To do this, it was used an attachment probability

   $$p_A = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N-1} k_j / r_j^{\alpha_A}} \tag{15}$$

   with $\alpha_A \geqslant 0$ (A stands for *attachment*), where r_i is the distance of the newly arrived site to the ith site of the pre-existing cluster, and the connectivity k_i is the number of links already arriving to the same ith site.

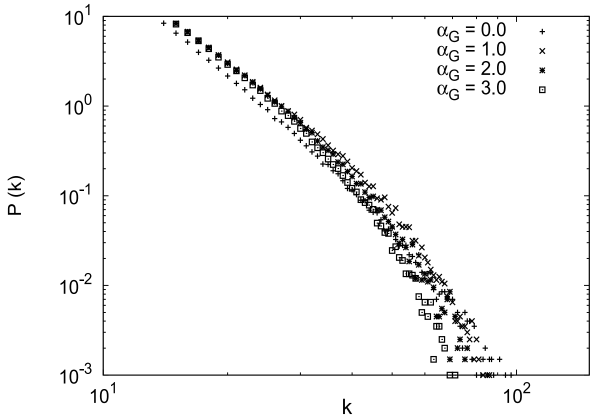

Fig. 1. This panel shows numerical curves for degree distribution for $\alpha_A = 2$ and typical values of α_G for networks of size $N = 10^4$ sites. This parameter has influence in the growth of network.

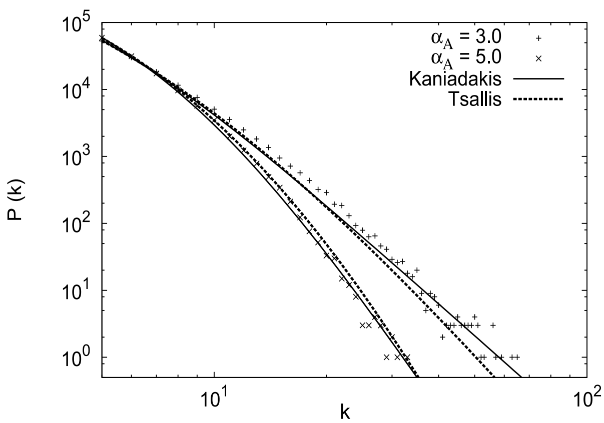

Fig. 2. Curves for degree distribution for typical values of α_A for networks of size 2×10^6. Points are our computer simulation results, solid lines are the best fit given by $P(k) = P_0 \exp_\kappa(-k/\eta_\kappa)$ and dashed lines are the best fit given by $P(k) = P_0 \exp_q(-k/\eta_q)$. This panel shows the log–log representation.

4. The earlier step (growth-attachment process) is sequentially repeated until the size wished of the lattice.

4.1.2. Results

Now, let us discuss the numerical results by implementing the Kaniadakis degree distribution, Eq. (11). In our simulations, we use the size of networks characterized by: (i) $N = 10^4$ and 2×10^3 samples for different values of α_G and $\alpha_A = 2$ and (ii) $N = 2 \times 10^6$ and 10 samples for different values of α_A and $\alpha_G = 2$. The rule of connection given by p_A, generates a competition between connectivity and distance between nodes. This competition breaks the hubs, by favoring a degree distribution more uniform on the lattice with increase of α_A and favors the Barabasi–Albert model, without metric, when the value of α_A tends to zero, by recovering completely this model when $\alpha_A = 0$.

Our numerical results are exhibited in Figs. 1 and 2. Here, we confirm that the α_G parameter controls the metrics of the emerging cluster and presents some influence on the κ- and q-degree distribution. On the other hand, these distributions are greatly influenced by the α_A parameter. Fig. 2 also shows that when α_A increases, the degree distribution $P(k)$ does obey a distribution like the Kaniadakis distributions given by Eq. (11); for completeness, we also show the Tsallis degree distribution, given by Eq. (6). The Kaniadakis degree distribution with $\kappa = 0$ provides an exponential behavior whereas for other values of κ admit a wider class of the power-law distributions.

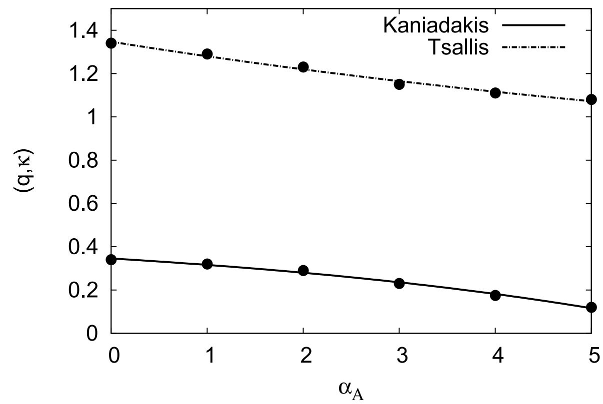

Fig. 3. Expressions $\kappa = \kappa(\alpha_A)$ and $q = q(\alpha_A)$ were obtained in the best fit calculated in Fig. 2. We can see an exponential decreasing behavior with different curvatures expressed by solid curve $\kappa = 0.410 - 0.134e^{0.200\alpha_A}$ and the dashed curve $q = 0.675 + 0.671e^{0.105\alpha_A}$.

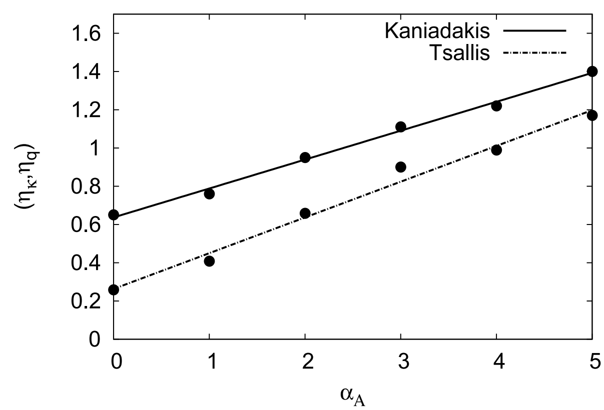

Fig. 4. $\eta = \eta(\alpha_A)$ used in the best fit calculated in Fig. 2. We can see a linear behavior in the interval $0 < \alpha_A < 5$, given by the solid curve $\eta_\kappa = 0.637 + 0.151\alpha_A$ and the dashed curve $\eta_q = 0.263 + 0.187\alpha_A$.

In Fig. 3, we compare the κ- and q-freedom distributions from a parametric spaces viewpoint. As result from the numerical simulations, we show the entropic parameters as function of α_A, as well as the function $\eta = \eta(\alpha_A)$. These simulations have shown that the function $\kappa(\alpha_A)$ and $q(\alpha_A)$ decay exponentially (see Fig. 3) whereas the function $\eta(\alpha_A)$ increases linearly (Fig. 4). The best fit for κ and q are given by $\kappa = 0.410 - 0.134e^{0.200\alpha_A}$ ($\eta_\kappa = 0.637 + 0.151\alpha_A$) and $q = 0.675 + 0.671e^{-0.105\alpha_A}$ ($\eta_q = 0.263 + 0.187\alpha_A$), respectively. By eliminating α_A of the above expressions, i.e. $q = 0.671\,(\frac{0.134}{0.401-\kappa})^{0.525} + 0.675$ and by using the asymptotic analytical behavior given by Eq. (13), we obtain $q = 1.048$. Therefore, we see that q and κ are correlated through the power law.[1]

Here, let us comment about the numerical constraint on the above parameters discussed. As one may be easily checked, for $\alpha_A = 0$, we have $\kappa_{\max} = 0.346$. On the other hand, in order to be mathematically consistent with the Kaniadakis statistics, we have that the entropic parameter must be constrained to interval $|\kappa| < 1$. Thus, by combining these results, it is possible to obtain the constraints: $\kappa \in [-1; 0.346]$, $\alpha_A \in [0; 12]$ and $\eta_\kappa \in [0.637; 2.451]$. Therefore, the Albert–Barabasi model is given by

[1] In contrast with the present investigation and considering other physical context, we stress that the linear relation between q and κ was also obtained through from the fit of radial velocity distributions for the data of 14 stellar open cluster [16]. In addition, a linear relation between q and κ was also calculated for the stellar polytropes [17].

degree distribution with $\kappa = 0.346$ and characteristics number of links $\eta_\kappa = 0.637$, i.e. $P(k) = P_0 \exp_{0.346}(-k/0.637)$.

It is worth emphasizing that a growth model by considering preferential attachment has been analytically and numerically studied in Ref. [5]. In this work, the degree distribution calculated analytically is precisely of the form q-exponential [11]. From the κ-framework viewpoint, it is not different, the expression (11) is also similar to Albert–Barabasi distribution [5] in the limit that the entropic parameter $\kappa \ll 1$. In this asymptotic regime, the κ-entropic index is given by $\kappa = m/[m(2r-3) - 1 + p + r]$, where (m, p, r) are parameters of the Albert–Barabasi model.

4.2. Growing network model

4.2.1. Analytical model

In this section, let us introduce the main aspects of growing network (GN) model investigated analytically in Refs. [13,14]. This model is based on an investigation of the rate equations for the densities of nodes of a given degree, i.e. nodes are added one at a time, and a link is established with pre-existent node following a attachment probability A_k relative of the degree of target node.[2]

The GN model presents the so-called the attachment kernel A_k as the probability that the newly added node links to a pre-existing node which already has k links. For the growing of the network, a degree distribution $N_k(t)$, i.e. the average number of nodes with k links accumulated. Krapivsky et al. [13,14] have introduced the general homogeneous model with $A_k = k^\gamma$ [3] and they also suggested that the degree distribution $N_k(t)$ crucially depends on the value of γ (for details on the dependence of $N_k(t)$ with γ, see Refs. [13,14]).

The time evolution of the degree distribution of the GN model is governed by rate equations given by

$$\frac{dN_k}{dt} = \frac{1}{A}[A_{k-1}N_{k-1} - A_k N_k] + \delta_{k1}. \tag{16}$$

In this equation, the first term on the right side means the process in which a node $k-1$ links is connected to a new node with a probability A_{k-1}/A normalized by factor $A(t) = \sum_{j\geqslant 1} A_j N_j(t)$. The second term presents a corresponding role to first one. Finally, the latter one represents the continuous introduction of new nodes with no incoming links. By assuming that the degree distribution and $A(t)$ grow linearly with time, and substituting

$$N_k(t) = t n_k, \tag{17}$$

and

$$A(t) = \mu t, \tag{18}$$

into Eq. (16), Krapivsky et al. [14] have calculated a recursing relation with a solution for n_k given by

$$n_k = \frac{\mu}{A_k}\prod_{j=1}^{k}\left(1 + \frac{\mu}{A_j}\right)^{-1}. \tag{19}$$

The complete solution is calculated using the definition of the amplitude, i.e. $\mu = \sum_{j\geqslant 1} A_j n_j$. The amplitude always depends on the entire attachment kernel.

[2] This growing model can be exemplified as the distributions of scientific citations and the structure of the worldwide web. From the citations viewpoint, is possible to interpret these nodes as publications, and the directed link from one paper to another as a citation to the earlier publication.

[3] Considering the value of γ, we have: (i) Sublinear kernels corresponds to $0 < \gamma < 1$ and are asymptotically homogeneous, (ii) Asymptotically linear kernels $\gamma = 1$ and (iii) Superlinear homogeneous kernels with $\gamma > 1$.

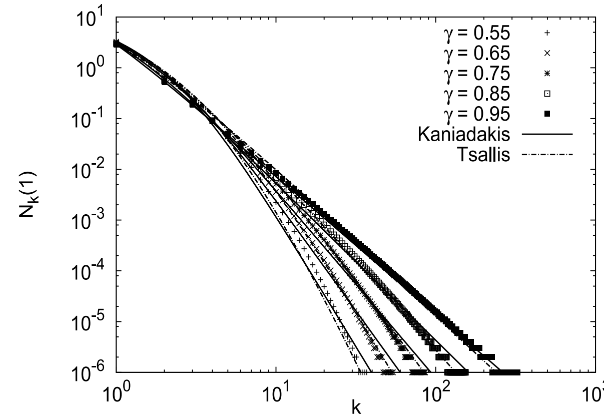

Fig. 5. Analytical curves for the degree distribution in the interval $1/2 < \gamma < 1$. Points are obtained of the analytical expression $N_k(t = 1)$ vs k. The solid lines are the best fit given by $N_k(1) = N_0 \exp_\kappa(-k/\eta_\kappa)$ and dashed lines are the best fit given by $N_k(1) = N_0 \exp_q(-k/\eta_q)$. This panel shows the log–log representation.

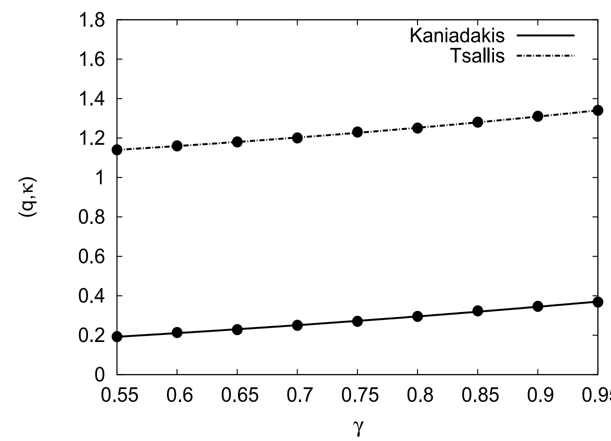

Fig. 6. Expressions $\kappa = \kappa(\gamma)$ and $q = q(\gamma)$ were obtained in the best fit calculated in Fig. 5. We can see an exponential behavior increasing smoothly unlike that one shown in Fig. 5. The exponential behavior is given by the best fits expressed by solid curve $\kappa = -0.179 + 0.216e^{0.983\gamma}$ and the dashed curve $q = 0.868 + 0.127e^{1.382\gamma}$.

4.2.2. Results

Here, we investigate the cases $\gamma < 1$ and $\gamma = 1$, also called sublinear and linear kernels, respectively. In the first one, we have $A_k \sim k^\gamma$, with $0 < \gamma < 1$. Furthermore, following the steps of Refs. [13,14], is possible to show that

$$n_k \sim k^{-\gamma}\exp\left[-\mu\left(\frac{k^{1-\gamma} - 2^{1-\gamma}}{1-\gamma}\right)\right] \tag{20}$$

for $\frac{1}{2} < \gamma < 1$. In order to compare with the κ- and q-degree distributions given by Eqs. (6) and (11), we consider only this case.

Looking at the expression (20), we plot n_k against k to different values of γ in the interval $0.5 < \gamma < 1$. We observe that n_k decreases exponentially with k irrespective t, therefore, we use $N_k(t) = n_k$, i.e. $t = 1$ in the expression (17). In Fig. 5, we show that Kaniadakis and Tsallis expressions, $N_k(1) = N_0 \exp_\kappa(-k/\eta_\kappa)$ and $N_k(1) = N_0 \exp_q(-k/\eta_q)$ provide fits for the GNR model, however we observe that some of the fittings are not very satisfactory. It is important to stress that is the first time that the Tsallis framework is investigated in the context of the Analytical Growing Network Model [13,14].

In Fig. 6 we compare both the Kaniadakis and Tsallis parameters as function of γ. These functions increase smoothly following the best fit given by $\kappa = -0.179 + 0.216e^{0.983\gamma}$ and $q = 0.868 + 0.127e^{1.382\gamma}$. Already the functions $\eta_\kappa(\gamma)$ and $\eta_q(\gamma)$ show two regimes. As we can see in Fig. 7, there exist a plateau for interval

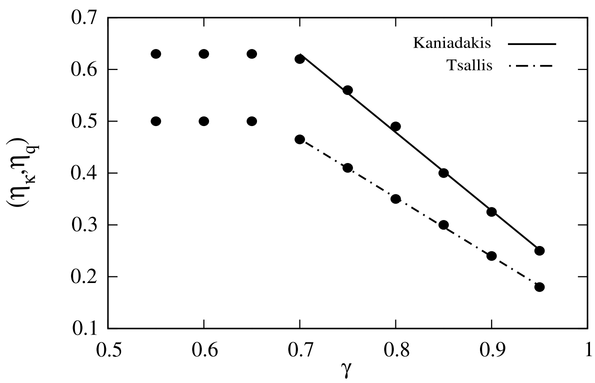

Fig. 7. $\eta = \eta(\gamma)$ used in the best fit calculated in Fig. 5. In the interval $0.5 < \gamma < 0.70$, we can see a plateau indicating that the characteristic number of link is a characteristic number. For interval $0.70 \leqslant \gamma < 1.0$, we have a decreasing linear regime expressed by the solid curve $\eta_\kappa = 1.689 - 1.511\gamma$ and the dashed curve $\eta_q = 1.260 - 1.134\gamma$ for Kaniadakis and Tsallis degree distribution, respectively.

$0.5 < \gamma < 0.70$, indicating that the number of links is given by a characteristic number. In this regime, the network behaves like a random classical network. For the interval $0.70 \leqslant \gamma < 1.0$, we have a decreasing linear regime expressed by $\eta_\kappa = 1.689 - 1.511\gamma$ and $\eta_q = 1.260 - 1.134\gamma$ for Kaniadakis and Tsallis degree distributions, respectively. Finally, eliminating γ of the above expressions, we have that $q = 0.127(\frac{\kappa+0.179}{0.216})^{1.406} + 0.868$. Here, considering the asymptotic analytical behavior expressed by $\kappa = 1 - q$, we obtain $q = 0.966$.

5. Summary and conclusions

In this Letter, we have investigated the effects of the Kaniadakis framework in the context of the complex networks. From the analytical viewpoint, the new κ-degree distribution has been calculated via the maximum-entropy method.

In order to check our proposal we have considered the viability of Kaniadakis degree distribution within the context of the preferential attachment growth model [11] and the growing network model [13,14]. In this regard, we have compared our results with ones calculated through the Tsallis framework, and we have observed that likely the Tsallis degree distribution, the Kaniadakis one was substantially influenced by α_A and independent of value of α_G.

Additionally, we have shown that the Kaniadakis degree distribution is numerically consistent with the κ-exponential function that emerges naturally in the frame of Kaniadakis statistics [7]. It was also shown that for $\alpha_A = 0$, the κ-degree distributions with $\kappa = 0, 346$ also belongs to the same universal class that the Barabasi–Albert model belongs. As discussed earlier, the combination of statistical and numerical constraints, i.e., $|\kappa| < 1$ and $\alpha_A = 0$ have provided the constraints $\kappa \in [-1; 0.346]$, $\alpha_A \in [0; 12]$ and $\eta_\kappa \in [0.637; 2.451]$.

Considering the analytical growing network (GN) model, we also discussed that the degree distribution N_k for the in the interval $0.5 < \gamma < 1.0$ is also consistent with both the κ and q-exponential functions. However, when we have compared the behavior of entropic parameters κ and q, as function of α_A and γ, the exponent γ has provided an inverse effect that one presented by exponent α_A.

Finally, it is worth mentioning that by comparing Tsallis and Kaniadakis degree distributions (Eqs. (6) and (11)) in the context of the preferential attachment growth model and growing network model (Figs. 2 and 5), they have furnished a very similar behavior, i.e., it decays as the power law. However, some of the fittings in Figs. 2 and 5 were not very satisfactory, mainly for $\alpha_A = 3.0$ in Fig. 2 and the fittings for $\gamma = 0.55, 0.65, 0.75$ and 0.85 in Fig. 5.

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