A study on the flexible multibody dynamic modeling of an axial piston pump considering housing flexibility

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Abstract

Axial piston type hydraulic pump is an important source of fluid power in hydraulic systems. In the present work, a methodology for modeling the dynamics of the pump in multibody framework considering component flexibility is studied. A multibody dynamic model of the pump is developed considering housing flexibility and pressure forces acting on the pistons. The flexibility of the housing is modeled using the relative nodal coordinate formulation (RNCF) with a floating frame of reference. The bearing forces, control piston force, swashplate motion and torque ripple are studied. The bearing and control piston force results are compared with the case of a rigid housing assembly. The experiments are conducted to measure the end cover vibration levels during operation of the pump. The vibration levels are also obtained from the simulation and compared with experiments. Numerical studies are conducted to better understand the influence of housing flexibility on the dynamics of the pump.

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Keywords: Axial piston pump; Flexible multibody dynamics; Vibration;

1. Introduction

The generation of vibration is an important consideration in the design and development of an axial piston pump. Figure (1) shows a schematic representation of an axial piston pump. This type of pump consists of pistons which translate in a cylinder barrel causing the displacement of the hydraulic fluid assuming a swashplate displacement angle. In previous studies [2,3], the dynamics of the pump have been studied considering only the internal

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components. The influence of housing flexibility has not been studied. The modeling and investigation of the dynamic characteristics of rotating machines is an important area of interest in engineering applications. A detailed understanding of the mechanism of generation and propagation of vibration help in the identification of critical factors influencing the vibration behavior and development of active and passive control methods. An axial piston type rotating machine is used as a source of fluid power in the hydraulic systems to meet the variable flow and high pressure requirements. The axial piston pump is a complex rotating machine consisting of pistons which undergo large translations relative to the rotor.

![Axial Piston Pump Diagram](image)

**Fig. 1.** A schematic representation of an axial piston pump [1].

In the present work, the multibody dynamic modeling of an axial piston pump with flexible housing assembly is studied. A numerical multibody model of the pump is developed with rigid internal components and housing assembly modeled as a flexible body. The multibody model is used to study the linear modes of the pump assembly, bearing forces, control forces, torque ripple, swashplate oscillations, and end cover acceleration. The end cover acceleration levels are compared with experimental measurements.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>([M_r])</td>
<td>Mass-inertia matrix in rigid MBD</td>
</tr>
<tr>
<td>([C_{qr}])</td>
<td>Constraint Jacobian matrix in rigid MBD</td>
</tr>
<tr>
<td>([q_r])</td>
<td>Generalized coordinates in rigid MBD</td>
</tr>
<tr>
<td>([\lambda_r])</td>
<td>Lagrange Multipliers in rigid MBD</td>
</tr>
<tr>
<td>([F_r])</td>
<td>External forces in rigid MBD</td>
</tr>
<tr>
<td>([Q_{dr}])</td>
<td>Quadratic velocity vector in rigid MBD</td>
</tr>
<tr>
<td>([U_i])</td>
<td>Interior displacements</td>
</tr>
<tr>
<td>([U_b])</td>
<td>Boundary displacements</td>
</tr>
<tr>
<td>([\Phi_f])</td>
<td>Fixed interface modes</td>
</tr>
<tr>
<td>([\Psi])</td>
<td>Constraint modes</td>
</tr>
<tr>
<td>([q_f])</td>
<td>Generalized coordinates of flexible MBD component</td>
</tr>
<tr>
<td>([M_f])</td>
<td>Mass-inertia matrix of flexible MBD component</td>
</tr>
<tr>
<td>([K_f])</td>
<td>Stiffness matrix of flexible MBD component</td>
</tr>
<tr>
<td>([D_f])</td>
<td>Damping matrix of flexible MBD component</td>
</tr>
<tr>
<td>([C_{qf}])</td>
<td>Constraint Jacobian matrix in flexible MBD</td>
</tr>
<tr>
<td>([\lambda_f])</td>
<td>Lagrange Multipliers in flexible MBD</td>
</tr>
<tr>
<td>([F_f])</td>
<td>External force vector in flexible MBD</td>
</tr>
</tbody>
</table>
2. Methodology

The rigid and flexible multibody dynamic modeling are the two important approaches in the multibody dynamics framework. In the present work, a combined rigid and flexible multibody modeling approach is adopted for developing the dynamic model of the pump. The figure (2) shows the schematic representation of the numerical modeling methodology.

In the rigid multibody dynamic modeling, the governing dynamic equations are developed using augmented Newton-Euler formulation with Lagrange multipliers[5]. This formulation leads to symmetric coefficient matrices with a sparse structure. The general form the equations is given by,

\[
\begin{bmatrix}
M_{qr} & C_{qr}^T \\
C_{qr} & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_r \\
\lambda_r
\end{bmatrix}
=
\begin{bmatrix}
F_r \\
Q_{dr}
\end{bmatrix}
\]

(1)

In the flexible multibody dynamic modeling approach, the equations of motion are developed using the relative nodal coordinate formulation (RNCF) with a floating frame of reference[4]. The component modes for the flexible body representation are obtained using the Craig-Bampton method[6]. In the Craig-Bampton method a finite element model of the flexible body is developed and the structural matrices are partitioned according to the boundary degrees of freedom and interior degrees of freedom. The undamped equation of motion is given by,

\[
\begin{bmatrix}
\bar{M}_{II} & \bar{M}_{IB} \\
\bar{M}_{BI} & \bar{M}_{BB}
\end{bmatrix}
\begin{bmatrix}
\ddot{\bar{U}}_I \\
\ddot{\bar{U}}_B
\end{bmatrix}
+
\begin{bmatrix}
\bar{K}_{II} & \bar{K}_{IB} \\
\bar{K}_{BI} & \bar{K}_{BB}
\end{bmatrix}
\begin{bmatrix}
\bar{U}_I \\
\bar{U}_B
\end{bmatrix}
=
\begin{bmatrix}
\bar{F}_I \\
\bar{F}_B
\end{bmatrix}
\]

(2)

The fixed interface modes are obtained by solving the eigenvalue problem \([\bar{K}_B][\Phi_k]=[ar{M}_B][\Phi_k][\Lambda]\) and the constraint modes are obtained as \([\Psi]=-\bar{K}_B\bar{K}_B^{-1}\bar{K}_B\bar{K}_B^{-1}\). The parameter k refers to the number of fixed interface modes. The constraint modes are the static reactions of the interior degrees of freedom, due to a unit displacement of one boundary degree of freedom and zero displacement of the remaining boundary degrees of freedom. Thus the transformation matrix \([B]\) is obtained which transforms the physical coordinates \([U_I]\) to the generalized flexible body coordinates \([\xi]\) as \([U_I]=[B][\xi]\). Thus, the equations of motion for the flexible body in the generalized coordinates is given by,
The equations (1) and (3) together constitute the governing equations of a combined rigid and flexible multibody system.

At the initial time $t=0$, the generalized position and velocity coordinates are known as the result of the initial conditions. Since the total vector of the generalized system coordinates are known, the equations of motion are solved to determine unknown accelerations and Lagrange multipliers. The vector of Lagrange multipliers are used to determine the generalized constraint reactions. The constraint Jacobian matrix is factorized using LU decomposition to identify a set of independent coordinates. The independent accelerations are to define the state-space equations which are numerically integrated to find the independent generalized position and velocity coordinates at a later time $t$. Using the values of the independent coordinates, the non-linear system of algebraic equations are solved using the Newton-Raphson algorithm to obtain the dependent coordinates. Using the values of independent velocities, the linear system of algebraic equations are solved to obtain the dependent velocities. Thus, the total vector of system coordinates are obtained at time $t$. Further, this process is continued until the final simulation time $t=t_f$.

The transformation from the time domain to frequency domain is performed using the Short Term Fourier Transform (STFT) algorithm. The number of points for the STFT are chosen considering the frequency range of interest and the Nyquist criterion. A rectangular type windowing function is used as it has coherence gain of 1.0 and does not need additional multiplication correction factors as required by other windowing functions. Thus, by adopting the above DFT procedure, the vectors consisting of constraint force $\{F_c\}$ or displacements $\{r\}$ at different time points obtained from the multibody simulation are converted into the spectrum $\{\hat{F}_c\}$ and $\{\hat{r}\}$ respectively in the frequency domain.

Figure (3) shows the multibody dynamic model of the axial piston pump. The internal and external components of the pump are considered in the development of the multibody dynamic model. The primary internal components of the pump viz., piston, shoe, swashplate, driveshaft and cylinder barrel are considered and treated as rigid bodies. The housing, end cover and end cover fittings are the primary external components which are considered together as a single flexible body representing the housing assembly. The flexible body representation of the housing assembly is obtained by developing a finite element model of the housing assembly and performing the Craig Bampton component mode synthesis to obtain the component modes. The housing and end cover are modeled using solid elements and the end cover fittings at the inlet and outlet ports are modeled as point concentrated masses. The fitting masses are connected to the corresponding inlet and outlet port nodes using the rigid multipoint constraints. The housing and end cover are connected at four bolt locations. The modeling of the materials in the pump is carried out using linear isotropic elastic properties. In the pump, the housing is made of ductile iron ($E_h, \rho_h, \mu_h$) and end cover is made of gray cast iron ($E_{ec}, \rho_{ec}, \mu_{ec}$). The boundary nodes for Craig Bampton synthesis are defined at the center of the high pressure swash bearing, low pressure swash bearing, driveshaft bearing in the housing, driveshaft bearing in the end cover, center of the housing flange and control piston location. The boundary nodes are connected to the
corresponding node sets in the pump finite element model using rigid multipoint constraints. The component modes of the housing assembly are obtained using Craig-Bampton technique with the boundary nodes as specified above and the number of fixed interface modes \( k=10 \).

The constraints are defined in the pump multibody model considering the nature of interaction between the different components. Table 1 lists the different constraints defined in the multibody model.

<table>
<thead>
<tr>
<th>Constraint type</th>
<th>Part ( i )</th>
<th>Part ( j )</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>Piston</td>
<td>Shoe</td>
<td>Ball end of piston</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>Piston</td>
<td>Cylinderbarrel</td>
<td>Center of cylinderbarrel bore</td>
</tr>
<tr>
<td>Planar</td>
<td>Shoe</td>
<td>Swashplate</td>
<td>Center of shoe</td>
</tr>
<tr>
<td>Fixed</td>
<td>Cylinderbarrel</td>
<td>Driveshaft</td>
<td>Driveshaft spline</td>
</tr>
<tr>
<td>Revolute</td>
<td>Driveshaft</td>
<td>Housing</td>
<td>End cover bearing, Housing bearing</td>
</tr>
<tr>
<td>Spherical</td>
<td>Control piston</td>
<td>Swashplate</td>
<td>Cam surface of the swashplate</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>Swashplate</td>
<td>Housing</td>
<td>Center of HP/LP Swash bearing</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>Control piston</td>
<td>Housing</td>
<td>Center of mass of control piston</td>
</tr>
<tr>
<td>Fixed</td>
<td>Housing</td>
<td>Ground</td>
<td>Center of housing flange</td>
</tr>
</tbody>
</table>

The pressure forces are the external forces acting on the pumping mechanism. The pressure forces acting on the piston vary with the angle of rotation of the driveshaft. Figure (4) shows a typical pressure force variation acting on the piston initially located at the center of low-pressure inlet side. The pressure variation acting on the individual pistons is obtained at discrete points from a 1D AMESim simulation. The discrete pressure variation data is used for defining a cubic spline interpolation function.

![Fig. 4. Variation of the normalized pressure forces with angle.](image-url)

The linear states analysis and transient simulation are carried out using the multibody model. The linear states analysis is carried out at \( t=0.0 \) seconds to determine the eigen values and mode shapes of the linearized model at \( t=0.0 \) seconds. The transient simulation is conducted for time duration \( t=1.35 \) seconds (corresponding to fifty revolutions at 2200 RPM) with output at 51200 time points. The numerical integration of the equations of motion is
carried out using the multistep backward difference formula method with normalized error tolerance of $1.0 \times 10^{-6}$ and normalized maximum step size of $1.96 \times 10^{-5}$. The bearing forces, control piston force, torque ripple, swashplate motion, and acceleration levels at the end cover are obtained from the numerical simulations.

The different bearing and control piston constraint forces are the vertical end cover bearing force ($F_1$), vertical housing bearing force ($F_2$), horizontal high pressure swash bearing force ($F_3$), horizontal low pressure swash bearing force ($F_4$), vertical high pressure swash bearing force ($F_5$), vertical low pressure swash bearing force ($F_6$) and control piston force ($F_7$). The Figure (5) shows the graphical representation of the locations and directions of the seven constraint forces responsible for the dynamic excitation of the pump. The torque ripple is obtained on the pump driveshaft about its axis of rotation. A local coordinate system attached to the swashplate is created at the center of the swash plate to study the motion of the swashplate. The angular oscillations of the swashplate about the global X, Y, and Z directions are obtained in the time domain using the local coordinate system. Similarly, a local coordinate system on the flexible housing assembly is created at the end cover location to obtain the accelerations along the X, Y, Z directions. The overall acceleration level at the end cover is obtained by the square root of the sum of squares of frequency domain accelerations along X, Y, and Z directions.

Fig. 5. The different constraint reaction forces determined in the pump.

3. Results and discussion

Figure (6) shows the dynamic excitation forces at the bearing and control piston locations in the frequency domain at the 9th, 18th, 27th and 36th orders. The figure (6a) shows the dynamic excitation forces obtained by considering the housing assembly to be rigid and figure (6b) shows the results obtained by considering the housing assembly to be flexible.

Fig. 6. Comparison of frequency spectrum of constraint forces obtained with (a) rigid housing (b) flexible housing at the four pumping orders at 2200 RPM.
It is observed that there are differences in the housing and end cover bearing force magnitudes between the rigid and flexible multibody model. In the rigid multibody model the housing and end cover bearing forces have almost identical magnitudes whereas in the flexible multibody model the housing bearing forces are larger than the end cover bearing forces. This is because of the dynamic stiffness effect. The housing bearing is located closer to pump flange location than the end cover bearing. Since the pump is bolted at the flange locations, the stiffness at the housing bearing is greater than the end cover bearing. Thus, the flexibility of the housing assembly leads to a redistribution of the forces at the bearing locations. Figure (7) shows the variation of the angular oscillations of the swashplate about the X axis which is perpendicular to the driveshaft axis. This is because the flexibility of the housing assembly leads to vibrations at the swashplate bearing locations which further cause the swashplate to oscillate together with the housing.

![Swashplate motion about the axis perpendicular to the driveshaft.](image)

Fig. 7. Swashplate motion about the axis perpendicular to the driveshaft.

The torque ripple of the pump is one of the important characteristics of the dynamic performance of the pump. Figure (8) shows the torque ripple of the pump for the rigid and flexible multibody dynamic models in the frequency domain. The flexibility of the housing is found to maximum influence on the torque ripple at the 9th order. It is also found that the torque ripple has a peak between the 9th and 18th order frequencies in flexible multibody dynamic model.

Figure (9) shows the variation of the overall acceleration levels at the end cover location at the 9th, 18th, 27th and 36th orders. It is seen that the overall acceleration levels predicted by the simulation agree well with the experimental results at the 9th, 18th and 27th orders. However, at the 36th order the simulation results are significantly higher than the experiment. In order to understand the over-prediction by the simulation results, the component modes are studied. The linear modes are determined at the initial time t=0.0 secs. The first 18 eigen values are obtained to be zero. This corresponds to the 18 degrees of freedom of the pumping mechanism.

![Comparison of the frequency spectrum of the torque ripple for rigid and flexible housing.](image)

Fig. 8. Comparison of the frequency spectrum of the torque ripple for rigid and flexible housing.
The figure (10) shows the mode shapes of the 19th, 20th, 21st and 22nd modes. The four mode shapes are vertical bending, lateral bending, torsional and axial modes of the pump assembly. Thus, the axial mode of the pump assembly is found to be excited at 36th order frequency leading to the larger acceleration levels in the simulation.

(a) (b) (c) (d)

Fig. 10. The first four mode shapes with non-zero frequency obtained from the linear states analysis of the flexible multibody model (a) Mode 28, (b) Mode 29, (c) Mode 30, and (d) Mode 31.

4. Conclusions

In the present work a multibody dynamic model of an axial piston pump with flexible housing assembly is developed. The multibody model is used to predict the linear modes, bearing forces, control forces, torque ripple, swashplate oscillations, and end cover vibration levels. The comparison of the rigid and flexible multibody dynamic modeling showed an increase in magnitude of the housing bearing forces and reduction in the end cover force magnitudes due to the flexibility of the housing assembly. The vibration levels predicted from the simulation were compared with the experiment and the linear modes analysis is found to be helpful in explaining the vibration behavior. It is seen that the vibration levels predicted by the simulation agree well with the experimental results at the 9th, 18th and 27th orders. However, the excitation of the axial mode of the pump assembly at the 36th order causes the predictions of the simulation to be higher than the experiment. The numerical studies are being conducted with the axial piston pump multibody model with flexible housing assembly to study the influence of stiffness, mass and damping properties on the vibration behavior of the pump.

References