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www.elsevier.com/locate/physletbOn the κ -Dirac oscillator revisitedF.M. Andrade^{a,*}, E.O. Silva^b, M.M. Ferreira Jr.^b, E.C. Rodrigues^b^a Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa, 84030-900 Ponta Grossa, PR, Brazil^b Departamento de Física, Universidade Federal do Maranhão, Campus Universitário do Bacanga, 65085-580 São Luís, MA, Brazil

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ABSTRACT

This Letter is based on the κ -Dirac equation, derived from the κ -Poincaré–Hopf algebra. It is shown that the κ -Dirac equation preserves parity while breaks charge conjugation and time reversal symmetries. Introducing the Dirac oscillator prescription, $\mathbf{p} \rightarrow \mathbf{p} - im\omega\beta\mathbf{r}$, in the κ -Dirac equation, one obtains the κ -Dirac oscillator. Using a decomposition in terms of spin angular functions, one achieves the deformed radial equations, with the associated deformed energy eigenvalues and eigenfunctions. The deformation parameter breaks the infinite degeneracy of the Dirac oscillator. In the case where $\varepsilon = 0$, one recovers the energy eigenvalues and eigenfunctions of the Dirac oscillator.

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1. Introduction

In 1989, in a seminal paper by Moshinsky and Szczepaniak [1] the basic idea of a relativistic quantum mechanical oscillator, called Dirac oscillator, was proposed. Such oscillator behaves as an harmonic oscillator with a strong spin-orbit coupling in the non-relativistic limit. Since the time of its proposal it has been the object of considerable attention in various branches of theoretical physics. For instance, it appears in mathematical physics [2–11], nuclear physics [12–14], quantum optics [15–18], supersymmetry [19–21], and noncommutativity [22–25]. Recently, the first experimental realization of the Dirac oscillator was realized by J.A. Franco-Villafañe et al. [26], which should draw even more attention for such system. Moreover, C. Quibay et al. proposed that the Dirac oscillator can describe some electronic properties of monolayer and bilayer graphene [27] and show the existence of a quantum phase transition in this system [28].

The Dirac oscillator has also been discussed in connection with the theory of quantum deformations [29]. Some of these deformations are based on the κ -deformed Poincaré–Hopf algebra, with κ being a masslike fundamental deformation parameter, introduced in Refs. [30,31] and further discussed in Refs. [32–35]. The κ -deformed algebra is defined by the following commutation relations:

$$[p_\nu, p_\mu] = 0, \quad (1a)$$

$$[M_i, p_\mu] = (1 - \delta_{0\mu})i\epsilon_{ijk}p_k, \quad (1b)$$

$$[L_i, p_\mu] = i[p_i]^{\delta_{0\mu}} [\delta_{ij}\varepsilon^{-1} \sinh(\varepsilon p_0)]^{1-\delta_{0\mu}}, \quad (1c)$$

$$[M_i, M_j] = i\epsilon_{ijk}M_k, \quad [M_i, L_j] = i\epsilon_{ijk}L_k, \quad (1d)$$

$$[L_i, L_j] = -i\epsilon_{ijk} \left[M_k \cosh(\varepsilon p_0) - \frac{\varepsilon^2}{4} p_k p_l M_l \right], \quad (1e)$$

where ε is defined by

$$\varepsilon = \kappa^{-1} = \lim_{R \rightarrow \infty} (R \ln q), \quad (2)$$

with R being the de Sitter curvature, q is a real deformation parameter, and $p_\mu = (p_0, \mathbf{p})$ is the κ -deformed generator for energy and momenta. Also, the M_i, L_j represent the spatial rotations and deformed boosts generators, respectively. The coalgebra and antipode for the κ -deformed Poincaré–Hopf algebra was established in Ref. [36].

Several investigations have been developed in the latest years in the context of this theoretical framework on space-like κ -deformed Minkowski spacetime. The interest in this issue also appears in field theories [37–40], quantum electrodynamics [41–43], realizations in terms of commutative coordinates and derivatives [44–47], relativistic quantum systems [48–52], doubly special relativity [53], noncommutative black holes [54] and the construction of scalar theory [55].

The aim of this Letter is to suitably describe the κ -Dirac oscillator making use of the κ -Poincaré–Hopf algebra, tracing a comparison with the results of Ref. [29], where it was argued that usual approach for introducing the Dirac oscillator, $\mathbf{p} \rightarrow \mathbf{p} - im\omega\beta\mathbf{r}$, in the κ -Dirac equation [32,33], has not led to the Dirac oscillator

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spectrum in the limit $\varepsilon \rightarrow 0$. This result, however, contradicts the well-known fact that the κ -Dirac equation recovers the standard Dirac equation in this limit. In this context, this Letter reassessed the κ -Dirac oscillator problem yielding a modified oscillator spectrum that indeed regains the Dirac oscillator behavior in the limit $\varepsilon \rightarrow 0$.

The plan of our Letter is the following. In Section 2 we introduce the κ -Dirac analyzing its behavior under \mathcal{C} , \mathcal{P} , \mathcal{T} (discrete) symmetries. In Section 3 the oscillator prescription is implemented in order to study the physical implications of the κ -deformation in the Dirac oscillator problem. Using a decomposition in terms of spin angular functions, we write the relevant radial equation to study the dynamics of the system. Section 4 is devoted to the calculation the energy eigenvalues and eigenfunctions of the κ -Dirac oscillator and to the discussion of the results. A brief conclusion is outlined in Section 5.

2. κ -Dirac equation and discrete symmetries

In this section, we present κ -Dirac equation, invariant under the κ -Poincaré quantum algebra [32], considering $O(\varepsilon)$ [33]:

$$\left\{ (\gamma_0 p_0 - \gamma_i p_i) + \frac{\varepsilon}{2} [\gamma_0 (p_0^2 - p_i p_i) - m p_0] \right\} \psi = m \psi, \quad (3)$$

which recovers the standard Dirac equation in the limit $\varepsilon \rightarrow 0$.

An initial discussion refers to the behavior of this deformed equation under \mathcal{C} , \mathcal{P} , \mathcal{T} (discrete) symmetries. Concerning the parity operator (\mathcal{P}), in the context of the Dirac equation, $\mathcal{P} = i\gamma^0$, with $\mathcal{P}\gamma^\mu\mathcal{P}^{-1} = \gamma_\mu$ and $\psi_{\mathcal{P}} = \mathcal{P}\psi$ being the parity-transformed spinor. Applying \mathcal{P} on the Dirac deformed equation, we attain

$$\left\{ (\gamma_0 p_0 - \gamma_i p_i) + \frac{\varepsilon}{2} [\gamma_0 (p_0^2 - p_i p_i) - m p_0] \right\} \psi_{\mathcal{P}} = m \psi_{\mathcal{P}}, \quad (4)$$

concluding that it is invariant under \mathcal{P} action.

We can now verify that this equation is not invariant under charge conjugation (\mathcal{C}) and time reversal (\mathcal{T}). As for the \mathcal{C} operation, the charge-conjugated spinor is $\psi_{\mathcal{C}} = U_{\mathcal{C}}\psi^* = \mathcal{C}\gamma^0\psi^*$, with $\mathcal{C} = i\gamma^2\gamma^0$ being the charge conjugation operator, and $U_{\mathcal{C}}\gamma^\mu U_{\mathcal{C}}^{-1} = -\gamma^\mu$. On the other hand, the time reversal operator is, $\mathcal{T} = i\gamma^1\gamma^3$, so that $\psi_{\mathcal{T}}(x, t') = \mathcal{T}\psi^*(x, t')$, and $\mathcal{T}\gamma^\mu\mathcal{T}^{-1} = (\gamma^0, -\gamma^i)$. Applying $U_{\mathcal{C}}$ and \mathcal{T} on the complex conjugate of Eq. (3), we achieve:

$$\left\{ (\gamma_0 p_0 - \gamma_i p_i) + \frac{\varepsilon}{2} [-\gamma_0 (p_0^2 - p_i p_i) - m p_0] \right\} \psi_{\mathcal{C}} = m \psi_{\mathcal{C}}, \quad (5)$$

$$\left\{ (\gamma_0 p_0 - \gamma_i p_i) + \frac{\varepsilon}{2} [(\gamma_0) (p_0^2 - p_i p_i) + m p_0] \right\} \psi_{\mathcal{T}} = m \psi_{\mathcal{T}}. \quad (6)$$

These equations differ from Eq. (3), revealing that the \mathcal{C} and \mathcal{T} are not symmetries of this system. As a consequence, particle and anti-particle eigenenergies should become different. Further, note that under \mathcal{CT} or \mathcal{CPT} operations the original equation is modified as

$$\left\{ (\gamma_0 p_0 - \gamma_i p_i) - \frac{\varepsilon}{2} [\gamma_0 (p_0^2 - p_i p_i) - m p_0] \right\} \psi' = m \psi', \quad (7)$$

where $\psi' = \psi_{\mathcal{CT}}$ or $\psi' = \psi_{\mathcal{CPT}}$, showing that this equation is not invariant under \mathcal{CT} or \mathcal{CPT} operations, once the parameter ε is always positive.

3. κ -Dirac oscillator equation

Now, we derive the equation that governs the dynamics of the Dirac oscillator in the context of Eq. (3). The Dirac oscillator stems from the prescription [1]

$$p_0 \rightarrow p_0 = H_0, \quad (8a)$$

$$\mathbf{p} \rightarrow \mathbf{p} - im\omega\beta\mathbf{r}, \quad (8b)$$

where \mathbf{r} is the position vector, m is the mass of particle and ω the frequency of the oscillator. The κ -Dirac oscillator can be obtained by substituting Eq. (8) into Eq. (3). The result is

$$H\psi = E\psi, \quad (9)$$

with

$$H = H_0 - \frac{\varepsilon}{2} [p_0^2 - (\mathbf{p} - im\omega\beta\mathbf{r})(\mathbf{p} - im\omega\beta\mathbf{r}) - \beta m p_0], \quad (10)$$

where H_0 represents the undeformed part of the Dirac operator

$$H_0 = \boldsymbol{\alpha} \cdot (\mathbf{p} - im\omega\beta\mathbf{r}) + \beta m. \quad (11)$$

At this point it is important trace a comparison with the results of Ref. [29], in which it is argued that the prescription of Eq. (8), yielding the κ -deformed Hamiltonian of Eq. (10), does not lead to an oscillator-like spectrum even when $\varepsilon \rightarrow 0$. This result, however, is not correct, as properly shown in Section 4. Furthermore, another deformed wave equation is introduced without any kind of proof (see Eq. (15) in [29]). Here, instead of postulating a deformed wave equation, we follow a pragmatic approach obtaining the κ -Dirac oscillator equation (10) from basic principles.

In the four-dimensional representation, the matrices $\boldsymbol{\gamma}$ and $\boldsymbol{\alpha}$ are given by

$$\boldsymbol{\beta} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \boldsymbol{\gamma} = \boldsymbol{\beta}\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \quad (12)$$

and obey the anticommutation relations and the square identity,

$$\{\alpha_i, \alpha_j\} = 0, \quad i \neq j,$$

$$\{\alpha_i, \beta\} = 0,$$

$$\alpha_i^2 = \beta^2 = I.$$

In the representation (12), ψ may be written as a bispinor $\psi = (\psi_1, \psi_2)^T$ in terms of two-component spinors ψ_1 and ψ_2 . Thus, Eq. (9) leads to

$$\begin{aligned} & \left(1 + \frac{m\varepsilon}{2}\right) (\boldsymbol{\sigma} \cdot \boldsymbol{\pi}^+) \psi_2 \\ & = (E - m) \psi_1 + \varepsilon [im\omega(\mathbf{r} \cdot \mathbf{p}) + m\omega(\boldsymbol{\sigma} \cdot \mathbf{L}) + m^2\omega^2 r^2] \psi_1, \end{aligned} \quad (13)$$

$$\begin{aligned} & \left(1 - \frac{m\varepsilon}{2}\right) (\boldsymbol{\sigma} \cdot \boldsymbol{\pi}^-) \psi_1 \\ & = (E + m) \psi_2 - \varepsilon [im\omega(\mathbf{r} \cdot \mathbf{p}) + m\omega(\boldsymbol{\sigma} \cdot \mathbf{L}) - m^2\omega^2 r^2] \psi_2, \end{aligned} \quad (14)$$

where

$$\boldsymbol{\pi}^\pm = \mathbf{p} \pm im\omega\mathbf{r}. \quad (15)$$

Since we are interested in studying the κ -Dirac oscillator in a three-dimensional spacetime, Eqs. (13) and (14) above may be solved in spherical coordinates. First, using the property

$$\boldsymbol{\sigma} \cdot \mathbf{p} = (\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) \left(\hat{\mathbf{r}} \cdot \mathbf{p} + i \frac{\boldsymbol{\sigma} \cdot \mathbf{L}}{r} \right), \quad (16)$$

with $\boldsymbol{\sigma} \cdot \mathbf{r} = r\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}$, we rewrite the quantity $\boldsymbol{\sigma} \cdot \boldsymbol{\pi}^\pm$ as

$$\boldsymbol{\sigma} \cdot \boldsymbol{\pi}^\pm = (\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) \left(\hat{\mathbf{r}} \cdot \mathbf{p} + i \frac{\hat{\mathbf{K}} - 1}{r} \pm im\omega r \right), \quad (17)$$

where the operator $\hat{\mathbf{K}}$ is related to the orbital angular momentum operator $\hat{\mathbf{L}}$ as

$$\hat{\mathbf{K}} = \boldsymbol{\sigma} \cdot \hat{\mathbf{L}} + \mathbf{1}. \tag{18}$$

We seek solutions of the form

$$\psi = \begin{pmatrix} \psi_1(\mathbf{x}) \\ \psi_2(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} f(r)\chi_k^{m_j}(\theta, \phi) \\ ig(r)\chi_{-k}^{m_j}(\theta, \phi) \end{pmatrix} = \frac{1}{r} \begin{pmatrix} u(r)\chi_k^{m_j}(\theta, \phi) \\ iv(r)\chi_{-k}^{m_j}(\theta, \phi) \end{pmatrix}, \tag{19}$$

where $\chi_{\pm k}^{m_j}(\theta, \phi)$ are the spin angular functions [56], with

$$k = \begin{cases} -(\ell + 1), & \text{for } j = \ell + 1/2, \\ \ell, & \text{for } j = \ell - 1/2. \end{cases} \tag{20}$$

By substituting Eq. (19) into Eqs. (13) and (14), and using the relations

$$(\boldsymbol{\sigma} \cdot \hat{\mathbf{r}})\chi_{\pm k}^{m_j} = -\chi_{\mp k}^{m_j}, \tag{21}$$

$$\hat{\mathbf{K}}\chi_{\pm k}^{m_j} = \mp k\chi_{\pm k}^{m_j}, \tag{22}$$

we find a set of two coupled radial differential equations of first order:

$$\begin{aligned} \left(1 + \frac{m\varepsilon}{2}\right) \left[-v' + \left(\frac{k}{r} + m\omega r\right)v\right] \\ = \varepsilon m\omega u' + \{(E - m) + \varepsilon[m\omega k + m^2\omega^2 r^2]\}u, \end{aligned} \tag{23}$$

$$\begin{aligned} \left(1 - \frac{m\varepsilon}{2}\right) \left[u' + \left(\frac{k}{r} + m\omega r\right)u\right] \\ = -\varepsilon m\omega v' + \{(E + m) + \varepsilon[m\omega k + m^2\omega^2 r^2]\}v. \end{aligned} \tag{24}$$

After some algebra, the above equations are decoupled yielding a single second order equation for $u(r)$,

$$\begin{aligned} u'' + 2m^2\varepsilon\omega u' \\ - \left[\frac{\ell(\ell + 1)}{r^2} + (1 - 2m\varepsilon)m^2\omega^2 r^2 - \mu_\varepsilon\right]u = 0. \end{aligned} \tag{25}$$

A similar equation exists for $v(r)$. Here

$$\mu_\varepsilon = (E^2 - m^2) - [(2k - 1)(1 + m\varepsilon) + \varepsilon E]m\omega, \tag{26}$$

and we have used the result $k^2 + k = \ell(\ell + 1)$.

4. Eigensolutions for the problem

In this section, we calculate the energy eigenvalues and eigenfunctions of the κ -Dirac oscillator, making some comparisons with those in the literature and discussing the associate results. The regular solution for Eq. (25) is

$$\begin{aligned} u(r) = e^{-m^2\omega r^2/2} \left[(1 - m\varepsilon)m\omega r^2 \right]^{(\ell+1)/2} \\ \times M\left(\frac{1}{2}\left(\ell + \frac{3}{2} - a_\varepsilon\right), \ell + \frac{3}{2}, (1 - m\varepsilon)m\omega r^2\right), \end{aligned} \tag{27}$$

with

$$a_\varepsilon = \frac{\mu_\varepsilon - m^2\varepsilon\omega}{2(1 - m\varepsilon)m\omega}, \tag{28}$$

and $M(a, b, z)$ is the confluent hypergeometric function of first kind [57]. The energy eigenvalues of the κ -Dirac oscillator come from requiring that the first parameter in the confluent hypergeometric function of Eq. (27) is a negative integer, $-n$, with n a

nonnegative integer. By using $N = 2n + \ell$ as principal quantum number, and with μ_ε given by Eq. (26), one finds

$$E^2 - m^2 = 2m\omega \left[N + k + 1 + m\varepsilon \left(\frac{E}{2m} - N + k - \frac{3}{2} \right) \right]. \tag{29}$$

By solving Eq. (29) for E , we obtain

$$\begin{aligned} E_\pm = \pm \left[\sqrt{2m\omega(N + k + 1) + m^2 + [2(k - N) - 3]m^2\varepsilon\omega} \right. \\ \left. \pm \frac{m\varepsilon}{2}\omega \right], \end{aligned} \tag{30}$$

which for $j = \ell + 1/2$ implies

$$\begin{aligned} E_\pm = \pm \left[\sqrt{2m\omega(N - j + 1/2) + m^2 - [2(j + N) + 4]m^2\varepsilon\omega} \right. \\ \left. \pm \frac{m\varepsilon}{2}\omega \right], \end{aligned} \tag{31}$$

and

$$\begin{aligned} E_\pm = \pm \left[\sqrt{2m\omega(N + j + 3/2) + m^2 + [2(j - N) - 2]m^2\varepsilon\omega} \right. \\ \left. \pm \frac{m\varepsilon}{2}\omega \right], \end{aligned} \tag{32}$$

for $j = \ell - 1/2$. The fact that particle and anti-particle energies turn out to be distinct, $E_+ \neq E_-$, is a consequence of charge conjugation symmetry breaking.

The limit $\varepsilon \rightarrow 0$ exactly conducts to the undeformed Dirac oscillator [56], whose eigenenergies are

$$E_\pm = \pm \sqrt{2m\omega(N - j + 1/2) + m^2}, \tag{33a}$$

$$E_\pm = \pm \sqrt{2m\omega(N + j + 3/2) + m^2}, \tag{33b}$$

for $j = \ell + 1/2$ and $j = \ell - 1/2$, respectively. These undeformed energy expressions yield an infinity degeneracy, once for $j = \ell + 1/2$ all states with $N \pm q$, $j \pm q$ have the same energy, while for $j = \ell - 1/2$ the equal energy states are the one with $N \pm q$, $j \mp q$, being q an integer. This infinity degeneracy is now lifted by the terms involving the deformation parameter, ε , inside the square root of Eqs. (31) and (32). Note that, in the limit $\varepsilon \rightarrow 0$, the eigenfunction (27) also regains the undeformed Dirac oscillator counterpart exhibited in [56], revealing the consistency of the description here developed.

5. Conclusions

We have studied the κ -Dirac oscillator problem based on the κ -deformed Poincaré–Hopf algebra and the κ -Dirac equation. First, we have analyzed the behavior of the κ -Dirac equation under discrete symmetries. Further, we have shown that the usual prescription $\mathbf{p} \rightarrow \mathbf{p} - im\omega\beta\mathbf{r}$ leads to a modified spectrum that in fact recovers the undeformed Dirac oscillator result. Using a decomposition in terms of spin angular functions, we have derived the deformed radial equation whose solution has led to the deformed eigenenergies and eigenfunctions. We have verified that the deformation parameter implies the breakdown of charge conjugation, time reversal and \mathcal{CPT} symmetries, while preserving parity. The deformation parameter modifies the energy eigenvalues and eigenfunctions of the Dirac oscillator, breaking the infinite degeneracy of the energy eigenvalues as well.

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