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Application of Kalman filter in detecting pre-earthquake ionospheric TEC anomaly*

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Abstract: As an attempt, the Kalman filter was used to study the anomalous variations of ionospheric Total Electron Content (TEC) before and after Wenchuan *M*s8.0 earthquake, these TEC data were calculated from the GPS data observed by the Crustal Movement Observation Network of China. The result indicates that this method is reasonable and reliable in detecting TEC anomalies associated with large earthquakes.

Key words: Kalman filter; TEC; anomaly detecting; Wenchuan earthquake

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1 Introduction

Anomalous ionospheric changes have been found to occur within a few hours to a few days before earthquakes of magnitude greater than 5[1-13]. Various methods have been used to detect such anomalies, including the moving-average, the median, and the interquartile-range method[14]. Although some obvious total-electron-content (TEC) anomalies can be detected by such methods, the reliability of the results is affected by the way the background variation is determined, namely, from data recorded in a period of about one month that precedes the earthquake occurrence.

Another method that has become widely used in data processing for a variety of observations is Kalman filter[16-18]. It is a linear minimum-variance optimal-recursion estimation theory obtained by minimizing mean-squares error as the criterion for optimization[15]. When used in data processing, it does not need to retain the data which have been processed, thus it can save a lot of computer-storage space and improve computing efficiency. Also it can obtain a new estimator in accordance with the recursive formula based on new data, thus avoid reprocessing old data. In this study, we applied the Kalman filter to the ionospheric TEC data recorded by the GPS reference stations of Crustal Movement Observation Network of China prior to the Wenchuan *M*s8.0 earthquake to search for possible pre-earthquake anomalies.

2 Observation and methodology

The ionosphere, which is about 60 to 1000 km above the earth's surface, is usually represented by a thin spherical layer in the GPS TEC studies. In this paper, we take a height of 350 km for this layer, which is in the middle of the height range of the largest TEC value (300 - 400 km). The ionosphere is a dispersive medium and the slant TEC (STEC) can be derived from GPS data and converted to VTEC by using a spherical harmonics mapping function at the ionospheric pierce point (see reference [19] for the specific calculation). Both VTEC and STEC are expressed in TECu unit (1

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TECu = 10^{16} el/m^2). By using GPS dual-frequency carrier phase smoothing of the P code observations and spherical-harmonic function as the fitting model for ionospheric TEC, we obtained the coefficients of the spherical harmonic by the least-squares fit[19,20]:

$$P = P_1 - P_2 =$$
$$a(1/f_1^2 - 1/f_2^2)F_1(z)\sum_{n=0}^{n_{\max}}\sum_{m=0}^{n}\bar{P}_{nm}(\sin\beta)(a_{nm}\cos(ms) + b_{nm}\sin(ms)) + b_4 \qquad (1)$$

where P_i is the code observations, $a = 4.03 \times 10^{17}\,\mathrm{ms}^{-2}\,\mathrm{TECU}^{-1}$, $F_1(z) = 1/\cos(z)$, Z is the satellite zenith angle, β is the geographical latitude of the ionospheric puncture point, S is the solar longitude, $s \approx UT + \lambda - \pi$, UT is the Universal Time, λ is the longitude, $n_{\max}$ is the maximum degree of the spherical harmonics (SH), b_4 is the deviation of hardware delay, $\bar{P}_{nm}(\sin\beta)$ is the orthogonal Legendre function, and a_{nm} and b_{nm} are the SH coefficients to be determined, $n = m = 5$. We selected the 36-parameter model of spherical harmonics to calculate the SH coefficients, and used equation (1) to estimate the the deviation of hardware-delay by least-squares method and to obtain the TEC values of each point at different time.

We used the GPS data from 24 stations of the Crustal Movement Observation Network of China (Fig. 1) during April 12 to May 22. As shown in figure 1, the station distribution is quite uniform, which is essential in obtaining accurate TEC values over China. The sampling interval is 30 seconds, and the cut-off angle is 15°. The calculated TEC values are evenly distributed over each grid (1° × 1°) in the range of 70°E – 140°E and 15°N – 55°N. In order to check the accuracy of the calculated TEC values, we selected an arbitrary access point (35°N, 105°E) in China and considered the 12 daily TEC values (at an interval of 2 hours) calculated by Center for Orbit Determination in Europe (CODE) as the true values. The time series of RMS difference between our calculated TEC values and the true values is shown in figure 2. The differences shown in the figure are mostly within 2 TECu, thus verifying the accuracy and the reliability of our calculated TEC values.

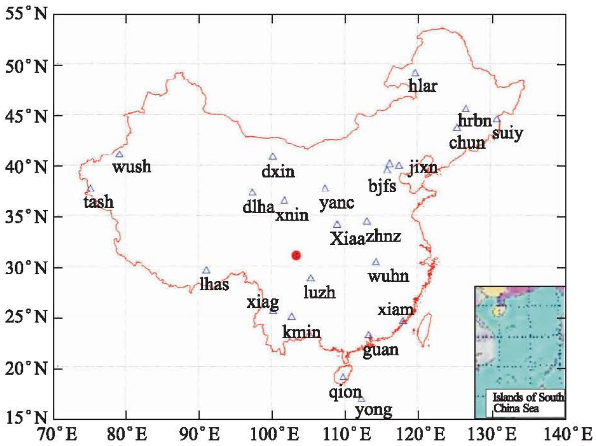


Figure 1 GPS station network in China

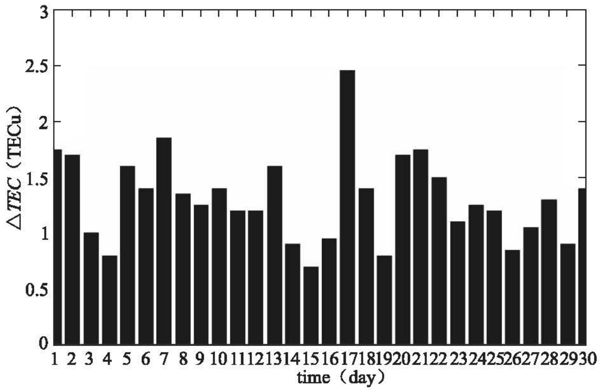


Figure 2 Time series of RMS differences between our TEC values and CODE values

3 Anomaly-detecting method based on Kalman filter

3.1 Kalman filter

The calculation with Kalman filter is a continuous process of correction, beginning with an initial estimation of a previous state to obtain the predicted value of the next step according to the equation of state. This value is then modified according to the currently observed value. The best estimate may be obtained by multiple iteration of this calculation. Assuming the state equation of the dispersive Kalman filter[21] to be

$$X_{k+1} = \Phi_{k+1|k}X_k + \Gamma_k u_k \qquad (2)$$

The observation equation is

$$Y_k = H_k X_k + v_k \tag{3}$$

Where X_{k+1} is the state vector, $\Phi_{k+1|k}$ is the transfer matrix, Γ_k is the system driver matrix, Y_k is the observation vector, H_k is the observed coefficient vector, v_k is the noise of observation, and u_k is the noise-of-state sequence, the mean value of which is 0. The best estimate of the state vector can be calculated according to the equation of state and noise statistics. The specific algorithm can be found in reference[21].

3.2 Anomaly detection

As mentioned above, in using this anomaly-identification algorithm, the first step is to determine an initial value of the state and its covariance matrix. In this study, we selected the TEC values of the first 10 days of observation, between April 12 and 21, as the initial values for the Kalman filter, and proceeded step by step, using prediction-residual difference as a measure to judge the abnormality of the disturbance. Assuming that Y_{k+1} is the observation vector and X_{k+1} is the state vector, then the observation equation can be expressed as follows

$$Y_{k+1} = H_{k+1} X_{k+1} + v_{k+1} \tag{4}$$

Assuming that the variance component of v_{k+1} is $\sigma_{k+1}^2(i)$, the forecast value of the state vector X_{k+1} is $\hat{X}_{k+1/k}$, and the prediction residual is e_{k+1}, then

$$e_{k+1} = Y_{k+1} - H_{k+1}\hat{X}_{k+1/k} \tag{5}$$

Under the null hypothesis, v_{k+1} is a zero-mean Gaussian random quantity whose variance matrix is

$$E[v_{k+1}v_{k+1}^{\mathrm{T}}] = H_{k+1}P_{k+1/k}H_{k+1}^{\mathrm{T}} + R_{k+1} \tag{6}$$

In equation (6), $P_{k+1/k}$ is the forecast variance matrix. By using statistics of the prediction residuals, each component of Y_{k+1} can be detected. The basis for anomaly detection is as follows:

$$|e_{k+1}(i)| \leqslant c\sqrt{(H_{k+1}P_{k+1/k}H_{k+1}^{\mathrm{T}})_{i,i} + \sigma_{k+1}^2(i)} \tag{7}$$

where $e_{k+1}(i)$ represents the ith component of $e_{k+1}(i)$ $(1 \leqslant i \leqslant m)$ and c is the quantile of the normal random quantity determined by α, which is 3 or 4 in practical application.

If $e_{k+1}(i) \leqslant -c\sqrt{(H_{k+1}P_{k+1/k}H_{k+1}^{\mathrm{T}})_{i,i} + \sigma_{k+1}^2(i)}$, then we considered Y_{k+1} as a negative anomaly; if $e_{k+1}(i) > c\sqrt{(H_{k+1}P_{k+1/k}H_{k+1}^{\mathrm{T}})_{i,i} + \sigma_{k+1}^2(i)}$, then we considered Y_{k+1} as a positive anomaly.

4 Calculated pre-earthquake TEC anomalies

We processed and analyzed the ionospheric TEC data over China before the *M*s8.0 Wenchuan earthquake, which occurred at 14:28 (06:28 UT) on May 12, 2008 at 31.0°N and 103.4°E and with a depth of 14 km[14]. The result shows several significant anomalous disturbances a few days before the earthquake. The mapping function for anomaly distribution is given in reference[10]. Figure 3 shows, for example, the two-dimensional distribution of $\Delta VTEC$ at 08:00 UT and 10:00 UT on May 9, three days before the earthquake. From figure 3 we can see that the TEC values increased anomalously by as much as about 5 TECu around the epicenter, and the anomaly was drifting from east to west, as found in some previous studies[10-12].

In order to see the distribution of the ionospheric TEC anomalies at different time more clearly, we show in figure 4 how the calculated distribution of the ΔTEC values vary with time over the epicenter of the earthquake from April 22 to May 22, or 20 days before to 10 days after the earthquake. In this figure, the horizontal axis shows time before and after the earthquake in unit of day and the vertical axis shows the time of each day in unit of hour.

From figure 4, we may see several features of the anomaly distribution over the epicenter: (1) The TEC disturbance may be either positive or negative. There were significant anomalous decreases (beyond the limits of about 3 TECu) on 13 days and 6 days before the earthquake and a larger anomalous increase of 5 TECu on the 3rd day before the earthquake. (2) The time intervals when these disturbances occurred were mainly between afternoon and evening, or 12:00 – 18:00 (LT). By studying the global ionospheric TEC grid data released by the IGS data center in the same way, we found that similar disturbances occurred on 13 days and 6 days before the earthquake also. This result indicates that these disturbances were associated with the

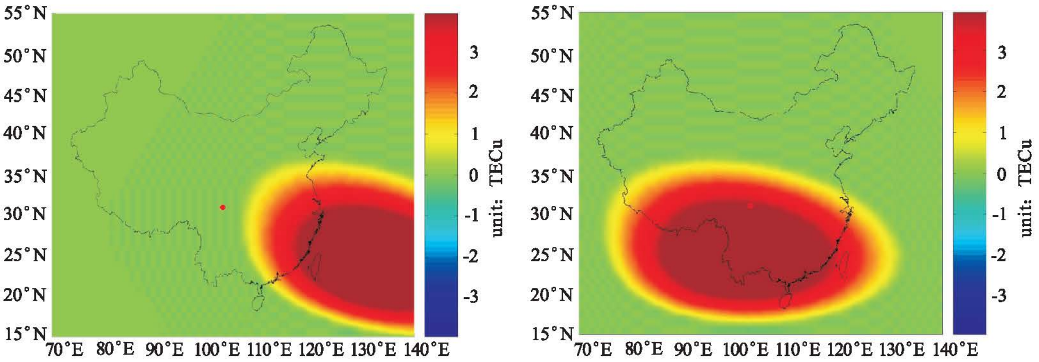


Figure 3 Distribution of △*TEC* values on May 9 over China at(lefc)08:00UT and(right)10:00 UT

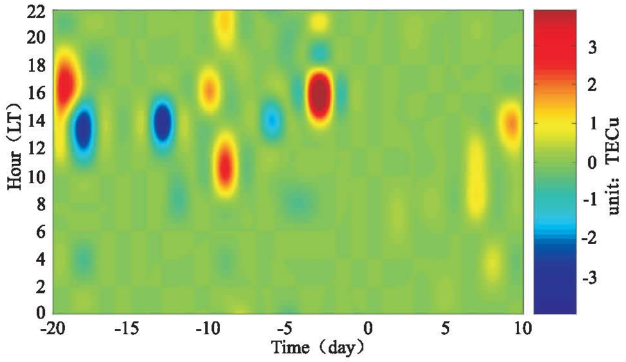


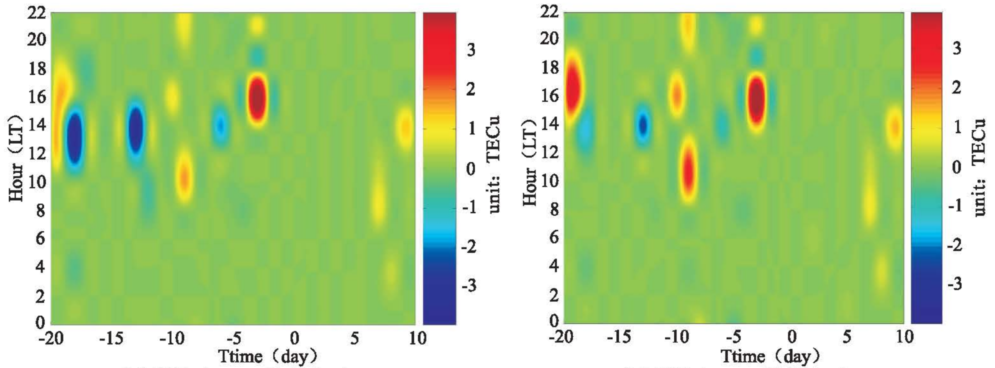
Figure 4 Temporal distribution of △*TEC* values over the epicenter

geomagnetic activity rather than the earthquake. However, the geomagnetic activity was quiet after May 6. Thus we consider the abnormal increase on the 3rd days before the earthquake to be earthquake-related.

To check whether the assumed initial condition may have affected the result of calculation, we changed the initial condition by reducing and increasing the TEC values to 50% and 150% , respectively, in similar calculations. The results are shown in figure 5. It may be seen that the TEC-variation patterns remain basically the same. Thus the influence of the prior abnormal observation to the detecting result gradually weakened, the results are not significantly affected.

5 Discussion and conclusion

The methods used currently in detecting TEC anomalies, including the moving-average, the median, and the interquartile-range[14], require highly reliable background-variation information. In this study, we attempted to use Kalman filter to establish a model of detecting the TEC anomalies. By using GPS data from the reference stations of Crustal Movement Observation Network of China, we have processed and analyzed the ionospheric TEC data before and after the Wenchuan *M*s8.0 earthquake. The result showed the occurrence of some pre-earthquake TEC anomalies, which are similar to those of previous findings[11-13]. Thus the result shows the usefulness of this method. However, this method also has some shortcomings: the calculated results are affected by abnormal background variation, especially when estimating the filter value; if the selected model noise and measurement noise covariance matrix are different, then the filter value and the detection result will be different also.



(a) With decreased initial values (b) With increased initial values

Figure 5 Temporal distribution of △*TEC* values over the epicenter by using different initial conditions

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References

[1] Leonard R S and Barnes R A. Observation of ionospheric disturbances following the Alaska earthquake. Journal of Geophysical Research, 1965, 70(5): 1250 – 1253.

[2] Calais E M. GPS detection of ionospheric TEC perturbations following the January 17, 1994, Northirdge earthquake. Geophys Res Lett., 1995, 22, 1045 – 1048.

[3] Chuo Y J. A study of seismo-ionospheric precursor. Taiwan: NCU Institute of Space Science, 2002. 31 – 33. (in Chinese)

[4] Liu J Y, et al. Seismo-ionospheric signature prior to $M \geqslant 6.0$ Taiwan earthquakes. Geophys Res Lett., 2000, 27(19): 3113 – 3116.

[5] Chuo Y J, et al. The anomalies in the f0Es prior to *Ms* 6.0 Taiwan earthquakes. Seismo Electromagnetic: Lithosphere-atmosphere-ionospherecoupling. Terrapub, 2002, 09 – 312.

[6] Liu J Y, et al. Pre-earthquake ionospheric anomalies registered by continuous GPS TEC measurements. Annales Geophysicae, 2004, 22: 1585 – 1593.

[7] Zhang Qiang. The ionospheric TEC detected by ground-based GPS and the relationship to the earthquake. Wuhan: Institute of Seismology, CEA, 2004, 43 – 51. (in Chinese)

[8] Wu Yun, et al. Preseismic ionospheric TEC anomaly detected by ground-based GPS. Journal of Geodesy and Geodynamics, 2005, 25(2): 36 – 40. (in Chinese)

[9] Yang Jian, Wu Yun and Zhou Yiyan. Research on seismo-ionospheric anomalies using GPS radio occultation data. Journal of Geodesy and Geodynamics, 2008, 28(2): 16 – 22. (in Chinese)

[10] Zhu Fuying, et al. Anomalous response of ionospheric VTEC before the Wenchuan earthquake. Acta Seismologica Sina, 2009, 31(2): 180 – 187. (in Chinese)

[11] Zhao Biqiang, et al. Recent advances on the ionospheric pre-cursors of earthquakes and ionospheric variations prior to Wenchuan earthquake. Science and Technology Review, 2008, 26(11): 30 – 33. (in Chinese)

[12] Zhou Yiyan, et al. Ionospheric anomalies detected by ground-based GPS before the Mw7.9 Wenchuan earthquake, China. Journal of Atmospheric and Solar-Terrestrial Physics, 2009, 71: 959 – 966.

[13] Yu Tao, et al. Study of the ionospheric anmaly before the Wenchuan earthquake. Chinese Science Bulletin, 2009, 54(6): 1080 – 1086.

[14] Zhu Fuying, et al. Study on method of detecting ionospheric TEC anomaly before earthquake. Journal of Geodesy and Geodynamics, 2009, 29(3): 50 – 54. (in Chinese)

[15] Cui Xizhang, et al. Generalized surveying adjustment. Wuhan: Wuhan University of Surveying and Mapping Press, 2005. (in Chinese)

[16] Zhang C Y. Application of Kalman filter in modeling multi-dimensional AR series. Journal of Geodesy and Geodynamics, 2003, 23(2): 92 – 95. (in Chinese)

[17] Liu Guolin, et al. Application of Kalman filter to noise eliminating and phase unwrapping of InSAR. Journal of Geodesy and Geodynamics, 2006, 26(2): 66 – 69. (in Chinese)

[18] Qi Gongyu, Qiu Weining and Hua Xianghong. The method of gross error modification in Kalman filtering and its application. Engineering of Surveying and Mapping, 2010, 19(1): 50 – 52. (in Chinese)

[19] Yuan Y B, et al. Preliminary results and analyses of using IGS GPS data to determine global ionospheric TEC. Progress in Natural Science, 2003, 13(8): 446.

[20] Wilson B D, et al. Subdaily northern hemisphere ionospheric maps using an extensive network of GPS receivers. Radio Science, 1999, 30: 639.

[21] Kalman R E. A new approach to linear filtering and prediction proble. Journal of Basic Engineering, 1960. 82(series D): 35 – 45.

[22] Fu Ningbo. Determination of TEC based-on GPS tracking station data and research on seismo-ionospheric disturbance. Institute of Seismology, CEA 2010, 59 – 64. (in Chinese)