# Optimization on Combination of Transport Routes and Modes on Dynamic Programming for a Container Multimodal Transport System 

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#### Abstract

Along with the swift boom of the carriage of container and logistic economy, multimodal transport becomes more and more important economically. Container multimodal transport is a form of combined transport organization aimed to optimize the overall cargo transport. The optimal organization of various transport modes in container multimodal transport system directly concern the time, cost and quality of the cargo transport. In order to describe the optimal organization problem, an optimization model based on dynamic programming is presented in the paper, and is satisfied with reality constraints. Then, a dynamic programming algorithm is proposed to obtain the optimal combination strategy of transport modes. Finally, an empirical study is used to show the feasibility and efficiency of the proposed model.


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## 1. Introduction

With the fast development of the process in economic globalization and regionalization, the industry supply chains in all fields become more and more complicated, the transport network is becoming more and more perplexing. Thus the requirements on the manner of cargo transport have significant changes, namely to meet customers' needs firstly, and then achieve the cargo transport quickly and efficiently with the trend of the pursuit of

[^0]low cost. A single container transport modes cannot meet the requirements of the development of today's global supply chain management, so the multimodal transport has become the mainstream in the field of the modern cargo transport, it is also the main operation mode of container transport, and has been widely recognized in the world. Creating the efficient and convenient container multimodal transport has become a global proposition that can't wait to solve.

Multimodal transport is the joint coordination of various modes of transport, it emphasizes close coordination and seamless of rail, road, water, air and other means. Multimodal container transport is a transport mode aiming at achieving benefit optimization of integrated goods transportation. It has become the leading means of transport in international logistics. The problem of multimodal container transport can be attributed to get the minimized cost of logistics through a reasonable transport routes and modes on the basis of meeting customer's needs. Therefore, careful studies on system optimization of Chinese multimodal container transport are needed according to its current development.

With modern companies increasingly focusing on the pursuit of low cost and high time-efficient transport mode, it makes container multimodal transport increasingly concerned. Transportation management has become the focus of academic research, particularly combination of transportation problems for each segment of the whole trade. Milan Janic established a model which contains inner cost and external cost model, and achieve the external cost internalization, and the model mainly focus on the impact of policies on the multimodal transport network [1]; Tsung-Sheng Chang described the optimal cargo transport route as a commodity flow problem of containing multitarget problem of time window and convex cost, finally the solving algorithm are given [2]; Min build a multi-goal programming model with chance constrains, the model can meet the various constrains on time request and take the minimize cost and risk as the optimization objective [3]; Haugen and Hervik established the game model for carrier choosing the transport mode from the point of congestion cost [4].

On the basis of summarizing the relevant literature, the paper takes further research and study on the multimodal transport. Dynamic programming model of the minimum generalized transport costs is found on limiting trade time and related examples are given to verify and support the model and its major conclusion. Also, the result could offer scientific evidence for multimodal transport operator to determine the transport scheme.

## 2. Problem Description

To examine the effects of different transport models in the multi-modal network, we will first introduce a simple example to explain the problem clearly, and later consider how the results obtained can be generalized. This simple network is shown in Fig.1, and it consists of four nodes (one origin and destination, two intermediate nodes of one route) [5], which are connected by three segments. Each segment consists of three links ("water", "road" and "railway"). From the Fig. 1, we can know that there are more than one choices of the trade, and our purpose is to get the mode of the lowest cost.


Fig. 1. Route.
The Fig. 1 shows a simple network, but in the real life, the network is complex. From the Fig. 2, we can see that there will be more than one route to be chosen, and every route has some different transport mode combinations, which will further increase the complexity of the problem. So, that how to complete the trade with the lowest cost become the focus of the article [6].

When there are many nodes in the trade, we introduce the matrix $\left[\mathrm{a}_{\mathrm{ij}}\right]$ and the topology graph of structure. We can easily get the topology graph of structure through the real life, then we can get the matrix $\left[a_{\mathrm{ij}}\right]\left(\mathrm{a}_{\mathrm{ij}}=1\right.$, the node $i$ can directly get the node $j$ without transit; $\mathrm{a}_{\mathrm{ij}}=0$, the node $i$ cannot directly get the node j .) according to the topology graph of structure. The matrix can help us get a convenient calculation [7].


Fig. 2. Network.
The Fig 2.2 can be seen as a topology graph of structure, and its corresponding matrix representation is as follows:
1
2
3
4
5
6
7
8
9
10 $\left[\begin{array}{llllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]$

### 2.1. The features of the problem

(1) Multi-objective problem: in the trade of container multimodal transport, there are a number of factors to be considered, including transportation costs, the total time of transport, transport quality, etc. In the process of modelling, we must take appropriate mathematical models to effectively address these problems, which will also result in the complex of the model calculating.
(2) Transportation cost problem: in practice, regardless of road, rail and air transport, sea transport, the unit cost of transportation is not immutable, and the transportation cost is nonlinear. With distance increasing, the unit cost will be gradually smaller, which should be considered in the process of establishing the model. Thus, the complexity of the model solution becomes higher.
(3) There are many factors producing incalculable effect in the delay of the arriving of the container, such as, the operation quality of each station and the collection and distribution system, especially the none soundness of the transport network [8].

### 2.2.Properties of the problem

The articles referring to the line optimization model of the container multimodal transport are relatively not many. When dealing with the line optimization model, the exiting problems as follows:
(1) We can only choose one kind of mode of transportation between cities. When the cargo arrive a node of the
trade network, there are a variety of modes of transportation to be selected.
(2) Assuming that the convergence between the various modes of transport can only occur in the nodes, which should be a must condition in practice. The transit and change of intermodal cargo occur at the node, such as, the logistics parks, container transfer station or port.
(3) Most of the models do not consider the effect of time on the container multimodal transport costs. The pursuit of the lowest cost is the most basic goal of the multimodal transport operator, but it must have a premise constraint: to ensure that the goods reach the destination in accordance with the time required. If this premise is not met, the multimodal transport operator must bear the necessary liability because of the delay, thus may actually increase the MTO people's total transportation costs even though the multimodal transport operator have get the lowest transport cost, which will also make the credit of the enterprise loosing [9].

## 3. Dynamic Model

### 3.1. The composition of the container multimodal transport cost

According to the container freight business accounting standard of various modes in our country, the railway freight of the international standard container $=$ (after base price ( $215 y$ yan $)+$ run base price $(0.9274 y u a n$ per container kilometer) * freight mileage) * container quantity; highway container transportation cost $=$ charged volume*base price (6yuan per container kilometer)*charged mileage; as for the cost of water transport, we produce the container liner cost accounting standard in our country for reference, all in charges for 950 yuan per TEU.

When adopting different modes of transport, the time value cost of container transport mainly reflects on the loss of working cost, occupancy costs in liquidity, storage fee.

$$
\begin{equation*}
C_{k}=\left(\frac{V_{g} \times m}{365}+V_{c}\right) \times\left(T_{d}+\frac{d_{i j}}{24 V_{k}}\right) \tag{1}
\end{equation*}
$$

$C_{k}$ __the time value cost of container transport when using transportation mode K ;
$m —$ the loan interest rate;
$V_{g} \quad$ The value of the transport cargo;
$V_{c} —$ Occupancy expense of the container;
$T_{d}$ _-Storage time during the transshipment of goods, a daily basis, take 1 ;
$d_{i, j} \quad$ The distance between the two transport nodes $i, j$;
$V_{k}$ _-The transport speed between the nodes $i, j(\mathrm{~km} / \mathrm{h})$.

### 3.2.The optimization model

There are some assuming conditions:
(1) There are three alternative modes of transport (rail, road and water transport) between the two cities;
(2)The transportation of goods between the two cities can only use a mode of transport, namely the traffic volume can not be divided;
(3)The impact of cargo size on the unit freight rate does not take into account, so the cargo freight of the specific mode of transport between two cities is a certain value;
(4) The model does not consider the cost of cargo damage generated in the process, and it is a transport model for a single transceiver points.

To simplify the model, we put the minimization of the generalized transport cost as the objective function.

$$
\begin{equation*}
Z=\operatorname{Min} \sum_{i} \sum_{k} x_{i_{1}, i_{2}}^{k} C_{i_{1}, i_{2}}^{k}+\sum_{i} \sum_{k} \sum_{l} y_{i_{1}}^{k l} C_{i_{2}}^{k l}+\sum_{i} \sum_{k} x_{i_{1}, i_{2}}^{k} T_{i_{1}, i_{2}}^{k}+\sum_{i} \sum_{k} \sum_{l}\left(u_{i_{1}}^{k l}-1\right) M \tag{2}
\end{equation*}
$$

s.t :

$$
\begin{align*}
& x_{i_{0}, i_{1}}^{k}+x_{i_{1}, i_{2}}^{l} \geq 2 y_{i_{1}}^{k l} \forall i, k, l  \tag{3}\\
& \sum_{k} x_{i_{1}, i_{2}}^{k}=1 \forall i  \tag{4}\\
& \sum_{k} \sum_{l} y_{i_{1}}^{k l}=1 \forall i  \tag{5}\\
& \sum_{\mathrm{i}} \sum_{k} t_{i_{1}, i_{2}}^{k} x_{i_{1}, i_{2}}^{k}+\sum_{i} \sum_{k} \sum_{l} a_{i_{1}}^{k l} y_{i_{2}}^{k l} \leq T  \tag{6}\\
& q \leq f_{i_{1}, i_{2}}^{k} \forall i, k, l  \tag{7}\\
& \left(i_{1}, i_{2}\right)=1  \tag{8}\\
& x_{i_{1}, i_{2}}^{k}, y_{i_{1}}^{k l} \in\{0,1\} \forall i, k, l
\end{align*}
$$

Z ——The generalized transport cost for different transportation routes;
$i \quad$ —The set of the urban nodes in the intermodal transport network, and $i_{1}, i_{2} \in i$;
$k, l-$ The optional modes of transportation;
$C_{i_{1}, h_{2}}^{k}$ ——The transport cost between city $\mathrm{i}_{1}$ iand $\mathrm{i}_{2}$ when using transport mode k ;
$C_{i_{1}}^{k l}$ ——The transit cost for the translation of the transport mode k and l in the city $\mathrm{i}_{1}$;
$T_{i_{1}, i_{2}}^{k}$ ——The time value cost between city $\mathrm{i}_{1}$ iand $\mathrm{i}_{2}$ when using transport mode k ;
$T_{d} \quad$ —The time of goods transshipment and storage at the city node;
$f_{i_{1}, i_{2}}^{k}$ —The transport capacity with the transport mode k between the city $\mathrm{i}_{1}$ and $\mathrm{i}_{2}$;
$t_{i, 1}^{k} i_{2} \quad$ —The transport time with the transport mode k between the city $\mathrm{i}_{1}$ and $\mathrm{i}_{2}$;
$q$ ——The amount of cargo;
$M$ ——A sufficiently large penalty factor;
$T$ ——The allowed time of the transport from origin to the destination;
$x_{i 1, i 2}^{k}=\left\{\begin{array}{l}1 \text { Choosing the transport mode } \mathrm{k} \text { between the city } \mathrm{i}_{1} \text { and } \mathrm{i}_{2} ; \\ 0 \text { Choosing other transport modes }\end{array} ;\right.$
$y_{i 1}^{k l}=\left\{\begin{array}{l}1 \text { The conversion from the transport mode } \mathrm{k} \text { to } 1 \text { in the city } \mathrm{i}_{1} ; \\ 0 \text { There is no conversion of the transport mode; } ;\end{array}\right.$
$\mu_{i 1}^{k l}=\left\{\begin{array}{l}1 \text { The facilities of the city } \mathrm{i}_{1} \text { can meet the requirements of the conservation } \\ \text { of the transport mode } \mathrm{k} \text { to } \mathrm{l} ; \\ 0 \text { The facilities cannot meet the requirements; }\end{array}\right.$

The explanation of the objective function:
The first part: the total transport cost from the starting point to the destination;
The second part: the cost produced by the transshipment in transit city;
The third part: the time value cost of the whole trade of the cargo;
The forth part: the punishment cost.
The explanation of the constraint condition:
The explanation of the objective function:
The first part: the total transport cost from the starting point to the destination;
The second part: the cost produced by the transshipment in transit city;
The third part: the time value cost of the whole trade of the cargo;
The forth part: the punishment cost.
The explanation of the constraint condition:
(3)Ensure continuity of transport;
(4)It shows that the transport of goods between city $i_{1}$ and $i_{2}$ can only use a mode of transport;
(5)It shows that the trade in city $i_{1}$ can only use one transshipment mode;
(6)The goods must be shipped to the destination within the prescribed period;
(7)The volume of goods can not exceed the capacity of a cargo conveyance;
(8)It means that there is no transit city between the city $i_{1}$ and $i_{2}$, it refers to the formulation (1).
(9)It says that the decision variables $X_{i 1, i 2}^{k}, y_{i 1}^{k l}$ are the $0-1$ variables.

### 3.3. The model solution

For the problem of Fig. 2, we first find the total possible paths with the breadth - first search algorithm, then calculate the lowest generalized transport cost with the dynamic programming algorithm to solve. In the process of solving, the each city in the selected transport route is equivalent to a stage of dynamic programming, so there is $n$ stages if the route has $n$ cities [10].

The solution is as follows:
As for the city n , the total cost $f_{n}=0$, the total time $T_{n}=0$;
As for the city $\mathrm{n}-1$, the total cost of the transport can be expressed as:

$$
\begin{equation*}
f_{n-1}(k, l)=C_{n-1, n}^{l}+C_{n-1}^{k l}+T_{n-1, n}^{l} \tag{10}
\end{equation*}
$$

The total time can be expressed as:

$$
\begin{equation*}
T_{\mathrm{n}-1}=\frac{d_{n-1, n}^{l}}{v_{n-1, n}^{l}}+T_{n} \tag{11}
\end{equation*}
$$

From the city 2 to city $\mathrm{n}-2$, the cost can be expressed as follows:

$$
\begin{equation*}
f_{i 1}(k, l)=C_{i 1, i 2}^{l}+C_{i 1}^{k l}+T_{i 1, i 2}^{l}+f_{i 2} \tag{12}
\end{equation*}
$$

The total time can be expressed as:

$$
\begin{equation*}
T_{\mathrm{i} 1}=\frac{d_{i 1, i 2}^{l}}{v_{i 1, i 2}^{l}}+T_{d}+T_{i 2} \tag{13}
\end{equation*}
$$

From the city 1 to city 2, the total cost can be expressed as follows:

$$
\begin{equation*}
f_{1}(s)=C_{1,2}^{k}+f_{2} \tag{14}
\end{equation*}
$$

The total time can be expressed as:

$$
\begin{equation*}
T_{1}=\frac{d_{1,2}^{k}}{v_{1,2}^{k}}+T_{2} \tag{15}
\end{equation*}
$$

Through the comparison and analysis of the lowest generalized transport costs, we can choose the best mode of transport. The factor of time plays an important role in this model, so in the process of solving the model, we do not need to select the best mode of transport at every stage, and we can just choose the mode of the lowest cost at the end of the solution within the time limit, which achieves the lowest generalized total cost of within the stipulated time frame.

This chapter in the experience of other scholars, based on the theory of dynamic programming, construct a mathematical model, where the container multimodal transportation cost is the objective function, to optimize the system for intermodal containers, while the model can meet the requirements of the given time constraints.

## 4. The case study

In the case study, we assume that the shipper has ten standard containers, and the stated period is two days. The goods are required to be shipped from Shenyang to Chengdu, via nodes of Beijing, Tianjin, Dalian, Jinan, Shanghai, Zhengzhou, Wuhan, Xian, Chongqing. There are more than ten routes to be selected.


Fig. 3. The topology graph of structure for the case study.
In this paper, the value of the goods for each container is about RMB300000, and the container occupancy fee is RMB38.5yuan per day $\bullet$ TEU.

When finding the best solution, we know that there is transit cost at the node, so we can assume that the city node is also a small path between two cities, and its cost consists of transit cost, transit time value cost. The cost of the real paths between the real cities consists of transport cost and time value cost. So we can get a bigger topology
graph of structure. Then we will get a shortest path problem, and we will solve the shortest path problem with dynamic programming [11].
var
s:the set of non-visisted cities;
dist[i,j]: The distance between any two cities;
function search(city):integer; \{ The shortest distance from city S to city E \}
begin
if city $=$ E then search $\leftarrow 0$;
else begin min:=maxint;
for i)from all over the city do
if dist[city, i$]>0$ and $(\mathrm{i} \in \mathrm{s})$
then begin
$\mathrm{s} \leftarrow \mathrm{s}$-[i];
j $\leftarrow$ dist[city,i]+search(i); \{ Recursively search process \}
$\mathrm{s} \leftarrow \mathrm{s}+[\mathrm{i}] ;$ if $\mathrm{j}<$ min then $\min \leftarrow \mathrm{j}$;
end; search $\leftarrow$ min;
end;
begin
$\mathrm{s} \leftarrow$ all the cities except E ;
dist[A,E] $\leftarrow \operatorname{search}(\mathrm{A})$;
end. [12]
Through the calculation, we can find that the best route is Shenyang-Beijing-Xian-Chengdu, and the best intermodal is rail-rail-rail, and total cost is 35849 yuan, the total time is 40 hours.

Table 1. The distance with different modes of transport (km).

| city | Road | Rail | Water |
| :--- | :--- | :--- | :--- |
| Shenyang-Beijing | 688.3 | 703 | -- |
| Shenyang-Tianjin | 663.7 | 666 | -- |
| Shenyang-Dalian | 390 | 383 | -- |
| Beijing-Xian | 1090 | 1199 | -- |
| Beijing-Zhengzhou | 724 | 689 | -- |
| Beijing-Jinan | 426.8 | 406 | -- |
| Tianjin-Jinan | 325.3 | 284 | -- |
| Tianjin-Shanghai | 1092 | 1196 | 1296.4 |
| Dalian-Shanghai | 2000 | 2238 | 1053 |
| Jinan-Zhengzhou | 439.5 | 668 | -- |
| Jinan-Wuhan | 842.6 | 936 | -- |
| Zhengzhou-Xian | 479.4 | 505 | -- |
| Zhengzhou-Chongqing | 1153.7 | 1361 | -- |
| Xian-Chongqing | 685.7 | 734 | -- |
| Shanghai-Wuhan | 833.9 | 888.2 | 876 |
| Wuhan-Chongqing | 711.1 | 842 | 1125 |
| Xian-Chengdu | 313 | 888.2 |  |
| Chongqing-Chengdu | -- |  |  |

Table 2. The average speed of different modes of transport.

|  | Road | Rail | Water |
| :--- | :--- | :--- | :--- |
| speed $(\mathrm{km} / \mathrm{h})$ | 100 | 80 | 37 |

Table 3. The transfer time between transport modes (unit: h).

|  | Road | Rail | Water |
| :--- | :--- | :--- | :--- |
| Road | 0 | 3 | 3 |
| Rail | 7 | 0 | 7 |
| Water | 18 | 18 | 0 |

Table 4. The transshipment cost between transport modes.

|  | Road | Rail | Water |
| :--- | :--- | :--- | :--- |
| transshipment cost (yuan) | 100 | 170 | 350 |

## 5. Conclusions

In this paper, we firs explain the meaning of the problem, and the optimization problem of the combination of transport routes and modes for a container multimodal transport system is formulated to be a mixed integer programming problem at first. Then, a dynamic programming algorithm is proposed to obtain the optimal combination strategy of transport modes. Finally, a real life problem was solved to show the feasibility and efficiency of the proposed model. In the future research, we will focus on further improvement for the practical constrains of the model, and we try to make the optimization problem of the combination of transport routes and modes visualized with the simulation technology.

## References

[1] Ma, Y.X., 2013, Optimization Research on time and cost model of multimodal transport under uncertain environment, Chongqing University.
[2] Wang, Q., 2014. Research on Inland Container Transportation Networks Optimization, Dalian Maritime University.
[3] Yu, R.J., Wang, J.R., Li, J., 2009. Research on Combination Optimization of Multimodal Transportation System of Containers, 5(4), 272-275.
[4] Song, B.L., 2003. Analysis of International Multi-Modal Transportation and Eastern Coastal Ports Competitiveness. China Voyage, (3), 61-64.
[5] Han, J., Xu, Q., Jin, Z.H., 2010. Optimization on Combination of Transport Modes Base on Dynamic Programming for a Container Multimodal Transport System, Journal of Wuhan University: Transportation Science and Engineering.
[6] Han, J.X., Xu, Q., Jin, Z.H., 2010. Research on the Path Optimization of Multimodal Transport of Bulky Cargo. Journal of Wuhan University of Science and Technology: transportation science and engineering, 34(4), 661-664.
[7] Han, Z.X., 2011. Research on the Optimal Routes and Modes Selection in Container Multimodal Transportation Networks. Dalian Maritime University.
[8] Van der, W., A. H., Verhoef, E. T., van den Berg, V. A. C., 2013. Competition in multi-modal transport networks: A dynamic approach. Transportation Research Part B: Methodological, 53, 31-44.
[9] Swinkels, J., Fleuron, T., 2014. Operational planning of a large-scale multi-modal transportation system.European Journal of Operational Research, 156(1), 41-53.
[10] Athanasios, Z., Whitney, W., 2000. An inter-modal optimum path algorithm for multi-modal networks with dynamic are travel times and switching delays. European Journal of Operational Research, 125(3), 486-502.
[11] Thomas, A. G., Mei. F.L., 2003. A Multimodal Transportation simulation Model for us costal Container Ports.TRB2003AnnualMeeting: USA: 2003.
[12] Wei, H., Li, J., Liu, N.Z., 2006. Analgorithm for shortest path with multi-modal in time-varying network.Chinese Journal of Management Science, 14(4), 56-63.


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