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Differences Between HCM Procedures and Fundamental Diagram Calibration for Operational LOS Assessment on Italian Freeways

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Abstract

A not clear national framework and the uncertainty as to the transferability of the U.S. HCM require an assessment of standard methodologies more calibrated on Italian freeways. The study is aimed at testing the Levels of Service assessment for a sample freeway segment on the basis of a calibrated Fundamental Diagram, and at evaluating its consistency with respect to the most recent methodologies from HCM2010. The research shows a test calibration of the Fundamental Diagram according to the Longitudinal Control Model and Van Aerde Model. The comparative analysis shows how standard procedures and ranges could underestimate operational congestion levels on the test section. Therefore, the results suggest that operators should to use carefully HCM standard procedures and that transferability issues should be further analyzed.

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Keywords: freeway LOS assessment; speed – flow curves; fundamental diagram calibration;

Nomenclature

q	flow rate [veh/h or pc/h]
v	space mean speed [km/h]
v_t	time mean speed [km/h]
k	density [veh/km or pc/km]
e	traffic flow efficiency [pc*km/h]
q_c	maximum flow rate, or capacity [veh/h or pc/h]

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k_c	critical density, or density value correspondent to capacity [veh/km or pc/km]
v_c	critical speed, or space mean speed value correspondent to capacity [km/h]
k_j	jam density [veh/km or pc/km]
v_f	free-flow speed [km/h]
FD	Fundamental Diagram
LC	Longitudinal Control
PCE	Passenger Car Equivalent coefficient
q_b	base flow rate, with passenger cars only [veh/h]
q_{mix}	mixed flow rate, with both passenger cars and heavy vehicles [veh/h].
P_t	heavy vehicles proportion in the mixed stream
(k_i, v_i^{85})	sample data point from binning procedure
γ	average aggressiveness of driver population [s²/m]
τ	average time response of driver population [s]
L	average effective length of the vehicles [m]
$RMSE$	random mean square error
$NRMSE$	normalized random mean square error
c_1, c_2, c_3	Van Aerde Model constants
E_t	Passenger Car Equivalent coefficient in HCM
FFS	free flow speed in HCM [km/h or mph]
BP	Break Point flow in HCM [pc/h]
f_{LW}	adjustment for lane width in HCM [mph]
f_{LC}	adjustment for right - side lateral clearance in HCM [mph]
TRD	total ramp density in HCM [ramps/mi]
f_p	adjustment for driver population in HCM
k_{LOSi}	max density for Level Of Service i (LOSi) [pc/km]
q_{LOSi}	max flow rate for Level Of Service i (LOSi) [pc/h]
v_{LOSi}	min speed for Level Of Service i (LOSi) [km/h]

1. Introduction

Important decisions of investment in roads and motorways are significantly influenced by the results of the functionality analysis of one or more existing or planned segments, which form the road network. Theories about traffic flow enable us to describe the phenomena related to vehicular traffic, and the same theories represent an indispensable construct to the implementation of models and tools for the design and management of road infrastructures and highways.

Starting from traffic flow theory and from the experiences due within the Committee on Traffic Flow Theory of the U.S. TRB, methodologies specified by the Highway Capacity Manual have become common at an international level. The Highway Capacity Manual, shortly HCM, collects procedures for computing the capacity and the quality of service of various highway facilities by the definition of the speed - flow curves and of Levels of Service, which describes a range of operating conditions on a freeway facility. Even the Italian context draws on the HCM procedures for the analysis of existing and planned infrastructure, especially highways. As early as 2001, the Italian legislation while summoning the concept of Level Of Service about the minimum conditions of functionality on a portion of infrastructure and in relation to type and location, does not explain methodological and procedural references by which these checks must be performed. It only refers to 1994 HCM edition, but it is however rejected by the common practice that refers to the 2000 edition of HCM. Recently TRB has released its latest issue of HCM 2010, which presents changes about the method for Free Flow Speed (FFS) calculation and for speed-flow curves identification, which are a critical part of Level of Service estimation.

The lack of a clear national framework and the uncertainty as to the transferability of the HCM require an assessment of the methodologies that is calibrated on situations that really happen in Italy. On the other hand, the experience of the Italian concessionary companies in using HCM methodologies as an analytical basis for the calculation of LOS, shows significant concerns in relation to the representativeness of the results, highlighting a gap between the outcomes of the analysis and the perception by the same operators of what is the real degree of congestion of the network. Accordingly, a specific study could be useful to work out a systematic way to evaluate the applicability of these methodologies. Indeed, the analysis of functionality represents not just a regulatory requirement and a test element in the planning and design phase, but also a business valuation matter. Therefore, it has to meet two basic criteria: adequacy and representativeness.

These considerations outline the purpose of the research, which is focused on several aspects concerning the experimental analysis of the Levels Of Service on the Italian motorway network. More clearly, this research is aimed at testing the applicability of traffic flow models to calibrate the Fundamental Diagram of traffic for a sample segment of the Italian motorway network, to calculate its performance by the calibrated FD, to evaluate the consistency of these findings with respect to the most recent methodologies from HCM2010.

Section 2 is devoted to the general topics of the Traffic Flow Theory and of the Fundamental Diagram, highlighting the main experiences in the assessment of the circulation quality using calibrated speed – flow - density functions. Section 3 presents the Longitudinal Control Model and the Van Aerde Model that are used in the paper for the calibration of the Fundamental Diagram. Section 4 presents the data sample, collected on a segment of an Italian freeway, and shows calibration procedures and results obtained by applying the two models. Finally, section 5 presents an analysis of the levels of service carried out by using the two calibrated models, which in turn is compared with the references suggested by HCM2010 and with the results that are obtained by applying the related procedures.

2. Traffic stream models and Fundamental Diagram

2.1. General issues of a Fundamental Diagram

The fundamental relationship of traffic flow represents the relation among density, space mean speed and flow, respectively k, v and q, which are related by the identity:

$$q = k \cdot v \tag{1}$$

The keyword "Fundamental Diagram" (FD), introduced by Haight (1963), indicates the empirical relationship expressed by the flow – density function $q = f(k)$. From the fundamental relationship (1), with a substitution for q, FD can be expressed as a speed – density curve; with a substitution for k it can be expressed as speed – flow curve. Therefore, with the keyword FD we consider the three equivalent forms related to the fundamental relationship of traffic flow:

$$q = f(k), \qquad v = f(k), \qquad q = f(v), \tag{2}$$

The identification of the FD, which is expressed through the pairwise relationships under the hypothesis of stationary, has involved several studies starting from the experiences of Greenshields (1935). In order to obtain a FD that is consistent with experimental data, different steady-state models have been proposed, which are based on assumptions of the single or multi regime, respectively with one or more valid relationships within certain ranges of variables. Indeed, if the k – q - v relationships are usually related to the macroscopic level, many macroscopic models, along with their FDs, can be derived from microscopic models. New models have been developed, which highlight an adequate representation of the behaviour of a single unit of traffic, together with an appropriate description of the macroscopic aggregate characteristics of the flow, explained in the FD. Among the most recent models, we can mention Van Aerde Model (Van Aerde, 1995) and Longitudinal Control Model (Ni, 2011), two single-regime steady-state models derived from microscopic field, which result flexible and consistent to represent the different conditions of the traffic flow, and to describe a broad spectrum of freeway segments with high data-fitting capabilities. In the multiplicity of functional forms which have been identified, relationships are characterized by certain key values, which often represent parameters in modeling or which can be derived from the analysis of the same functions. Below the key values of the FD:

- v_f free-flow speed FFS: for low densities the traffic speed, i.e. space mean speed, is at its maximum;
- q_c maximum flow rate: usually referred to as the capacity;
- k_c critical density: it is density value correspondent to capacity;
- v_c critical speed: it is space mean speed value correspondent to capacity;
- k_j jam density: maximum density with a speed value equal to 0.

Whatever is the functional form chosen on the basis of a defined theoretical formulation, the equations of the FD express the link between the characteristics of the flow, in dependence on certain parameters. These parameters can be estimated on the basis of experimental data of speed, flow and density, using mathematical and statistical methods for curve fitting.

In general, flow refers to traffic volume, which is defined as the number of vehicles passing per unit of time, i.e. an hour; with flow rate, we refer to hourly flows that occur for a period less than one hour. Speed for a traffic unit is the distance covered per unit of time, but as in a moving stream each particle has its speed, the flow speed can be considered as an average speed. Thus, for traffic stream the average speed can be computed as time mean speed v_t, i.e. the average speed of all vehicles passing in a specified time period, or as space mean speed v, i.e. the average speed of vehicles which occupy a given section in a specified time instant. Both time and space mean speed can be calculated from a series of vehicle speed measures by point sensors (loop or radar devices), the former as the arithmetic mean and the latter as harmonic mean. Knowing flow and space mean speed, the fundamental relationship provides an estimate of the value of the density (Knoop et al., 2009), which is normally difficult to obtain directly without using space sensors (e.g. video cameras, aerial filming).

As specified above, fundamental relationship relates q and v with density k, which is the number of vehicle occupying a given section with unit length. Indeed, a survey of the vehicles and related speeds in a specified section allows to calculate, within established time intervals, the flow rate in vehicles/h and the space mean speed in km/h, and to estimate the density in vehicles/km. These experimental points can be used for the estimation of the FD expressed in the three equivalent forms related to the fundamental relationship of traffic flow. In this way, a common approach is to calibrate the model considering the speed-density curve or the flow-density curve, which are bijective functions, and to estimate the remaining curves by using the underlying relationship (Wang et al., 2009) (Lu et al., 2013).

As previously said, the equations of the FD represent steady-state relations between the macroscopic quantities of the flow, while the measures from experimental surveys highlight scatter plot of speed, flow and density more or less extensive. Under low density conditions, and therefore of free flow speed, the interactions between the vehicles are rare and the experimental points are concentrated; as density and the interactions between vehicles increase, on the contrary, the points appear to be more dispersed. These differences between the steady-state relation and the scatter of experimental data are due to the fact that in reality the traffic is neither stationary nor homogeneous, as observed by Moerivot and De Moor (2005). To overcome this problem Del Castillo and Benitez (1995) have proposed the separation of non-stationary from stationary states. The restriction proposed, together with the choice of measurements made in homogeneous situations of the traffic and of external variables that can influence the flow (time, weather conditions, etc.) is a requirement for the correspondence of the theoretical model to the experimental data. However, in this research as in many other studies that address the calibration of an FD on the basis of experimental data, the restriction is removed and it is considered the whole scatter plot obtained in such a short time interval, e.g. 5 minutes. This choice reduces the probability of multiple traffic state combination and captures most of the trends in the varying traffic conditions. Hence, the space mean speed and the density in selected time intervals are used in order to obtain, by means of optimization techniques, the regression model of the FD.

2.2. Fundamental Diagram and circulation quality

The link between the circulation quality and the macroscopic variables of traffic flow is an issue that interested the research over the last 50 years. From this point of view the HCM is, along with the methods that share its setting, a reference tool enormously widespread and applied worldwide for the evaluation of the operating conditions of the traffic. HCM is published by the Transportation Research Board (TRB) of the U.S. National Academies of Science until 1950's and it collects procedures for computing the capacity and the quality of service of various highway facilities. As in the 1950's freeways were not common, only with the second edition of 1965 the TRB approached the study of the traffic on a motorway facility under the guidance of the Highway Capacity and Quality of Service Committee from US Bureau of Public Roads, introducing speed - flow curves as an expression of the FD for freeways. Passing through several editions, in 2010 TRB provided the last version of the manual, i.e. HCM2010. Statistical analysis conducted on a significant database of speed – flow data, and judgment criteria expressed by a special Committee have led to a set of FD curves for freeways developed with respect to Free Flow Speed (FFS). As FFS is estimated by a specified methodology, FD curves cover a range of capacity from 2250 pc/h/ln for 55 mi/h FFS to 2400 pc/h/ln for 70 -75 mi/h FFS. For all the curves it was identified a constant value for density at capacity, which is equal to 45 pc/mi/ln (28 pc/km/ln). The equations consider two-segment curves: the first segment for flow values between 0 and a Break Point flow (BP) identified for the specific FFS, with constant speed equal to FFS; the second segment for flow values between the same BP and the capacity, and speed which vary according to the formula:

$$v = FFS - a \cdot (q - BP)^b \tag{3}$$

Closely linked to the definition of the speed - flow curves in the different editions of the HCM is the definition of Levels of Service, which describes a range of operating conditions on a freeway facility. Level of Service, or LOS, is indexed by the letters from A to F, where A stands for free-flow conditions and F stands for jam conditions. LOS classifications are based on a Measure Of Effectiveness, MOE, which is the density for HCM2010.

LOS A represents free flow conditions: individual users are virtually unaffected by the presence of others in the traffic stream. LOS B is in the range of stable flow, but the presence of other users in the traffic stream begins to be noticeable. Also LOS C is in the range of stable flow, but it marks the beginning of the range of flow in which the operations of individual users become significantly affected by interactions with other vehicles in the traffic stream. LOS D represents high-density but still stable flow; small increases in traffic flow can cause operational problems. LOS E represents operating conditions close to the capacity level; operations are usually unstable and small increases in flow or minor perturbations within the traffic stream will cause breakdowns. LOS F is used to define forced or breakdown flow. LOS classifications are based on some measures of effectiveness, MOE, changeable with the various HCM editions.

Anyhow, the reference values reported in HCM are derived from statistical analysis and from judgment criteria expressed on U.S. highways, in relation to the characteristics of infrastructure, traffic conditions, driver behavior and vehicle characteristics, which are typical in the United States. Due to the fact that different site, different rules and different behavior can show different relationships between traffic variables, a critical review of HCM assumptions must be taken into account. In recent years some studies have proposed methods for calibrating HCM curves, with the possibility of identifying the specific relationship between flow and speed, the capacity and the relative values for density and critical speed. In some cases, as Khazraee et al. (2012), these values are significantly different compared to HCM standards, with values for critical density and lower capacity than the standards in the manual.

An analysis of capacity and speed - flow curves outside the U.S. territory and an assessment of the differences compared to the previous editions of HCM has been proposed by Bertini et al. (2006). According to the study, the values for the capacity used by HBS 2001, analogous to HCM in German territory, are lower (12-17%) than those of HCM. The authors attribute these disparities to different strategies in freeways operations and control, and to different driver behavior and vehicle operating characteristics between Europe and U.S.. Wu (1998) justifies the lower values for the capacity defined by the HBS with the inhomogeneous division of traffic on the lanes that occurs in Germany. The inhomogeneous lane split is put in relation with driving rules that consider jointly the command of driving on right and the prohibition of overtaking on the right, which are valid in Germany and in most European countries like Italy, against the homogeneous distribution on the lanes ensured by the "keep in lane" rule in the U.S.. Also for these reasons speed-flow relationships in HBS are referred to the whole direction carriageway, and not to a single lane as in HCM. In the draft revision of HBS Wu (2009) reaffirms lower values for the capacity of a roadway, which are 25% lower compared to the U.S. for three lanes and flat ground carriageway. Other authors, such as Brilon et al. (2011), have suggested some alternatives to the current practice of using the relationships defined by HCM, especially outside the U.S., proposing models calibrated with experimental data.

Although most calculation procedures for circulation quality, capacity and level of services are based on traffic flow models derived from measured data, as mentioned above, some significant approaches are based on the concept of reliability of traffic flow. Studies on traffic flow reliability have been conducted in Europe using: the principal component analysis (Ferrari, 1982) that identifies a saturation flow rate; the breakdown analysis that characterizes capacity as a random variable (Brilon, 2005); the instability analysis that deduces capacity from the inverse procedure identified for the formulation of instability probability (Ferrari, 1991) (Mauro, 2011).

In this way, calibrated methods must be investigated to test HCM standard practices, often applied uncritically and regardless without considering the non homogeneity of the contexts.

3. Models for Fundamental Diagram calibration

As mentioned before, recent studies on traffic models have provided a significant contribution to bring both micro and macro approaches in a simple and efficient structure, making it possible to synthesize the phenomena associated with traffic flow. Longitudinal Control Model (LC Model) proposed by Ni (2011), is a single-regime steady-state model derived from microscopic field, which results simple, flexible and consistent to represent the different conditions of the traffic flow, and to describe a broad spectrum of freeway segments with high data-fitting capabilities. Under steady-state conditions, the macroscopic version of the LC Model is derived from the equation of the microscopic model, which describes the control and the longitudinal motion of a vehicle subject to a force field determined by the presence of the road infrastructure and of the other vehicles in motion on the same roadway. This research aims at investigating and use the LC Model proposed by Ni (2011), which is given by:

$$v = v_f \left[1 - e^{1 - \frac{k^*}{k}} \right] \tag{4}$$

where $$k^* = \frac{1}{v^2 \gamma + v\tau + L} \tag{5}$$

In this way LC Model is expressed by a relationship between space mean speed and density that depends on two parameters v_f free flow speed (FFS) and k^*, which in turn is a function of the speed and a function of the three parameters: average aggressiveness γ and average time response τ of driver population, and average effective length of the vehicles L. The other two relationships, density – flow and speed – flow, will be obtained from the fundamental equation of traffic flow, providing for the complete definition of the LC Model FD. The use of LC Model allows estimating the FD getting the key values, i.e. free flow speed, the critical values of flow, velocity and density corresponding to the capacity, and the jam density. The model also allows having an estimate of parameters useful to assess and compare the driving behavior, in terms of average time response and drivers' aggressiveness.

In order to test the adequacy of the LC Model, it was considered useful to make a comparison with the previously mentioned Van Aerde Model, widely used in the calibration of the Fundamental Diagram and for the evaluation of flow characteristics of freeway sections in U.S., Canada, (Van Aerde & Rakha, 1995) (Washburn et al., 2010), Netherlands (Van Aerde & Rakha,1995), Germany (Brilon & Lohoff, J. (2011), Iceland, (Erlingsson et al., 2006). Van Aerde Model is based on a simple car following model which considers the minimum distance headway between consecutive vehicles as a combination of a constant term, a term which depends on the difference between the space mean speed v and the free flow speed v_f, and a term which depends on the space mean speed v.

The model is given by:

$$k = \frac{1}{c_1 + \dfrac{c_2}{v_f - v} + c_3 \cdot v} \tag{6}$$

where c_1, c_2, c_3, respectively fixed distance headway constant (km), first variable distance headway constant (km^2/h) and second variable distance headway constant (h), are the parameters for the respective three terms. Van Aerde and Rakha (1995) demonstrate the fitting capabilities of the model comparing the traffic flow parameters to other single- and multi-regime fits, by using data from the literature and data from both European and North American freeways.

4. Test section and Diagram calibration

4.1. Data sample

The two models have been estimated using a set of data, collected in July 2012 by the concessionaire company in a 3-lanes test section of the Italian A4 Motorway between Padua West and Padua East, at km 361 east bound. The freeway section is on level terrain with a vertical profile that is not characterized by significant slopes, and it is located on a straight stretch far from ramps (on-ramps or off-ramps). Each lane is 3,75 m wide, with 3.0 m shoulder lane, and there are no special restrictions on overtaking or speed, except those provided for by general law (130 km/h for light vehicles and 80-100 km/h for heavy vehicles). Referring to Hall (2001), type location for test section is B, which can provide information on the uncongested portion and on capacity operation.

Each lane is equipped with a radar detection device, placed in an elevated position with respect to the roadway. Radar devices allow the concessionaire company to store the individual transits, with the date and time of the passage, speed, vehicle length and headway.

For the whole period, from July 1 to July 31, 2012, the database includes 1’384’177 record, one for each detected vehicle, with an ADT equal to 44’650 vehicle/day and a maximum value equal to 56’800 vehicle/day. On Friday and Saturday the traffic exceeds 50’000 vehicles per day, due to the presence of a significant component of tourists, which goes to the holiday destinations over the weekends. Due to monitoring period, it can be stated that sample data have been collected under prevailing good weather conditions.

Figure 1 (left) shows the time series of total transits per hour (blue line), and the related space mean speeds during the whole monitoring period (red line). Figure 1 (right) shows the scatter plot of speed - length of the vehicles which highlights a threshold in length distribution and in the relative speed behaviors. This value, which can be placed on 5m vehicle length, has been assumed for the classification of vehicles in short vehicles, likely light vehicles (<= 5m) and long vehicles, likely heavy vehicles (> 5m).

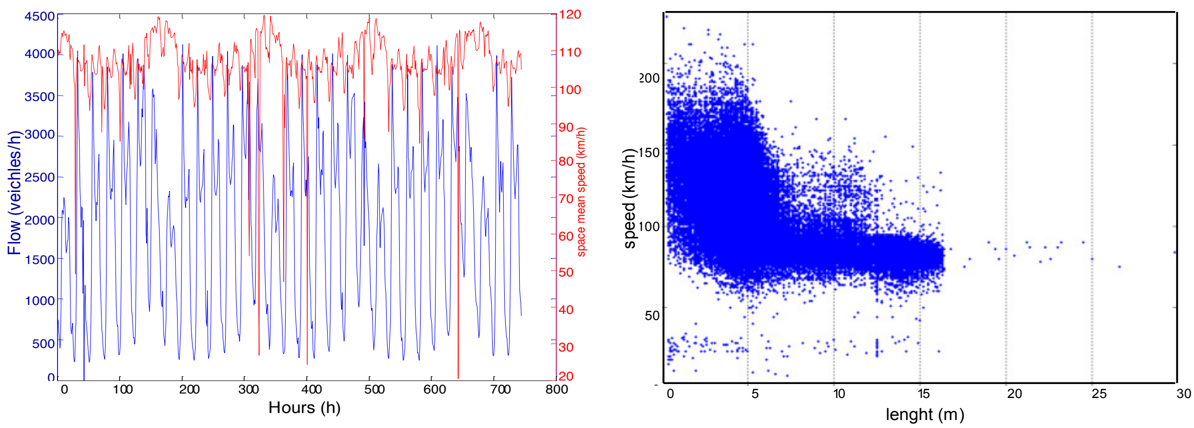

Fig. 1. Total hourly flow and space mean speed (left); Speed - length of vehicle scatter plot (right)

Hourly flow rate and average speed have been aggregated every 5-minute intervals for each lane, respectively for light and heavy vehicles. In particular, the average speed was aggregated as the harmonic mean of the speed of individual vehicles in each interval, to take into account the space mean speed required by the fundamental relationship of traffic. Flow rates have been transformed into equivalent flow rates (passenger car units per hour: pc/h) by using the Passenger Car Equivalent coefficients, PCEs. The PCEs have been estimated specifically for the test section by using the method already proposed by Sumner (1984), Elefteriadou (1994), Demarchi and Setti (2003). It considers the base flow rate q_b (passenger cars only), and the mixed flow rate q_{mix} with varying proportion for heavy vehicles, and different PCEs have been estimated for each range of heavy vehicles proportion P_t in the mixed stream by using the following definition for PCE:

$$PCE(P_t) = \frac{1}{P_t}\left(\frac{q_b}{q_{mix}} - 1\right) + 1 \quad (7)$$

Technical calculations yield PCE values that can be approximated to 3 for $P_t < 5\%$, to 2 between 5% and 10% and to 1.5 for $P_t > 10\%$. By using estimated PCE values, the macroscopic characteristics of the flow have been transformed in characteristics of the passenger cars equivalent flow (flow-speed-density).

Because of non-homogeneous traffic conditions related to driving rules and driver behaviors that exist on the Italian network, flow rates and harmonic mean speed have been aggregated over the whole three-lane carriageway for the equivalent flow and, using the same values, density has been estimated for each 5-minute interval by the fundamental relationship (1). Figure 2 shows the scatter plot k - v, q - v and k - q. Color gradient displays the frequency with which the different experimental points occurred during the monitoring period.

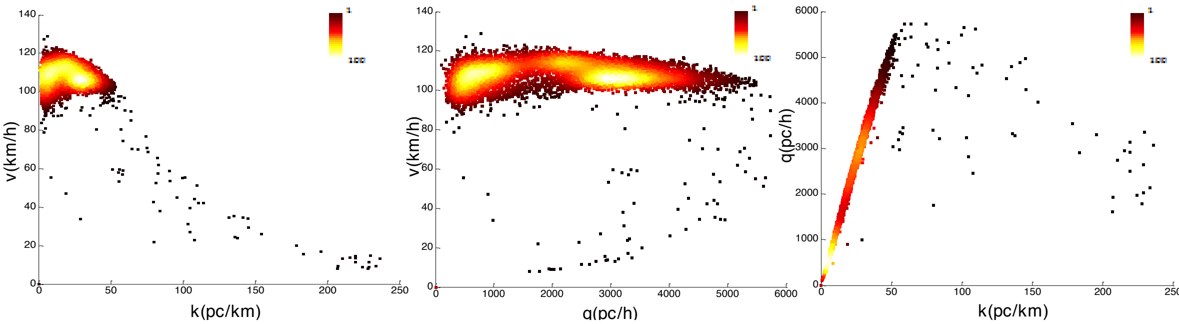

Fig. 2. Density – speed, flow – speed, and density – flow frequency plots combination

As the scatter plots show, speed covers a variable range, up to a maximal value for a given density. For low density conditions, the upper limit represents free-flow speed, which can be estimated as the 85th percentile of related speed observations (Moses et al, 2013). The upper limit decays for larger density value and it can be considered as the speed at which drivers' behaviors get restricted by the surrounding vehicles in the flow, estimated by the same percentile of speed observations.

In order to consider the upper end of the distributions of speed and density, an approximate quantile regression method (Chow et al, 2008), (Dervisoglu et al, 2009), is implemented. Using a binning procedure (Wickham, 2013), density range is partitioned into bins, in order to condense the original dataset, with real density and speed values, into a smaller one, with summary density and speed values. The method involves a fixed bin condensation, so that each bin involves intervals of amplitude 1 for density. For each window the mean density k_i and 85th percentile v_i^{85} is determined among values in the bin, as summary value for them. As we choose the speed - density function for curve fitting, the other two relationships density – flow and speed – flow will be obtained from the fundamental relationship among the three principal variables: $q = v * k$. As v_i is the 85th percentile value v_i^{85}, i.e. the threshold speed for a given density k_i, we remember that q_i is the consequent threshold flow for v_i^{85} and k_i. The resulting points (k_i, v_i^{85}) have been assumed as sample data points for model estimation using Non Linear Regression (Smyth, 2002).

Furthermore, if on one hand the set of observed points is indispensable for a curve fitting procedure, on the other hand the existence of outliers may lead to incorrect results. It means that one or more outlier data can have a big influence on the resulting curve and on estimated parameters. For this reason, to obtain a robust Non Linear Regression, the points (k_i, v_i^{85}) have been filtered before model estimation, in order to mitigate the influence of outliers in regression. The filtering method (Vivatrat, 1979) subdivides the range of densities in bins containing a minimum of 5-10 points. In this application the bins have been chosen with an amplitude $\Delta= 10$. Within each bin, the values are filtered out of range $\mu_i \pm A^*S_r$, where μ_i is the median of the points, S_r is the representative standard deviation and A is the coefficient that determines the amplitude semi-interval considered acceptable for values, here equal to 2. S_r is defined as the minimum value between the following expressions:

$$S_r = \frac{1}{2}(S_{i+1} + S_i) \qquad S_r = \frac{1}{2}(S_{i-1} + S_i) \qquad S_r = \frac{1}{2}(S_{i+1} + S_{i-1}) \tag{8}$$

where S_{i-1}, S_i and S_{i+1} are the standard deviations calculated for the bins *(i-1)*, *i* and *(i+1)* respectively. The filtering procedure identifies outliers and assigns them a zero weight in the fitting process.

4.2. Longitudinal Control Model calibration

The four parameters of LC Model have been estimated through a numerical procedure of curve-fitting based on Non Linear Regression. The basic idea of Non Linear Regression is the same as that of Linear Regression, characterized by a prediction equation non-linearly dependent on one or more unknown parameters. Non Linear Regression model has the form:

$$v_i = f(k_i, v_f, \gamma, \tau, L) + \varepsilon_i = f(k_i, \theta) + \varepsilon_i \tag{9}$$

where f is non linear in v_f, γ, τ, L and ε_i are random, uncorrelated errors with mean zero and constant variance.

A Non Linear Regression model problem cannot be solved in one step, but it must be solved iteratively starting from an initial estimate of each parameter. In this case the physical meaning of each parameter makes it not difficult to estimate a first guess for free flow speed v_f, average aggressiveness γ, average time response τ and average effective length L of the vehicles.

By using non outliers points, the four parameters have been estimated through a numerical procedure of curve-fitting, based on the Non Linear Least Squares Method and adapted to a non-explicit function. Least Squares Method is a very common tool for the Non Linear Regression curve fitting problem. As $f(k_i, \boldsymbol{\theta})$ is a non linear function and (k_i, v_i^{85}) are sample data points, the objective of Least Squares Method is to find $\boldsymbol{\theta}=(v_f, \gamma, \tau, L)$ that minimizes the sum of squared errors $s(\boldsymbol{\theta})$:

$$\min_\theta \sum_i [v_i - f(k_i, \theta)]^2 \tag{10}$$

The optimization algorithm used for the Least Squares minimization is the Trust Region Reflective, an iterative procedure for minimization of the sum of squared errors $s(\boldsymbol{\theta})$ in order to find, for each step, a $\boldsymbol{\theta}_{\iota+1}$ with a smaller value for s than the previous $\boldsymbol{\theta}_\iota$ by a quadratic approximation of s in the neighborhood N of $\boldsymbol{\theta}_\iota$ from its Taylor expansion around $\boldsymbol{\theta}_\iota$. N is called trust region and both $\boldsymbol{\theta}_\iota$ and $\boldsymbol{\theta}_{\iota+1}$ will be in the region. Trust Region Reflective is one of the algorithm options for NLLSR in Matlab7.11 (release 2010b).

Non Linear Least Squares Regression algorithms, like Trust Region Reflective, must be iterative and an initial estimate of the value of each parameter must be specified. Actually, it is an important issue in a Non Linear curve fitting problem: a bad selection of initial values may have important consequences, like wrong or no solution for the problem. To obtain a good first guess for parameters, we have considered typical values for γ and τ. Average driver aggressiveness γ has a value of -0.035 s^2/m, typical of platoons of vehicles with medium aggressive drivers, while average time response τ has a value of 2 s.

The initial values for v_f and L are estimated by analyzing the trend of the flow characteristics, in correspondence of the extremes of the domain of k. For $k = 0$, in fact, speed matches v_f, while for $k = k_j$ we get the effective length L, that is equal to the reciprocal of jam density on the single lane. The values are obtained considering two Linear Regressions on the scatter plots q_i - k_i, where $q_i = v_i^{85} * k_i$. The first Linear Regressions considers $k < 30$ *pc/km*, i.e. presumably not conditioned traffic for three-lane carriageway, allows to obtain v_f, and the second Linear Regressions considers $k > 90$ *pc/km*, i.e. presumably conditioned traffic, allows to obtain k_j and then L. Figure 4 reports the two regression lines, and the initial values for the two parameters.

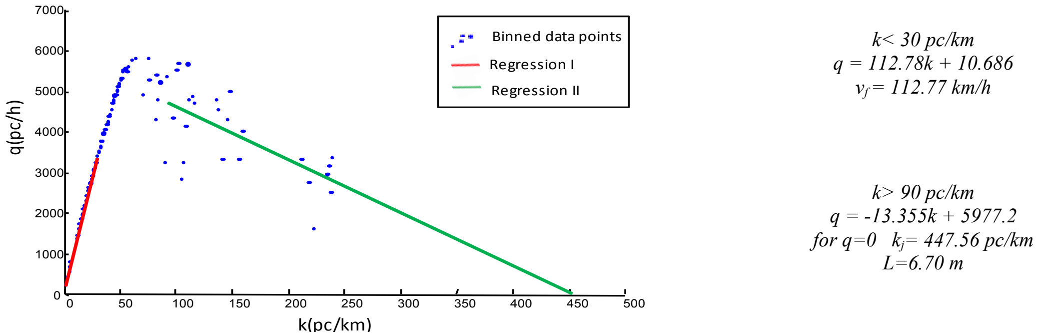

Fig. 3. Linear regression for estimating initial values for v_f and L

Table 1 contains the estimated parameters by Non Linear Least Squares Regression, and their 95% confidence interval, evaluated by Monte Carlo Method with 1000 iterations. The Monte Carlo Method for confidence interval estimation uses the standard error of the fit of the Non Linear Regression model to the observed data, to produce sets of virtual data. These virtual data are modeled using the same Non Linear Regression model and a new group of parameters is estimated for each virtual set of data. Confidence intervals can be obtained from the statistical distribution of the parameters

Table 1. Estimated LCM parameters and asymptotic 95% confidence interval

Segment	v_f (km/h)	γ (s^2/m)	τ (s)	L (m)
361_EB	111.65 (110.22|113.61)	-0.0377 (-0.0305|-0.0421)	1.3858 (1.1448|1.5207)	7.5709 (5.7561|8.6959)

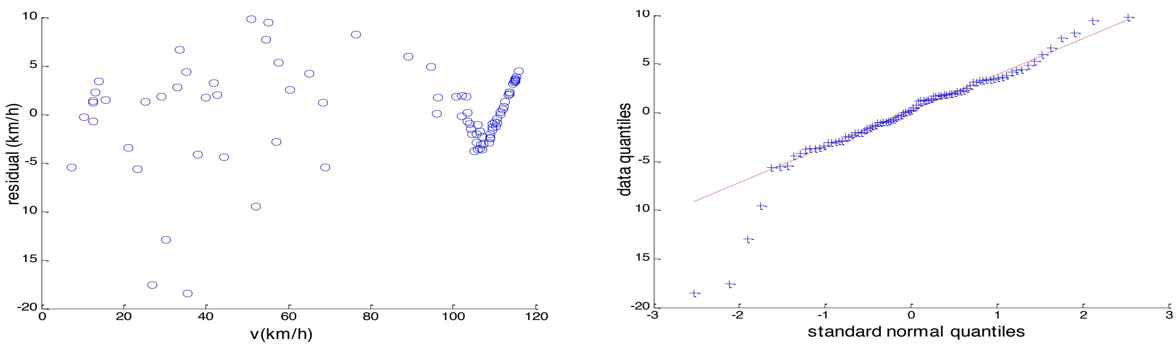

Fig. 4. Residual plot (left) and normality checks for residuals (right)

Confidence intervals appear quite narrow and parameter estimation can be considered sufficiently precise, with a free flow speed (FFS) v_f which is slightly less than 112 km/h, an average aggressiveness γ equal to -0.038 s^2/m, an average time response τ equal to 1.39 s, and average effective length L of the vehicles of 7.6 m.

Model estimation requires the assumption that the errors are uncorrelated random variables with mean zero and constant variance. Figure 4 shows the residual plots and a normality distribution check, while the Table 2 shows some accuracy measures for Non Linear Least Squares Regression curve fitting.

Table 2. Accuracy of curve fitting by Non Linear Regression

Data points	Degrees of freedom	Sum of Squares Regression	Sum of Squares Total	RMSE	NRMSE	R^2
87	4	1885.0691	119088.3294	4.7657	4.4%	0.98

Table 3 illustrates the main key values from the estimated parameters of LC Model and their 95% confidence interval. Confidence intervals have been evaluated by using a bootstrap technique, which involves parameter sets estimated in each iteration of Monte Carlo Method. Figure 5 contains a superimposition of the relationships of the calibrated FD, and scatter plots of data points (original and binned data).

Table 3. Estimated LCM Fundamental Diagram key values and 95% confidence interval.

Segment	**q_c (pc/h)**	**k_c (pc/km)**	**v_c (km/h)**	**k_j (pc/km)**
361_EB	5489 (4981|5978)	59 (56|62)	93 (88|96)	396 (332|468)

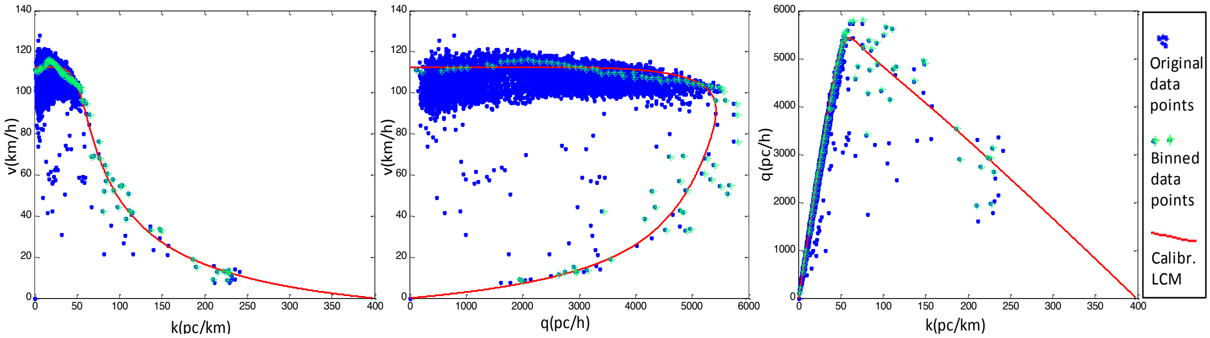

Fig. 5. Scatter plot and calibrated LC Model - FD relationships

4.3. Van Aerde Model calibration

The parameters of Van Aerde Model have been calibrated by a Non Linear Regression analysis on the basis of a speed-density relationship (Van Aerde and Rakha, 1995). The regression analysis procedure that performs the fitting of the Van Aerde Model is implemented in the software program SPD_CAL (Rakha, 2007). The software program calibrates the parameters for the sample data, which form the dataset already used for LC Model calibration, by a heuristic hill-climbing technique to determine the optimum parameters. The calibrated parameters are used to determine the free-flow speed v_f, speed at capacity v_c, capacity q_c, and the jam density k_j. From the optimization technique, the model parameters are calculated from the following:

$$c_1 = m \cdot c_2 \qquad m = \frac{2 \cdot v_c - v_f}{\left(v_f - v_c\right)^2} \tag{12}$$

$$c_2 = \frac{1}{k_j \cdot \left(m + \frac{1}{v_f}\right)} \qquad c_3 = \frac{\frac{v_c}{q_c} - c_1 - \frac{c_2}{v_f - v_c}}{v_c} \tag{13}$$

Table 4 illustrates the main key values from the estimated parameters of Van Aerde Model, comparing them with the corresponding values obtained by LC Model calibration. The results show the LC Model estimated values for capacity q_c, critical speed v_c, and critical density k_c, which are very close to Van Aerde Model ones (Figure 6).

Table 4. Estimated Van Aerde and LCM Fundamental Diagram key values.

Segment	v_f (km/h)	q_c (pc/h)	k_c (pc/km)	v_c (km/h)	k_j (pc/km)
Van Aerde	109.30	5528	60	92.5	396
LCM	111.65	5489	59	93.2	424

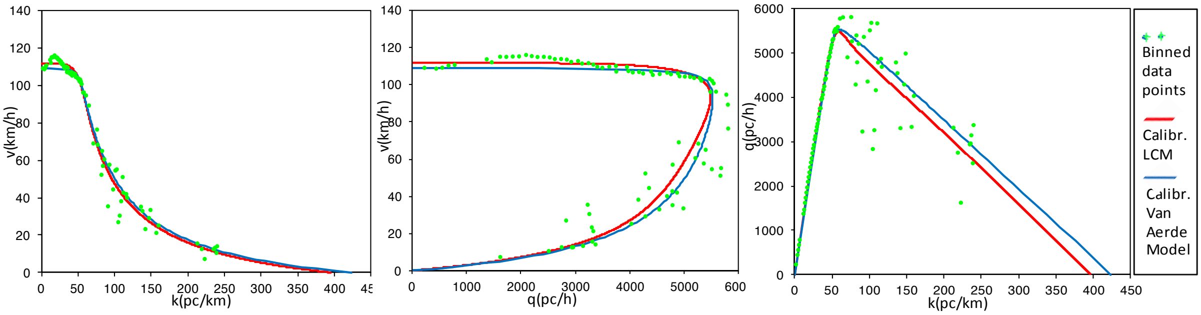

Fig. 6. Binned data plot and calibrated LC and Van Aerde FDs relationships

5. Comparative LOS analysis

The estimated LC Model and Van Aerde Model can be used in capacity and LOS analysis for the test segment. For the definition of the LOS, specific density ranges have been identified in analogy with what has been proposed by HCM. The upper threshold k_{LOSE} of LOS E is represented by capacity threshold, and then by k_c. For the upper threshold of LOS D we used the density corresponding to the maximum value of the efficiency *e* (Brilon, 2000). In analogy with mechanical systems and replacing the concept of force with the flow of vehicles, efficiency can be defined as the traffic flow power, i.e. the work done by the flow *q* moving with velocity *v* in the unit of time T:

$$e = q \cdot v \cdot T \tag{14}$$

In these terms, the efficiency expresses the total kilometers traveled per unit of time by the flow that runs through the section. Maximum efficiency is an important MOE for the performance of the section, jointly in terms of flow and speed; furthermore, for a toll road, it can be linked to the productivity of the flow in terms of toll collection, being related to the total of the distances traveled. For density values above the efficiency maximum, the freeway segment has not yet reached the limit of capacity, but it is no longer operating at its highest possible level. These are the reasons why it is useful to use the density corresponding to the maximum efficiency as the limit between LOS D and LOS E, i.e. k_{LOSD}. For the thresholds of the remaining LOS C, B and A, the density range 0-k_{LOSD} has been divided into four equal parts.

Table 5. Estimated LOS ranges (Longitudinal Control Model and Van Aerde Model)

Max value of	A		B		C		D		E	
	LC	Van Aerde	LC	Van Aerde	LC	Van Aerde	LC	Van Aerde	LC	Van Aerde
k_{LOS} (pc/km) ≤	12.9	13.0	25.8	26.0	38.6	39.1	51.5	52.1	58.9	59.8
v_{LOS} (km/h) ≥	111.6	109.3	111.5	108.7	110.1	108.1	103.0	103.1	93	92.5
q_{LOS} (pc/h) ≤	1437	1421	2873	2832	4253	4221	5304	5370	5489	5528

Table 5 shows the density ranges for each level of service for the segment; flow and speed limits for each LOS are shown as well, with fairly small differences, both for LC and Van Aerde models. By using the identified density ranges, the percentage of occurrence of each Level of Service are calculated during the whole period of monitoring.

The last edition of the Manual, i.e. HCM2010, estimates FFS (mph) by using the equation

$$FFS = 75.4 - f_{LW} - f_{LC} - 3.22TRD^{0.84} \tag{15}$$

where f_{LW} is the adjustment for lane width (mph), f_{LC} is the adjustment for right - side lateral clearance (mph), and *TRD* is the total ramp density (ramps/mi).

For the test section, as described in section 4.1, HCM2010 suggests f_{LW} = 0 and f_{LC} =0. Whereas *TRD* = 0.5 ramps/mi, *FFS* value is equal to 73.56 mph, i.e. 118.40 km/h. As HCM 2010 recommends to round the FFS to the nearest 5 mi/h avoiding interpolation of curves, the free flow speed reference curve is identified for *FFS* = 75 mph.

HCM2010 identifies 2400 pc/h/lane as basic capacity, i.e. 7200 pc/h on three lanes carriageway, with a critical density equal to 28 pc/km/lane, i.e. 84 pc/km on three-lane carriageway, and a speed to capacity equal to 53.3 mph, i.e. 85.8 km/h. As the Manual also suggests, if a measured *FFS* is available this value can be assumed for speed-flow curve identification; using LC Model *FFS* estimation (111.65 km/h i.e. 69.37 mph) or Van Aerde *FFS* estimation (109.3 km/h, i.e. 67.79 mph) the free flow speed reference curve is the one for 70 mph. Critical value at capacity are the same suggested for 75 mph curve.

Figure 7 reports the overlap between the speed-flow curves in HCM 2010 for 70 mph (113 km/h) and 75 mph (120 km/h), and the curves calibrated for the LC Model and Van Aerde Model. The calibrated FDs show a capacity and a critical density to values lower than those proposed as basic by HCM. The clear deviation between the two curves can be attributed to several specific conditions of the section and differences in characteristics of vehicles or driving behavior of drivers with respect to situations taken as standard from the Manual.

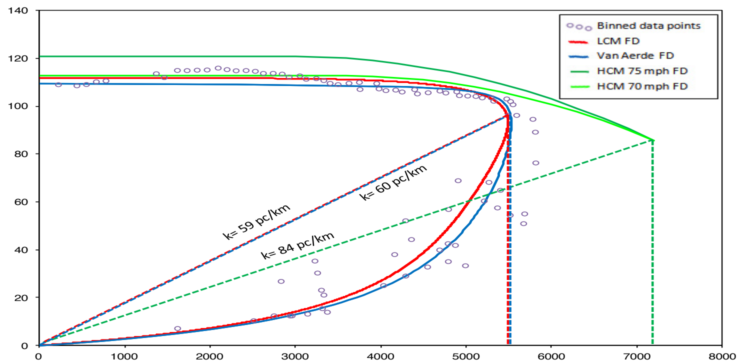

Fig. 7. Calibrated LC and Van Aerde speed – flow curves and HCM 2010 70 -75 mph curves

The identification of Levels of Service according to the HCM2010 involves the average density per lane as the ratio of the equivalent flow and the average speed, and then the assignment of LOS according to specific ranges, which are declared in Table 6.

For calculating the equivalent flow rate, in each time interval it is considered the total flow rate, the related proportion of heavy vehicles P_t and the passenger car equivalent E_t, which is set equal to 1.5 as suggested for level terrain. Due to the short duration of time intervals, Peak-Our Factor (PHF) is set equal to 1. As regards the adjustment for driver population, we have considered two situations: case [1] with $f_p = 1$ for mostly commuters and case [2] with $f_p = 0.9$ to take into account non-commuters.

Table 6. HCM2010 LOS definition for 70 mph and 75 mph speed – flow curves (total for three-lane carriageway)

LOS	A		B		C		D		E	
FFS(mph)	70	75	70	75	70	75	70	75	70	75
k_{LOS} (pc/km) ≤	20.5	20.4	33.3	33.3	48.3	48.2	65.1	65.2	83.9	83.9
v_{LOS} (km/h) ≥	112.7	120.7	112.6	118.9	107.4	110.1	97.2	98.0	85.8	85.8
q_{LOS} (pc/h) ≤	2310	2460	3750	3960	5190	5310	6330	6390	7200	7200

The density required to identify the LOS has been determined by following two different approaches: the first one, named HCM2010, estimates the density for each time interval by considering the equivalent flow rate, calculated as described above for the cases [1] and [2], and the mean speed as obtained by the speed – flow function suggested for FFS = 75 mph (i.e. FFS estimated using the eq. 15 and the parameters provided by the Manual). The second approach, named HCM2010* estimates the density by considering the same equivalent flow rate and the space mean speed from sample data.

On the basis of the identified density ranges for each time interval, the proportion of occurrence of each Level of Service has been calculated for HCM2010 and HCM2010* during the whole period of monitoring, and both for cases [1] and [2]. The results are illustrated in Table 7 and compared with the evaluations obtained by the LC and Van Aerde models. Although the comparative analysis is limited to a single segment highway, it can be observed how the maximum densities for each level of service estimated by calibrated FDs are significantly lower than those of HCM2010, with a density to capacity (k_{LOSE}) 30% lower, and with a density limit for LOS D (k_{LOSD}) 23% lower. Actually, for the test segment, HCM analysis represents better circulation conditions than those obtained by the FD calibrated using the LC and Van Aerde models. The comparative analysis shows how HCM density ranges could underestimate operational congestion levels on the test section.

Table 7. Percentage of occurrence of LC Model, Van Aerde Model and HCM2010 LOS

% LOS	A+B	C+D	E+F
LC Model	61%	37%	2%
Van Aerde Model	63%	36%	1%
HCM 2010 (case [1]-case [2])	91% - 85%	9% - 15%	0% - 0%
HCM 2010* (case [1]-case [2])	83% - 74%	17% -25%	0% - 1%

6. Conclusion and recommendations

Highway Capacity Manual is the main reference for planning, design and analysis of a highway infrastructure, and its methods and procedures are invoked by technical regulations of many countries and used in the practice by many technicians. It should be remembered, however, that the reference values reported in HCM are derived from analysis on U.S. highways and from judgments expressed by a special Committee, in relation to the characteristics of infrastructure, traffic conditions, driver behavior and vehicle characteristics, which are typical in the U.S..

Even the Italian context draws on the HCM definitions for the analysis of existing and planned infrastructure, especially highways, but technical regulations don’t explain methodological and procedural references by which these checks must be performed. The lack of a clear national framework and the uncertainty as to the transferability of the HCM, moreover shown by the experience of the Italian concessionary companies, call for an assessment of methodologies more calibrated on actual situations occurring in Italy.

These are the reasons why this research has proposed a comparison between the FD calibrated on a test section of the Italian highway network and the speed-flow curves suggested by the HCM 2010 for the same section. This study has focused on the Longitudinal Control Model (Ni, 2011) and on Van Aerde Model (Van Aerde, 1995), which result consistent to represent the different conditions of the traffic flow, and to describe a broad spectrum of freeway segments with high data-fitting capabilities. The parameters of the models have been estimated using a set of monthly data collected by radar devices in a 3-lanes test section of the Italian A4 Motorway. Flow rates and harmonic mean speed have been aggregated over the whole three-lane carriageway for the equivalent flow, by specific Passenger Car Equivalent coefficients, and density has been estimated for each 5-minute steps. In order to consider the speed at which driver behaviors get restricted by the surrounding vehicles in the flow, an approximate quantile regression (Chow et al, 2008), (Dervisoglu et al, 2009) has been implemented by the 85th percentile of speed observations for unitary density bins.

By using non outliers points, the parameters of LC Model have been estimated through a numerical procedure of curve-fitting, based on the Non Linear Least Squares Method and adapted to a non-explicit function, while the parameters of Van Aerde Model have been estimated by a heuristic hill-climbing technique implemented in the software program SPD_CAL (Rakha, 2007).

The results have shown that the LC Model and Van Aerde Model estimate values for capacity q_c, critical speed v_c, and critical density k_c that are very close to each other, lower than that provided by the HCM 2010, and close to German values in HBS. Although the comparative analysis limited to a single segment highway, the functional shapes have shown how the curves defined by HCM 2010 follows the trend of the data in a definitely worse way than the calibrated ones. The clear deviation can be attributed to several specific conditions of the section and differences in characteristics of vehicles or driving behavior and rules, such as the command of driving on the right and the prohibition of overtaking on the right, which are valid in Italy, against the "keep in lane" rule in the U.S..

The estimated Longitudinal Control and Van Aerde FD can be used in capacity and LOS analysis for the test segment. For the definition of the LOS, specific density ranges have been identified in analogy with what has been proposed by HCM. The upper threshold of LOS E is represented by capacity threshold. For the upper threshold of LOS D we used the density corresponding to the maximum value of the efficiency *e*. For the thresholds of the remaining LOS C, B and A, the density range 0-k_{LOSD} has been divided into four equal parts. The maximum densities for each level of service are significantly lower than those of HCM2010, with a density to capacity 30% lower, and with a density limit for LOS D 23% lower.

The comparative analysis has shown how HCM2010 density ranges could underestimate operational congestion levels on the test section. Therefore, the results suggest that operators should to use carefully HCM standard procedures and that transferability issues should be further analyzed. Failure to use calibrated procedures and models and the alternative use of procedures which seem too general and coming from heterogeneous contexts may not bring the actual congestion levels. Therefore, it might invalidate the requirements of adequacy and representativeness of LOS in assessing the quality of service. FD calibration methodology, with the correspondent LOS analysis, and HCM2010 comparisons should be tested with an extensive set of data from Italian freeways, with different characteristics of the roadway and traffic volumes. In this way the variations that exist in the shape of the FD and in the definition of the range of LOS at different conditions can assist in understanding how the function of the HCM procedures may be adequate in the Italian context.

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