Physics Letters B 678 (2009) 470–476

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Magnetic and quadrupole moments of light spin-1 mesons in light cone QCD sum rules

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ARTICLE INFO

Article history:
Received 26 February 2009
Received in revised form 6 May 2009
Accepted 30 June 2009
Available online 4 July 2009
Editor: A. Ringwald

PACS:
11.55.Hx
13.40.Em
14.20.Jn

ABSTRACT

The magnetic and quadrupole moments of the light-vector and axial-vector mesons are calculated in the light cone QCD sum rules. Our results for the static properties of these mesons are compared with the predictions of lattice QCD as well as other approaches existing in the literature.

1. Introduction

The electromagnetic form factors of the mesons and baryons represent an important tool for understanding their internal structure in terms of quarks and gluons. Investigation of the electromagnetic form factors of the nucleons, both theoretically and experimentally, is one of the main research areas in particle physics [1–3]. The electromagnetic form factors of the pseudoscalar mesons, especially the pion, has been extensively studied (see [4–9] and references therein). Unfortunately, the form factors of the vector mesons have received less interest (see [10–15] for recent studies). The electromagnetic form factors of vector mesons is also the subject of recent lattice QCD calculations (see [9,10] and [16]). In the present work we calculate the magnetic and quadrupole moments of the light-vector and axial-vector mesons in light cone QCD sum rule (LCSR) (for more about light cone QCD sum rule method, see [17] and [18]). Note that the magnetic moment of ρ meson is calculated in this framework in [15]. Here in this work we present improved calculations for the ρ meson including the distribution amplitudes which are neglected in [15], as well as a new result on quadrupole moment of the ρ meson. Furthermore, it should be noted that, the magnetic moment of ρ mesons calculated in the framework of QCD sum rules using the external field technique in [11].

The Letter is organized as follows. In Section 2 the LCSR for the magnetic and quadrupole moments of the light-vector and axial-vector mesons are derived. Section 3 is devoted to the numerical analysis. Furthermore, this section contains out conclusions and comparison of our results with the ones obtained from lattice QCD calculations.

2. Light cone QCD sum rules for the magnetic and quadrupole moments of light-vector and axial-vector mesons

In this section we derive the LCSR for the magnetic and quadrupole moments of the light-vector and axial-vector mesons. For this aim we consider the correlation function of the two vector meson currents in the presence of the external electromagnetic field, which is the main object in LCSR.

$$\Pi_{\mu\nu} = i \int d^4x\, e^{ipx} \langle 0|\mathcal{T}\{J_\nu(x) J_\mu^\dagger(0)\}|0\rangle_\gamma, \tag{1}$$

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doi:10.1016/j.physletb.2009.06.073

where γ is the external electromagnetic field, $J_\nu(x) = \bar{q}_1(x)\Gamma_\nu q_2(x)$ is the interpolating current of the light-vector (axial-vector) mesons when $\Gamma_\nu = \gamma_\nu$ $(\gamma_\nu\gamma_5)$.

According to QCD sum rules method, the correlation function is calculated in two different ways:

- in terms of quark degrees of freedom interacting with the nonperturbative QCD vacuum (theoretical part),
- being saturated by the mesons (as is in our case) having the same quantum as the interpolating current (phenomenological part).

We start our analysis by calculating the phenomenological part of the correlation function. Inserting a complete set states with the same quantum numbers as the interpolating current and isolating the ground state meson, we get:

$$\Pi_{\mu\nu} = \frac{\langle 0|J_\nu|i(p)\rangle\langle i(p)|i(p')\rangle_\gamma\langle i(p')|J_\mu^\dagger(0)|0\rangle}{(p^2-m_i^2)(p'^2-m_i^2)} + \cdots, \tag{2}$$

where i represents light-vector or axial-vector mesons, $p' = p+q$, q is the photon momentum and the dots correspond to the contribution of higher states and continuum.

It follows from Eq. (2) that in order to calculate the phenomenological part of the correlation function, the matrix elements $\langle 0|J_\nu|i(p)\rangle$ and $\langle i(p)|i(p')\rangle_\gamma$ are needed. The matrix element $\langle 0|J_\nu|i(p)\rangle$ is defined as:

$$\langle 0|J_\nu|i(p)\rangle = f_i m_i. \tag{3}$$

Imposing the parity and time reversal invariance of the electromagnetic interaction, the electromagnetic vertex of the light-vector (axial-vector) is described in terms of the three form factors in the following way [19]:

$$\langle i(p,\varepsilon^r)|i(p',\varepsilon^{r'})\rangle_\gamma = -\varepsilon^\rho(\varepsilon^r)^\alpha(\varepsilon^{r'})^\beta\left\{G_1(Q^2)g_{\alpha\beta}(p+p')_\rho + G_2(Q^2)(q_\alpha g_{\rho\beta} - q_\beta g_{\rho\alpha}) - \frac{1}{2m_i^2}G_3(Q^2)q_\alpha q_\beta(p+p')_\rho\right\}, \tag{4}$$

where ε^ρ id the photon and $(\varepsilon^r)^\alpha$, $(\varepsilon^{r'})^\beta$ are the light-vector (axial-vector) meson polarization vectors. The covariant form factors G_1, G_2 and G_3 can be expressed in terms of the Sachs charge, magnetic and quadrupole form factors as follows [19,20]:

$$F_{\mathcal{C}} = G_1(Q^2) + \frac{3}{3}\eta F_{\mathcal{D}}(Q^2), \qquad F_{\mathcal{M}} = G_2(Q^2), \qquad F_{\mathcal{D}} = G_1(Q^2) - G_2(Q^2) + (1+\eta)G_3(Q^2), \tag{5}$$

where $\eta = Q^2/4m_i^2$.

The charge q_i, magnetic moment μ_i and quadrupole moment $\mathcal{D}_i$ are determined from $F_{\mathcal{C}}$, $F_{\mathcal{M}}$ and $F_{\mathcal{D}}$, respectively, at zero momentum transfer,

$$eF_{\mathcal{C}}^i(0) = q_i, \qquad eF_{\mathcal{M}}^i(0) = 2m_i\mu_i, \qquad eF_{\mathcal{D}}^i(0) = m_i^2\mathcal{D}_i. \tag{6}$$

Substituting the expressions in Eqs. (3) and (4) into Eq. (2), and performing summation over polarizations of the light-vector (axial-vector) meson, for the phenomenological part of the correlation function we have:

$$\begin{aligned}\Pi_{\mu\nu}^{ph} = f_i^2 m_i^2 \frac{\varepsilon^\rho}{(m_i^2-p^2)[m_i^2-(p+q)^2]}&\left\{G_1(Q^2)(p+p')_\rho\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_i^2} - \frac{p'_\mu p'_\nu}{m_i^2} + \frac{p'_\mu p_\nu}{2m_i^4}(Q^2+2m_i^2)\right]\right.\\
&+ G_2(Q^2)\left[q_\mu g_{\nu\rho} - q_\nu g_{\mu\rho} - \frac{p_\nu}{m_i^2}\left(q_\mu p_\rho - \frac{1}{2}Q^2 g_{\mu\rho}\right) + \frac{p'_\mu}{m_i^2}\left(q_\nu p'_\rho + \frac{1}{2}Q^2 g_{\nu\rho}\right) - \frac{p'_\mu p_\nu p_\rho}{m_i^4}Q^2\right]\\
&\left.- \frac{1}{m_i^2}G_3(Q^2)(p+p')_\rho\left[q_\mu q_\nu - \frac{p_\mu q_\nu}{2m_i^2}Q^2 + \frac{p'_\mu q_\nu}{2m_i^2}Q^2 - \frac{p'_\mu p_\nu}{4m_i^4}Q^4\right]\right\},\end{aligned} \tag{7}$$

where $Q^2 = -q^2$.

As has already been noted, in order to determine the magnetic and quadrupole moments, the values of the form factors are needed only at $Q^2 = 0$. Substituting Eq. (5), as well as the relations $p' = p+q$ and $q\varepsilon = 0$ into Eq. (7), we obtain the final result for $\Pi_{\mu\nu}$:

$$\begin{aligned}\Pi_{\mu\nu}^{ph} = f_i^2 m_i^2 \frac{\varepsilon^\rho}{(m_i^2-p^2)[m_i^2-(p+q)^2]}&\left\{2F_{\mathcal{C}}(0)p_\rho\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_i^2} - \frac{p_\mu q_\nu}{m_i^2}\right]\right.\\
&\left.+ F_{\mathcal{M}}(0)\left[q_\mu g_{\nu\rho} - q_\nu g_{\mu\rho} - \frac{p_\rho}{m_i^2}(p_\mu q_\nu - p_\nu q_\mu)\right] - \left[F_{\mathcal{C}}(0) + F_{\mathcal{D}}(0)\right]\frac{p_\rho}{m_i^2}q_\nu q_\mu\right\}.\end{aligned} \tag{8}$$

In determining magnetic and quadrupole magnetic moments, different structures are needed. For this purpose, we prefer to choose the structures which do not get contribution from the contact terms after Borel transformation (more about contact terms can be found in [21]). The structures $q_\mu\varepsilon_\nu$ and $q_\nu q_\mu(\varepsilon p)$ do not receive contributions from the contact terms, and for this reason we choose them for extracting the magnetic and quadrupole magnetic moments of the light-vector (axial-vector) mesons.

Our next task is the calculation of the correlation function from the QCD side in terms of the photon distribution amplitudes. Using the explicit expression of the interpolating current in x representation, for the correlation function we get

$$\Pi_{\mu\nu}^{th} = i\int d^4x\, e^{ipx}\langle\gamma(q)|S(x)\Gamma_\mu S(-x)\Gamma_\nu|0\rangle. \tag{9}$$

The correlation function contains three different combinations:

(a) perturbative contributions,
(b) "mixed contribution", i.e., photon interacts with the quark propagator perturbatively, and other quark fields organize the quark condensate,
(c) nonperturbative contribution, i.e., photon is emitted at long distance.

In calculating the contribution coming from (c), the propagator of the quark field is expanded near the light cone $x^2=0$, as a result of which appears the matrix elements of the nonlocal operators $\langle\gamma(q)|\bar{q}(x_1)\Gamma q(x_2)|0\rangle$, between the vacuum and one photon states, i.e., these matrix elements are expressed in terms of the photon distribution amplitudes (Da's).

In Eq. (9), $S(x)$ is the full propagator of the light quark expanded in the light cone, which has the following form [22,23]:

$$S(x)=\frac{i\not{x}}{2\pi^2x^4}-\frac{m_q}{4\pi^2x^2}-\frac{\langle\bar{q}q\rangle}{12}\left(1-\frac{im_q}{4}\not{x}\right)-\frac{x^2}{192\pi^2}m_0^2\langle\bar{q}q\rangle\left(1-\frac{im_q}{6}\not{x}\right)$$
$$-ig_s\int_0^1 du\left\{\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(ux)\sigma^{\mu\nu}-\frac{i}{4\pi^2x^2}ux^\mu G_{\mu\nu}(ux)\gamma^\nu(ux)-\frac{im_q}{32\pi^2}G_{\mu\nu}(ux)\sigma^{\mu\nu}\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right)+2\gamma_E\right]\right\}, \tag{10}$$

where Λ is the scale parameter

The matrix elements $\langle\gamma(q)|\bar{q}_1(x_1)\Gamma q(x_2)|0\rangle$ are determined in terms of the photon Da's in the following way [25]:

$$\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle=-ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int_0^1 du\,e^{i\bar{u}qx}\left(\chi\varphi_\gamma(u)+\frac{x^2}{16}\mathbb{A}(u)\right)$$
$$-\frac{i}{2(qx)}e_q\langle\bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)-x_\mu\left(\varepsilon_\nu-q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 du\,e^{i\bar{u}qx}h_\gamma(u),$$

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle=e_qf_{3\gamma}\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)\int_0^1 du\,e^{i\bar{u}qx}\psi^v(u),$$

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5q(0)|0\rangle=-\frac{1}{4}e_qf_{3\gamma}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta\int_0^1 du\,e^{i\bar{u}qx}\psi^a(u),$$

$$\langle\gamma(q)|\bar{q}(x)g_sG_{\mu\nu}(vx)q(0)|0\rangle=-ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{S}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}i\gamma_5(vx)q(0)|0\rangle=-ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\tilde{\mathcal{S}}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5q(0)|0\rangle=e_qf_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)g_sG_{\mu\nu}(vx)i\gamma_\alpha q(0)|0\rangle=e_qf_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_sG_{\mu\nu}(vx)q(0)|0\rangle$$
$$=e_q\langle\bar{q}q\rangle\left\{\left[\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\nu}-\frac{1}{qx}(q_\alpha x_\nu+q_\nu x_\alpha)\right)q_\beta-\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\nu}-\frac{1}{qx}(q_\beta x_\nu+q_\nu x_\beta)\right)q_\alpha\right.\right.$$
$$-\left(\varepsilon_\nu-q_\nu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\mu}-\frac{1}{qx}(q_\alpha x_\mu+q_\mu x_\alpha)\right)q_\beta$$
$$\left.+\left(\varepsilon_\nu-q_\nu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\mu}-\frac{1}{qx}(q_\beta x_\mu+q_\mu x_\beta)\right)q_\alpha\right]\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_1(\alpha_i)$$
$$+\left[\left(\varepsilon_\alpha-q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\right)q_\nu-\left(\varepsilon_\alpha-q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\nu\beta}-\frac{1}{qx}(q_\nu x_\beta+q_\beta x_\nu)\right)q_\mu\right.$$
$$\left.-\left(\varepsilon_\beta-q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\right)q_\nu+\left(\varepsilon_\beta-q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\nu\alpha}-\frac{1}{qx}(q_\nu x_\alpha+q_\alpha x_\nu)\right)q_\mu\right]\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_2(\alpha_i)$$
$$+\frac{1}{qx}(q_\mu x_\nu-q_\nu x_\mu)(\varepsilon_\alpha q_\beta-\varepsilon_\beta q_\alpha)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_3(\alpha_i)$$
$$\left.+\frac{1}{qx}(q_\alpha x_\beta-q_\beta x_\alpha)(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_4(\alpha_i)\right\}, \tag{11}$$

where χ is the magnetic susceptibility of the quarks, $\varphi_\gamma(u)$ is the leading twist 2, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}$ and $\mathcal{V}$ are the twist 3 and $h_\gamma(u)$, $\mathbb{A}$, $\mathcal{T}_i$ ($i=1,\ 2,\ 3,\ 4$) are the twist 4 photon distribution amplitudes. The explicit expressions of these Da's are given in [25]. The integral measure $\mathcal{D}\alpha_i$ is defined as

$$\mathcal{D}\alpha_i = \int_0^1 d\alpha_g \int_0^1 d\alpha_q \int_0^1 d\alpha_{\bar{q}}\, \delta(1-\alpha_g-\alpha_q-\alpha_{\bar{q}}). \tag{12}$$

Substituting Eqs. (10) and (11) into Eq. (9) and performing integration over x, one can obtain the expression for the correlation function $\Pi^{th}_{\mu\nu}$ in the momentum space. Matching two different representations Π^{ph} and Π^{th} of the correlation function via the dispersion relation and applying double Borel transformations on the variables p^2 and $(p+q)^2$, which suppresses higher states and continuum contributions, we obtain the sum rules for the form factors. Separating the coefficients of the structures $q_\mu \varepsilon_\nu$ and $(\varepsilon p) q_\mu q_\nu$, which are free of contact term contributions, we obtain the following sum rules for mesons containing u and d quarks, i.e., for ρ^+ and a_1^+ mesons:

$$\begin{aligned}
F_{\mathcal{M}}(0) = {}& \frac{1}{f_i^2 m_i^2} e^{m_i^2/M^2} \bigg\{ \mp \frac{1}{48M^4} \langle g_s^2 GG\rangle \big[e_u m_d \langle \bar{u}u\rangle \mathbb{A}(\bar{u}_0) - 2 e_u m_d \langle \bar{u}u\rangle \big(u_0 \tilde{i}_3(h_\gamma,1) + \tilde{i}_3(h_\gamma, u-\bar{u}_0)\big) \\
& - e_d m_u \langle \bar{d}d\rangle \big(\mathbb{A}(u_0) - 2u_0 \tilde{i}'_3(h_\gamma,1) + 2\tilde{i}'_3(h_\gamma, u-u_0)\big)\big] \\
& + \frac{1}{24M^2} \big[2m_0^2 \big(\mp 3 e_u m_u \langle \bar{d}d\rangle + 2 e_u m_d u_0 \langle \bar{d}d\rangle \pm 3 e_d m_d \langle \bar{u}u\rangle - 2 e_d m_u u_0 \langle \bar{u}u\rangle \big) \\
& \mp e_u m_d \langle g_s^2 GG\rangle \langle \bar{u}u\rangle \chi \varphi_\gamma(\bar{u}_0) \pm e_d m_u \langle g_s^2 GG\rangle \langle \bar{d}d\rangle \chi \varphi_\gamma(u_0)\big] - \frac{1}{8\pi^2}(e_d - e_u) M^4 (3+4u_0) E_1\big(s/M^2\big) \\
& + E_0\big(s/M^2\big) M^2 \chi \big(\mp e_u m_d \langle \bar{u}u\rangle \varphi_\gamma(\bar{u}_0) \pm e_d m_u \langle \bar{d}d\rangle \varphi_\gamma(u_0)\big) \\
& + \frac{1}{4}\big[E_0\big(s/M^2\big) M^2 f_{3\gamma} \big(4 e_u i_2(\mathcal{A},\bar{v}) - 4 e_u i_2(\mathcal{V},\bar{v}) - 4 e_d i'_2(\mathcal{A},v) - 4 e_d i'_2(\mathcal{V},v) \\
& + 4 e_u \tilde{i}_3\big(\psi^v,1\big) - 4 e_d \tilde{i}'_3\big(\psi^v,1\big) - e_u \psi^a(\bar{u}_0) + e_d \psi^a(u_0) + 4 e_u u_0 \psi^v(\bar{u}_0) \\
& - 4 e_d u_0 \psi^v(u_0) + e_u u_0 \psi^{a\prime}(\bar{u}_0) + e_d u_0 \psi^{a\prime}(u_0)\big)\big] + \langle \bar{u}u\rangle \bigg[\frac{1}{2} e_d(\pm 2m_d + m_u) + e_u m_d \big(\pm i_1(\mathcal{S},1) \pm i_1(\tilde{\mathcal{S}},1) \pm i_1(\mathcal{T}_1,1) \\
& \pm i_1(\mathcal{T}_2,1) - i_1(\mathcal{T}_3,1) - i_1(\mathcal{T}_4,1) + u_0 \tilde{i}_3(h_\gamma,1) + \tilde{i}_3(h_\gamma, u-\bar{u}_0)\big)\bigg] \\
& + \langle \bar{d}d\rangle \bigg[-\frac{1}{2} e_u (m_d \pm 2m_u) \mp e_d m_u \big(i'_1(\mathcal{S},1) + i'_1(\tilde{\mathcal{S}},1) - i'_1(\mathcal{T}_1,1) \\
& - i'_1(\mathcal{T}_2,1) + i'_1(\mathcal{T}_3,1) + i'_1(\mathcal{T}_4,1) \mp u_0 \tilde{i}'_3(h_\gamma,1) \pm \tilde{i}'_3(h_\gamma, u-u_0)\big)\bigg]\bigg\},
\end{aligned}$$

$$\begin{aligned}
F_{\mathcal{C}}(0) + F_{\mathcal{D}}(0) = {}& \frac{m_i^2}{f_i^2 m_i^2} e^{m_i^2/M^2} \bigg\{ \frac{1}{3M^4} m_0^2 u_0^2 \big(e_u m_d \langle \bar{d}d\rangle - e_d m_u \langle \bar{u}u\rangle \big) + \frac{1}{M^2} \big[-e_u m_d u_0 \langle \bar{d}d\rangle + e_d m_u u_0 \langle \bar{u}u\rangle \\
& \pm 4 e_u m_d u_0 \langle \bar{u}u\rangle \big(i_0(\mathcal{T}_1,1) + i_0(\mathcal{T}_2,1) - i_0(\mathcal{T}_3,1) - i_0(\mathcal{T}_4,1)\big) \\
& \pm 4 e_d m_u u_0 \langle \bar{d}d\rangle \big(i'_0(\mathcal{T}_1,1) + i'_0(\mathcal{T}_2,1) - i'_0(\mathcal{T}_3,1) - i'_0(\mathcal{T}_4,1)\big)\big] + \frac{1}{4\pi^2} M^2 u_0 (e_d - e_u) E_0\big(s/M^2\big)(3-4u_0) \\
& + 2 f_{3\gamma} \big[-e_u i_1(\mathcal{A},1) + e_u i_1(\mathcal{V},1-2v) + 2 e_u u_0 \tilde{i}_2\big(\psi^v,1\big) \\
& + e_d \big(i'_1(\mathcal{A},1) - i'_1(\mathcal{V},1-2v) - 2u_0 \tilde{i}'_3\big(\psi^v,1\big)\big)\big]\bigg\},
\end{aligned} \tag{13}$$

where the upper (lower) sign corresponds to the light-vector (axial-vector) meson, χ is the magnetic susceptibility, and the continuum contributions are described by the function

$$E_n(x) = 1 - e^{-x} \sum_{i=0}^{n} \frac{x^i}{i!}, \tag{14}$$

where $x = s_0/M^2$ and s_0 is the continuum threshold. The Borel parameters M_1 and M_2 are taken to be equal to each other, i.e., $M_1^2 = M_2^2 = 2M^2$, since we deal with a single meson, we have then

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} = \frac{1}{2}. \tag{15}$$

The functions i_n, i'_n, $\tilde{i}_n$ and $\tilde{i}'_n$ are defined as

$$i_0\big(\phi, f(v)\big) = \int \mathcal{D}\alpha_i \int_0^1 dv\, \phi(\alpha_{\bar q}, \alpha_q, \alpha_g) f(v) \theta(k - u_0),$$

$$i'_0\big(\phi, f(v)\big) = \int \mathcal{D}\alpha_i \int_0^1 dv\, \phi(\alpha_{\bar q}, \alpha_q, \alpha_g) f(v) \theta(k' - u_0),$$

$$i_1\big(\phi, f(v)\big) = \int \mathcal{D}\alpha_i \int_0^1 dv\, \phi(\alpha_{\bar q}, \alpha_q, \alpha_g) f(v) \delta(k - u_0),$$

$$i'_1\big(\phi, f(v)\big) = \int \mathcal{D}\alpha_i \int_0^1 dv\, \phi(\alpha_{\bar q}, \alpha_q, \alpha_g) f(v) (k' - u_0),$$

$$i_2\big(\phi, f(v)\big) = \int \mathcal{D}\alpha_i \int_0^1 dv\, \phi(\alpha_{\bar q}, \alpha_q, \alpha_g) f(v) \delta'(k - u_0),$$

$$i'_2\big(\phi, f(v)\big) = \int \mathcal{D}\alpha_i \int_0^1 dv\, \phi(\alpha_{\bar q}, \alpha_q, \alpha_g) f(v) \delta'(k' - u_0),$$

$$\tilde{i}_3\big(\phi, f(u)\big) = \int_0^{\bar u_0} du\, \phi(u) f(u), \qquad \tilde{i}'_3\big(\phi, f(u)\big) = \int_{u_0}^1 du\, \phi(u) f(u),$$

where $k = \alpha_q + \alpha_g \bar v$ and $k' = \alpha_{\bar q} + \alpha_g v$. The result for the K^{*0} (K^{*+}) meson can be obtained from Eq. (13) by making the replacement $u \to s$ ($d \to s$).

We would like to finish this section by the following remark. In expression of the quark propagator the terms linear in quark mass are taken into account. However, the numerical calculations are performed for massless u and d quarks. It is well known that the photon Da's presented in Eq. (11) is written only for massless quarks. Therefore it might be seen that the calculation of the magnetic moments of the mesons containing strange quark is incomplete. In this connection we note that in LCSR the main contribution to the sum rule comes from the leading twist Da's. In the first reference in [24], quark mass corrections to the leading twist Da's have been calculated. But, as can be seen in Eq. (13), the leading twist Da's is multiplied by the quark masses, and hence any quark mass correction to the Da's gives a second order mass correction to the form factors which are neglected.

3. Numerical analysis

In this section we calculate the magnetic and quadrupole moments of the light-vector and axial-vector mesons. The values of the input parameters we use in our numerical analysis are, $\langle \bar u u\rangle(1\ \text{GeV}) = \langle \bar d d\rangle(1\ \text{GeV}) = -(0.243)^3\ \text{GeV}^3$, $\langle \bar s s\rangle(1\ \text{GeV}) = 0.8\langle \bar u u\rangle(1\ \text{GeV})$, $m_0^2(1\ \text{GeV}) = 0.8$ [23], $\chi(1\ \text{GeV}) = -3.15\ \text{GeV}^{-2}$ [26], $\Lambda = 0.5$ GeV and $f_{3\gamma} = -0.0039\ \text{GeV}^2$ [25], $m_\rho = 0.77$ GeV, $f_\rho = 0.215$ GeV, $m_{K^*} = 0.892$ GeV, $f_{K^*} = 0.217$ GeV, $m_{a_1} = 1.260$ GeV, $f_{a_1} = 0.200$ GeV.

The photon Da's entering the sum rules are [25]:

$$\begin{aligned}
&\varphi_\gamma(u) = 6u\bar u\big(1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar u)\big),\\
&\psi^v(u) = 3\big(3(2u-1)^2 - 1\big) + \frac{3}{64}\big(15 w_\gamma^V - 5 w_\gamma^A\big)\big(3 - 30(2u-1)^2 + 35(2u-1)^4\big),\\
&\psi^a(u) = \big(1 - (2u-1)^2\big)\big(5(2u-1)^2 - 1\big)\frac{5}{2}\left(1 + \frac{9}{16} w_\gamma^V - \frac{3}{16} w_\gamma^A\right),\\
&\mathcal{A}(\alpha_i) = 360\alpha_q\alpha_{\bar q}\alpha_g^2\left(1 + w_\gamma^A \frac{1}{2}(7\alpha_g - 3)\right),\\
&\mathcal{V}(\alpha_i) = 540 w_\gamma^V(\alpha_q - \alpha_{\bar q})\alpha_q\alpha_{\bar q}\alpha_g^2, \qquad h_\gamma(u) = -10(1 + 2\kappa^+) C_2^{\frac{1}{2}}(u - \bar u),\\
&\mathbb{A}(u) = 40u^2\bar u^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)\big[u\bar u(2 + 13u\bar u) + 2u^3\big(10 - 15u + 6u^2\big)\ln(u) + 2\bar u^3\big(10 - 15\bar u + 6\bar u^2\big)\ln(\bar u)\big],\\
&\mathcal{T}_1(\alpha_i) = -120\big(2\zeta_2 + \zeta_2^+\big)(\alpha_{\bar q} - \alpha_q)\alpha_{\bar q}\alpha_q\alpha_g,\\
&\mathcal{T}_2(\alpha_i) = 30\alpha_g^2(\alpha_{\bar q} - \alpha_q)\big((\kappa - \kappa^+) + \big(\zeta_1 - \zeta_1^+\big)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)\big),\\
&\mathcal{T}_3(\alpha_i) = -120\big(3\zeta_2 - \zeta_2^+\big)(\alpha_{\bar q} - \alpha_q)\alpha_{\bar q}\alpha_q\alpha_g,\\
&\mathcal{T}_4(\alpha_i) = 30\alpha_g^2(\alpha_{\bar q} - \alpha_q)\big((\kappa + \kappa^+) + \big(\zeta_1 + \zeta_1^+\big)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)\big).
\end{aligned} \tag{16}$$

Table 1
The magnetic moments of light-vector and axial-vector mesons (in units of $e/2m_i$).

Meson	$\boldsymbol{\mu}$ (in $e/2m_i$)					
	present work	[9]	[10]	[16]	covariant quark model [27]	light cone quark model [12]
ρ^+	2.4 ± 0.4	2.01	2.2	2.7	2.14	1.92
K^{*+}	2.0 ± 0.4	2.23	2.08	2.36	–	–
K^{*0}	0.28 ± 0.04	−0.26	−0.08	−0.06	–	–
a_1^+	3.8 ± 0.6	–	–	2	–	–

Table 2
The quadrupole moments of light-vector and axial-vector mesons (in units of e/m_i^2).

Meson	$\mathcal{D}$ (in e/m_i^2)					
	present work	[9]	[10]	[16]	covariant quark model [27]	light cone quark model [12]
ρ^+	-0.85 ± 0.15	−0.41	–	–	−0.79	−0.043
K^{*+}	-0.8 ± 0.15	−0.38	–	–	–	–
K^{*0}	-0.008 ± 0.0004	0.01	–	–	–	–
a_1^+	-0.9 ± 0.3	–	–	–	–	–

The values of the constant parameters appearing in the Da's are [25]: $\varphi_2(1\ \text{GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$.

The Borel mass is the artificial parameter of the sum rules and the physically measurable quantities should be independent of them. For this reason, we must find "working region" of M^2, where the values of the magnetic and quadrupole moments are, practically, independent of M^2. In order to find the upper bound of M^2, we require that the contributions continuum and higher states be less than 30% of the total results. In other words, the Borel parameter M^2 should not be too large in order to guarantee that the above–mentioned contributions are exponentially suppressed. Moreover, the lower bound of M^2 is determined through the following argument. The Borel parameter could not be too small to satisfy the validity of the OPE of the correlation function near the light cone in the Euclidean region, since higher twist contributions are proportional to $1/M^2$. As a result of these constraints, the working regions of M^2 are determined to be:

$$0.8\ \text{GeV}^2 \leqslant M^2 \leqslant 1.8\ \text{GeV}^2 \quad (\rho\ \text{meson}),$$
$$1.0\ \text{GeV}^2 \leqslant M^2 \leqslant 2.0\ \text{GeV}^2 \quad (K^*\ \text{meson}),$$
$$1.5\ \text{GeV}^2 \leqslant M^2 \leqslant 3.0\ \text{GeV}^2 \quad (a_1\ \text{meson}).$$

For the continuum threshold s_0, we choose, $s_0^{(\rho)} = 1.7\ \text{GeV}^2$, $s_0^{(K^*)} = 2.0\ \text{GeV}^2$ and $s_0^{(a_1)} = 3.0\ \text{GeV}^2$.

The numerical results in analysis of the sum rules for the magnetic and quadrupole moments, which is the main task of the present work, are presented in Table 1.

The error in our predictions come from the variations in the Borel parameter, s_0 and uncertainties from the nonperturbative input parameters in the Da's of photon.

For a comparison, in these tables, we also present predictions of the other approaches. We see from Tables 1 and 2 that within the limits of errors, the predictions of different approaches on the magnetic moment of the ρ and K^{*+} mesons are very close to each other.

As we have already noted, the magnetic moment of the ρ meson is studied within the framework of LCSR in [15], which is slightly different from our prediction. This small difference can be attributed to the Da's that are neglected in [15].

Our prediction of $\mu_{K^{*0}}$ is more or less in agreement with the prediction of [5], except its sign, while it drastically differs from the predictions of [9] and [16]. Additionally, our result on the magnetic moment of a_1^+ differs considerably from the one given in [16]. The situation on the quadrupole moments can be summarized as follows. Our result on for $\mathcal{D}_{\rho^+}$ coincides with the prediction of [27]. But our results for $\mathcal{D}_{K^{*+}}$ and $\mathcal{D}_{K^{*0}}$ are almost twice as larger compared to the predictions of the other approaches.

Our final remark is that we have also calculated the magnetic and quadrupole moments of the ρ^0, ω and ϕ mesons, which are all equal to zero, as expected.

In conclusion, we have calculated the magnetic and quadrupole moments of the light- and axial-vector mesons within the frame work of LCSR method. Our results for the magnetic moments of the ρ^+ and K^{*+} mesons are in agreement with the predictions of the other approaches. Our prediction on the magnetic moment of the K^{*0} meson, except its sign, confirms the prediction of [10], while drastic differences are observed in comparison to the other approaches. Note also that, our prediction of the magnetic moment of axial-vector meson a_1^+, is 1.5–2 times larger compared to that given in [16].

In regard to the quadrupole moments of the light-vector mesons, we conclude that our predictions are approximately 2 times larger in comparison to the other models, except the one predicted by [23], which is in close agreement with ours.

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