



The optimal approach of detecting stochastic gravitational wave from string cosmology using multiple detectors

Xi-Long Fan, Zong-Hong Zhu *

Department of Astronomy, Beijing Normal University, Beijing 100875, People's Republic of China

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Abstract

String cosmology models predict a relic background of gravitational wave produced during the dilaton-driven inflation. Its spectrum is most likely to be detected by ground gravitational wave laser interferometers (IFOs), like LIGO, Virgo, GEO, as the energy density grows rapidly with frequency. We show the certain ranges of the parameters that underlying string cosmology model using two approaches, associated with 5% false alarm and 95% detection rate. The result presents that the approach of combining multiple pairs of IFOs is better than the approach of directly combining the outputs of multiple IFOs for LIGO, LIGOL, Virgo and GEO.

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1. Introduction

Stochastic gravitational wave background, which has two origins, is one target of gravitational wave interferometers (IFOs). It might result from an extremely large number of weak astrophysical gravitational wave sources, like compact stars in binary system (see, e.g., [1] for more details). It also might result from some processes of very early universe, like phase transitions or amplification of vacuum fluctuations in inflationary (see, e.g., [2–4] for reviews). In the latter case of origin, the gravitational waves carry the earlier information of the universe than that shown by electromagnetic waves. One of the most interesting processes in the early universe is from the string cosmology [5,6], which predicts a quite different gravitational wave background spectrum from that predicted by other cosmological models for early universe. That the energy density grows rapidly with frequency [7] means that the ground IFOs may be the best detectors. Several large scale ground IFOs are in operation: Laser Interferometric Gravitational

Wave Observatory (LIGO) [8] in Livingston (LIGOL) and in Hanford (LIGOH), Virgo [9] near Pisa and GEO [10] in Hanover.

Two approaches of combining $2N$ detectors to improve the detection ability to the stochastic gravitational wave background are proposed in [11]: (i) correlating the outputs of a pair of detectors, then combining the multiple pairs, and (ii) directly combining the outputs of $2N$ detectors. As shown in [12], for $2N$ detectors with equal noise level, the data observation time and the overlap reduction functions, the optimal approach is to combine multiple pairs of two detectors comparing to directly combining $2M$ ($M \leq N$) detectors. But the real detectors should not be of identical noise levels and overlap reduction functions. We plot the proposed noise curves of detectors in Fig. 1 and the overlap reduction functions in Fig. 2. A number of authors [13–16] have used the approach of combining LIGO and LIGOL to detect the string cosmology gravitational wave background. A recent work [17] has shown that the approach of combining multiple pairs of IFOs using Virgo and LIGO and GEO can improve the detection ability to the stochastic gravitational wave background illustrated by a simulated isotropic gravitational wave background generated with an astrophysically-motivated spectral shape.

* Corresponding author.

E-mail address: zhuzh@bnu.edu.cn (Z.-H. Zhu).

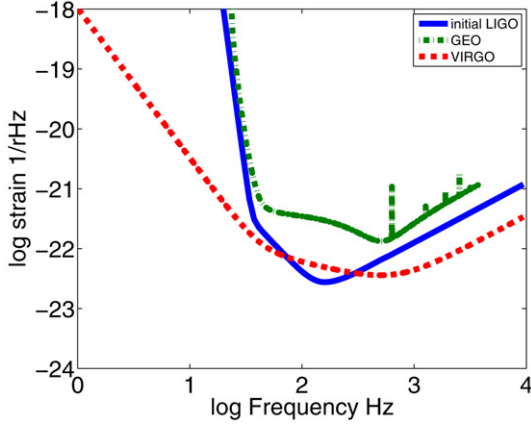


Fig. 1. The designed noise power spectrum of initial LIGO, Virgo and GEO. Data are taken from [22–24].

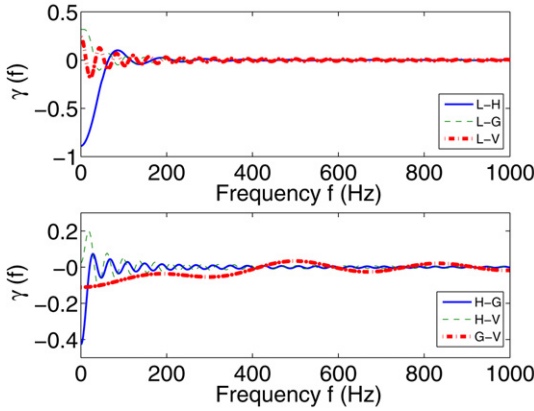


Fig. 2. The overlap reduction function. Data are taken from [22–25].

In this Letter we compare the two approaches by constraining the parameter space of gravitational wave background predicted from string cosmology using LIGO, LIGOL, Virgo, and GEO. Our result shows that the approach of combining multiple pairs of IFOs is better than the approach of directly combining the outputs of multiple IFOs for those real IFOs at their designed noise levels. Our Letter is organized as follows. In Section 2 we review the two approaches of detecting a stochastic background using multiple detectors. In Section 3 after a brief review of the gravitational wave background produced by string cosmology, we implement the two approaches using four IFOs: LIGO, LIGOL, Virgo and GEO at their designed noise levels. Our conclusion will be provided in Section 4.

2. Two approaches of detecting a stochastic background using multiple detectors

It has been shown [11] that after correlating signals of two detectors for time T (we take $T = 10^7$ sec = 3 months) the squared ratio of “Signal” (S) to “Noise” (N) is given by an integral over frequency f :

$$\left(\frac{S}{N}\right)^2 = \frac{9H_0^4}{50\pi^4} T \int_0^\infty df \frac{\gamma^2(f) \Omega_{\text{gw}}^2(f)}{f^6 P_1(f) P_2(f)}, \quad (1)$$

where $P_i(f)$ is the one-side noise power spectral density which describes the instrument noise $n_i(f)$ in frequency domain:

$$\langle \tilde{n}_i^*(f) \tilde{n}_i(f') \rangle = \frac{1}{2} \delta(f - f') P_i(|f|). \quad (2)$$

Eq. (1) is under the assumption that the noise of the detectors are (i) stationary, (ii) Gaussian, (iii) statistically independent of one another and of the stochastic gravitational wave background, and (iv) much larger in magnitude than the stochastic gravitational wave background.

$\gamma(f)$ is the so-called *overlap reduction function* first calculated by Flanagan [18], which shows the co-response of two detectors. This is a dimensionless function of frequency f , which is determined by the relative positions and orientations of two detectors. Explicitly,

$$\gamma(f) := \frac{5}{8\pi} \sum_A \int_{S^2} d\hat{\Omega} e^{i2\pi f \hat{\Omega} \cdot \Delta \vec{x}/c} F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega}), \quad (3)$$

where $\hat{\Omega}$ is a unit vector specifying a direction on the two-sphere, $\Delta \vec{x} := \vec{x}_1 - \vec{x}_2$ is the separation vector between the central stations of the two detector sites, and

$$F_i^A(\hat{\Omega}) := e_{ab}^A(\hat{\Omega}) \frac{1}{2} (\hat{X}_i^a \hat{X}_i^b - \hat{Y}_i^a \hat{Y}_i^b) \quad (4)$$

is the i th detector’s response to a zero frequency, unit amplitude, $A = +, \times$ polarized gravitational wave, where \hat{X}_i^a and \hat{Y}_i^a are unit vectors pointing in the direction of the detector arms. The overlap reduction function $\gamma(f)$ in Eq. (3) is normalized for coincident and coaligned detectors: $\gamma(0) = 1$. We refer the reader to [2,11] for more details about the overlap reduction function $\gamma(f)$. Two approaches were shown in [11] for multiple IFOs, the optimal approach of combining multiple detector pairs:

$$\left(\frac{S}{N}\right)_{\text{opt I}}^2 = \sum_{\text{pair}} \left(\frac{S}{N}\right)_{\text{pair}}^2, \quad (5)$$

and the optimal approach of directly combining $2N$ detectors:

$$\left(\frac{S}{N}\right)_{\text{opt II}}^2 \approx \binom{12}{2} \left(\frac{S}{N}\right)^{2(34)} \left(\frac{S}{N}\right)^2 \cdots \binom{2N-1, 2N}{2} \left(\frac{S}{N}\right)^2 + \text{all possible permutations.} \quad (6)$$

In order to detect a stochastic background with 5% false alarm and 95% detection rate, the total optimal signal to noise ratio SNR_{opt} threshold should be 3.29.

3. Detecting a string cosmology background using two approaches by four IFOs

String cosmology, also denoted in the literature as the “pre-Big-Bang (PBB) models” depicts a different view of PBB era from the “slow-roll” (standard) inflation. The dilaton-driven inflation phase is well understood, followed by a sting phase which is currently not known. The “minimal” PBB model [19] describes the sting phase following the dilaton-driven inflation phase is a constant curvature phase. At the end of the string

phase, the dilaton reaches the present vacuum expectation value and stops. The spectrum of gravitational wave background predicted during the periods of dilaton-driven inflation phase and string phase in string cosmology is discussed in [20]. The simplest model, in which the approximate form of the spectrum is [21]

$$\Omega_{\text{gw}}(f) = \begin{cases} \Omega_{\text{gw}}^s (f/f_s)^3, & f < f_s, \\ \Omega_{\text{gw}}^s (f/f_s)^\beta, & f_s < f < f_1, \\ 0, & f_1 < f, \end{cases} \quad (7)$$

where

$$\beta = \frac{\log[\Omega_{\text{gw}}^{\text{max}} / \Omega_{\text{gw}}^s]}{\log[f_1 / f_s]}$$

is the logarithmic slope of the spectrum produced in the string phase. The spectrum depends on four parameters: the frequency f_s and the fractional energy density Ω_{gw}^s produced at the end of the dilaton-driven inflation phase, the maximal frequency f_1 above which gravitational radiation is not produced and the maximal fractional energy density $\Omega_{\text{gw}}^{\text{max}}$ which occurs at frequency f_1 . In this Letter we follow [13] setting

$$f_1 = 1.3 \times 10^{10} \text{ Hz} \left(\frac{H_r}{5 \times 10^{17} \text{ GeV}} \right)^{1/2} \quad (8)$$

and

$$\Omega_{\text{gw}}^{\text{max}} = 1 \times 10^{-7} h_{100}^{-2} \left(\frac{H_r}{5 \times 10^{17} \text{ GeV}} \right)^2, \quad (9)$$

assuming no late entropy production and making reasonable choices for the number of effective degrees of freedom. H_r is the Hubble parameter at the string phase. The “reduced” Hubble parameter h_{100} is in the range $0.4 \leq h_{100} \leq 0.85$ by observations.

By virtue of Eqs. (5) and (6), using given multiple IFOs, for any given set of parameters we may numerically evaluate the optimal signal to noise ratio SNR_{opt} ; if this value is greater than 3.29 then with 5% false alarm and 95% detection rate, the background can be detected by those IFOs. We compare the two approaches for constraining the string cosmology gravitational wave background space parameter adopting LIGO, LIGOL, Virgo and GEO at their designed noise levels [22–24]. The location information of the different GW observatories were obtained from [25] and references therein. The regions of detection ability to parameters space for the approach of combining multiple pairs of IFOs (labeled “combining pairs”) and the approach of directly combining four IFOs (labeled “directly combining”) for spectrum in Eq. (7) (labeled “dilaton + string” case) are shown in Fig. 3. We have assumed $h_{100} = 0.63$ [26] and $H_r = 5 \times 10^{17} \text{ GeV}$.

The parameter β is determined by the basic physical parameters of string cosmology models which are not well known. Just to show the different detection abilities of two approaches, a phenomenological model is adopted. This model is the “dilaton only” case in [13], assuming NO stochastic background is produced during the string phase of expansion, i.e.,

$$\Omega_{\text{gw}}(f) = \begin{cases} \Omega_{\text{gw}}^s (f/f_s)^3, & f < f_s, \\ 0, & f_s < f. \end{cases} \quad (10)$$

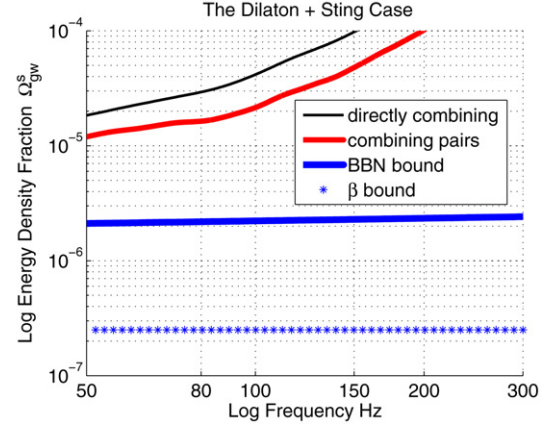


Fig. 3. The region of parameter space for which the gravitational wave stochastic produced in the “dilaton + string” case is detectable by the approach of combining multiple pairs of IFOs (labeled “combining pairs”) and the approach of directly combining four IFOs (labeled “directly combining”). With 5% false alarm and 95% detection rate, the region above of the curves shows the detectable parameter space of the stochastic gravitational wave background produced in the “dilaton + string” case by networks of detectors. It is clear that the “combining pairs” approach has higher detection ability for the gravitational wave background. But the “combining pairs” approach is still far away from observing the stochastic gravitational wave background spectrum.

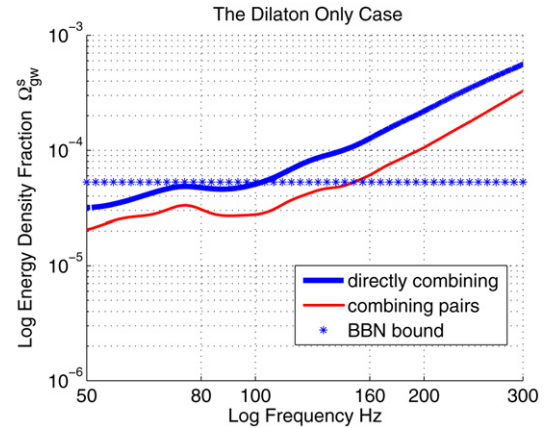


Fig. 4. The region in parameter space for which the gravitational wave stochastic background is observable by two approaches with 5% false alarm and 95% detection rate. The region above of the curves shows the detectable parameter space of the stochastic gravitational wave background produced in the “dilaton only” case by networks of detectors. It is clear that the “combining pairs” approach has more chance to detect the gravitational wave background.

It is phenomenologically interesting as it is a model whose spectrum peaks in the real IFOs band. Fig. 4 shows the regions of detection abilities to parameter space for “combining pairs” approach and “directly combining” approach.

We also plot the Big Bang nucleosynthesis (BBN) bounds [13,27] to see the detective chance for the assuming spectrum in Eqs. (7) and (10). Note that in the “dilaton + string” case, the slope β appearing in Eq. (7) must satisfy the constraint $\beta \geq 0$ (see e.g. [28]). As a consequence, $\Omega_{\text{gw}}^{\text{max}} = \Omega_{\text{gw}}(f_1)$ is always larger than $\Omega_{\text{gw}}^s = \Omega_{\text{gw}}(f_s)$. So, there is another bound for Ω_{gw}^s . In this Letter, by assuming $h_{100} = 0.63$ and $H_r = 5 \times 10^{17} \text{ GeV}$, $\Omega_{\text{gw}}(f_1) \simeq 0.25 \times 10^{-6}$ sets a tighter bound. We also plot this tighter bound (labeled “ β ” bound) in Fig. 3.

4. Conclusion

Two optimal approaches of combing multiple detectors to detect a stochastic gravitational wave background are proposed in [11]. As shown in Figs. 1 and 2, the real detectors should not be of identical noise levels and overlap reduction functions. It is necessary to compare two optimal approaches of detecting the stochastic gravitational wave background for any given real IFOs. In this short Letter, we have compared two approaches of combing four real ground IFOs (LIGO, LIGOL, Virgo, GEO) to show the detection ability of the stochastic gravitational wave background predicted by string cosmology of the early universe. As shown in Figs. 3 and 4, it is clear that the approach of combing multiple pairs of IFOs shows more detective chance to string cosmology in both “dilaton + string” case and “dilaton only” case than the approach of directly combing outputs of those four IFOs. In the “dilaton only” case, as shown in Fig. 4, both approaches have the chance to observe the spectral peak between 50 and 100 Hz. The approach of combing multiple pairs of IFOs could detect that background at a little higher frequency up to 160 Hz. In the “dilaton + string” case, both approaches are also far away from observing the stochastic gravitational wave background spectrum. We hope the next generation IFOs will constrain the parameter tighter.

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References

- [1] R. Schneider, V. Ferrari, S. Mataese, S.F. Portegies Zwart, *Mon. Not. R. Astron. Soc.* 324 (2001) 797.
- [2] M. Maggiore, *Phys. Rep.* 331 (2000) 283.
- [3] B. Allen, The stochastic gravity-wave background: sources and detection, in: *Proceedings of the Les Houches School on Astrophysical Sources of Gravitational Radiation*, Springer-Verlag, 1996, gr-qc/960403.
- [4] A. Buonanno, gr-qc/0303085.
- [5] G. Veneziano, *Phys. Lett. B* 265 (1991) 287.
- [6] M. Gasperini, G. Veneziano, *Astropart. Phys.* 1 (1993) 317; M. Gasperini, G. Veneziano, *Mod. Phys. Lett. A* 8 (1993) 3701.
- [7] M. Gasperini, M. Giovannini, *Phys. Lett. B* 282 (1992) 36.
- [8] A. Abramovici, et al., *Science* 256 (1992) 325, <http://www.ligo.org>.
- [9] B. Caron, et al., *Class. Quantum Grav.* 14 (1997) 1461, <http://www.virgo.infn.it>.
- [10] H. Lück, et al., *Class. Quantum Grav.* 14 (1997) 1471, <http://www.geo600.uni-hannover.de>.
- [11] B. Allen, J.D. Romano, *Phys. Rev. D* 59 (1999) 102001.
- [12] O. Malaspinas, R. Sturani, *Class. Quantum Grav.* 23 (2006) 319.
- [13] B. Allen, R. Brustein, *Phys. Rev. D* 55 (1997) 3260.
- [14] R. Brustein, gr-qc/9810063.
- [15] M. Gasperini, S. Nicotri, *Phys. Lett. B* 633 (2006) 155.
- [16] V. Mandic, A. Buonanno, *Phys. Rev. D* 73 (2006) 063008.
- [17] G. Cella, et al., arXiv: 0704.2983.
- [18] E. Flanagan, *Phys. Rev. D* 48 (1993) 2389.
- [19] M. Gasperini, M. Maggiore, G. Veneziano, *Nucl. Phys. B* 474 (1997) 315.
- [20] R. Brustein, M. Gasperini, M. Giovannini, G. Veneziano, *Phys. Lett. B* 361 (1995) 45.
- [21] R. Brustein, hep-th/9604159.
- [22] A. Lazzarini, et al., 1996, LIGO Science Requirement Documents, LIGO E950018-02, and http://www.ligo.caltech.edu/~lazz/distribution/LSC_Data/.
- [23] M. Punturo, Virgo note VIR-NOT-PER-1390-51, Issue 10 (2004). Digital format from <http://www.virgo.infn.it/senscurve/>.
- [24] <http://www.aei.mpg.de/jrsmith/geocurves.html>.
- [25] B. Allen, <http://www.lsc-group.phys.uwm.edu/ballen/grasp-distribution/>.
- [26] A. Sandage, G.A. Tammann, A. Saha, F.D. Macchetto, N. Panagia, *Astrophys. J.* 653 (2006) 843.
- [27] V.F. Schwartzmann, *JETP Lett.* 9 (1969) 184; T. Walker, et al., *Astrophys. J.* 376 (1991) 51.
- [28] M. Gasperini, *Elements of String Cosmology*, Cambridge Univ. Press, Cambridge, 2007, Chapter 7, p. 304.