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Comparison of LMA and LOW solar solution predictions in an $SO(10)$ GUT model

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Abstract

Within the framework of an $SO(10)$ GUT model that can accommodate both the LMA and LOW solar neutrino mixing solutions by appropriate choice of the right-handed Majorana matrix elements, we present explicit predictions for the neutrino oscillation parameters Δm_{21}^2 , $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and δ_{CP} . Given the observed near maximality of the atmospheric mixing, the model favors the LMA solution and predicts that δ_{CP} is small. The suitability of Neutrino Superbeams and Neutrino Factories for precision tests of the two model versions is discussed.

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Over the last few years the evidence for neutrino oscillations between the three known neutrino flavors (ν_e , ν_μ , and ν_τ) has become increasingly convincing. The atmospheric neutrino flux measurements from the Super-Kamiokande (Super-K) experiment exhibit a deficit of muon neutrinos which varies with zenith angle (and hence baseline) in a way consistent with $\nu_\mu \rightarrow \nu_\tau$ oscillations [1]. In addition, recent combined evidence from Super-K and the SNO experiments [2] indicate that some electron-neutrinos from the sun are oscillating into muon and/or tau neutrinos. While the atmospheric neutrino data with $\nu_\mu \rightarrow \nu_\tau$ oscillations points to a small region of the mixing parameter space [1], the solar neutrino data is consistent with at least two regions of parameter space [3], corresponding to either the Large Mixing Angle (LMA) or to the LOW MSW [4] solution.

Neutrino oscillation data constrain Grand Unified Theories (GUTs) which provide a theory of flavor and relate lepton masses and mixings to quark masses and mixings. It is known that the presently implied neutrino mass scales can be accommodated naturally within the framework of GUTs by the seesaw mechanism [5]. In practice finding an explicit GUT model for the LMA solution has been found challenging. However, one example has been constructed by Barr and one us [6]. In this model the Dirac and Majorana neutrino mass matrices are intimately related. It has been shown by the present authors [7] that by varying the Majorana mass matrix parameters any

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point in the presently-allowed LMA region can be accommodated. In the present Letter we show that the LOW region can also be realized in the model by choosing an appropriate texture for the right-handed Majorana mass matrix. In addition to the Majorana mass matrix, we also spell out the Dirac mass matrices, list the values of the associated input parameters and give results for the quark and charged lepton sectors. For each choice of the Majorana matrix we discuss the oscillation predictions. We use our results to illustrate how Neutrino Superbeams and Neutrino Factories [8] can further test this GUT model. Our results suggest that, independent of which is the preferred solution (LMA or LOW), Neutrino Superbeams and Factories will be necessary to identify the correct model, and that further investment in developing them should be encouraged.

Within the framework of three-flavor mixing, the flavor eigenstates ν_α ($\alpha = e, \mu, \tau$) are related to the mass eigenstates ν_j ($j = 1, 2, 3$) in vacuum by

$$\nu_\alpha = \sum_j U_{\alpha j} \nu_j, \quad U \equiv U_{\text{MNS}} \Phi_M, \quad (1)$$

where U is the unitary 3×3 Maki–Nakagawa–Sakata (MNS) mixing matrix [9] times a diagonal phase matrix $\Phi_M = \text{diag}(e^{i\chi_1}, e^{i\chi_2}, 1)$. The MNS matrix is conventionally parametrized by 3 mixing angles ($\theta_{23}, \theta_{12}, \theta_{13}$) and a CP-violating phase, δ_{CP} :

$$U_{\text{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\xi^* \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\xi & c_{12}c_{23} - s_{12}s_{23}s_{13}\xi & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\xi & -c_{12}s_{23} - s_{12}c_{23}s_{13}\xi & c_{23}c_{13} \end{pmatrix}, \quad (2)$$

where $c_{jk} \equiv \cos \theta_{jk}$, $s_{jk} \equiv \sin \theta_{jk}$ and $\xi = e^{i\delta_{\text{CP}}}$. The three angles can be restricted to the first quadrant, $0 \leq \theta_{ij} \leq \pi/2$, with δ_{CP} in the range $-\pi \leq \delta_{\text{CP}} \leq \pi$, though it proves advantageous to consider θ_{13} in the fourth quadrant for the LMA solutions.

The atmospheric neutrino oscillation data indicate that [1]

$$\Delta m_{32}^2 \simeq 3.0 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} = 1.0, \quad (\geq 0.89 \text{ at } 90\% \text{ c.l.}), \quad (3)$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and m_1, m_2 and m_3 are the mass eigenstates. The atmospheric neutrino oscillation amplitude can be expressed solely in terms of the U_{MNS} matrix elements and is given by $\sin^2 2\theta_{\text{atm}} = 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \simeq 4|U_{\mu 3}|^2|U_{\tau 3}|^2 = c_{13}^4 \sin^2 2\theta_{23}$. The approximation is valid because $|U_{e3}|$ is known to be small [10].

The solar neutrino oscillation data from Super-K indicate that, for the LMA solution, the allowed region is approximately bounded by

$$\Delta m_{21}^2 \simeq (2.2\text{--}17) \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{\text{sol}} \simeq (0.6\text{--}0.9), \quad (4)$$

while for the LOW solution,

$$\Delta m_{21}^2 \simeq (0.3\text{--}2) \times 10^{-7} \text{ eV}^2, \quad \tan^2 \theta_{12} \simeq (0.6\text{--}1.2), \quad (5)$$

where the solar neutrino oscillation amplitude is

$$\sin^2 2\theta_{\text{sol}} = 4|U_{e1}|^2(1 - |U_{e1}|^2) \simeq 4|U_{e1}|^2|U_{e2}|^2,$$

while $\tan^2 \theta_{12} = |U_{e2}/U_{e1}|^2$.

The GUT model we consider is based on an $SO(10)$ GUT with a $U(1) \times Z_2 \times Z_2$ flavor symmetry. The model involves a minimum set of Higgs fields which solves the doublet–triplet splitting problem. The Higgs superpotential exhibits the $U(1) \times Z_2 \times Z_2$ symmetry which is used for the flavor symmetry of the GUT model. Details of the model can be found in [6]. We simply note that the Dirac mass matrices U, D, N, L for the up quarks, down quarks, neutrinos and charged leptons, respectively, are found for $\tan \beta \simeq 5$ to be

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix},$$

$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix}, \quad (6)$$

where U and N are scaled by M_U , and D and L are scaled by M_D . All nine quark and charged lepton masses, plus the three CKM angles and CP phase, are well-fitted with the eight input parameters

$$\begin{aligned} M_U &\simeq 113 \text{ GeV}, & M_D &\simeq 1 \text{ GeV}, \\ \sigma &= 1.78, & \epsilon &= 0.145, \\ \delta &= 0.0086, & \delta' &= 0.0079, \\ \phi &= 126^\circ, & \eta &= 8 \times 10^{-6}, \end{aligned} \quad (7)$$

defined at the GUT scale to fit the low scale observables after evolution downward from Λ_{GUT} :

$$\begin{aligned} m_t(m_t) &= 165 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV}, \\ m_u(1 \text{ GeV}) &= 4.5 \text{ MeV}, & m_\mu &= 105.7 \text{ MeV}, \\ V_{us} &= 0.220, & m_e &= 0.511 \text{ MeV}, \\ V_{cb} &= 0.0395, & \delta_{\text{CP}} &= 64^\circ. \end{aligned} \quad (8)$$

These lead to the following predictions:

$$\begin{aligned} m_b(m_b) &= 4.25 \text{ GeV}, & m_c(m_c) &= 1.23 \text{ GeV}, \\ m_s(1 \text{ GeV}) &= 148 \text{ MeV}, & m_d(1 \text{ MeV}) &= 7.9 \text{ MeV}, \\ |V_{ub}/V_{cb}| &= 0.080, & \sin 2\beta &= 0.64. \end{aligned} \quad (9)$$

With no extra phases present, the vertex of the CKM unitary triangle occurs near the center of the presently allowed region with $\sin 2\beta \simeq 0.64$, comparing favorably with recent results [11]. The Hermitian matrices $U^\dagger U$, $D^\dagger D$ and $N^\dagger N$ are diagonalized with small left-handed rotations, U_U , U_D , U_N , respectively, while $L^\dagger L$ is diagonalized by a large left-handed rotation, U_L . This accounts for the small value of

$$|V_{cb}| = |(U_U^\dagger U_D)_{cb}|,$$

while

$$|U_{\mu 3}| = |(U_L^\dagger U_\nu)_{\mu 3}|$$

will turn out to be large for any reasonable right-handed Majorana mass matrix, M_R [12].

The effective light neutrino mass matrix, M_ν , is obtained from the seesaw mechanism once M_R is specified. While the large atmospheric neutrino mixing $\nu_\mu \leftrightarrow \nu_\tau$ arises primarily from the structure of the charged lepton mass matrix, the structure of M_R determines the type of $\nu_e \leftrightarrow \nu_\mu$, ν_τ solar neutrino mixing.

To obtain the LMA solution requires some fine-tuning and a hierarchical structure for M_R , but this can be explained in terms of Froggatt–Nielsen diagrams [13]. Here we restrict our attention to a slightly less general form for M_R than that considered in [6] and [7]:

$$M_R = \begin{pmatrix} b^2 \eta^2 & -b\epsilon \eta & a\eta \\ -b\epsilon \eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \quad (10)$$

where the parameters ϵ and η are those introduced in Eq. (6) for the Dirac sector. This structure for M_R can be understood as arising from one Higgs singlet which induces a $\Delta L = 2$ transition and contributes to all nine matrix elements while, by virtue of its flavor charge assignment, a second Higgs singlet breaks lepton number but modifies only the 13 and 31 elements of M_R . As shown in detail in [7], we can introduce additional CP violation by assigning

a relative phase to the two lepton number breaking Higgs singlets, whereby we set

$$a = b - a' e^{i\phi'}. \quad (11)$$

On the other hand, we find the LOW solution can be obtained with the simple hierarchical structure for M_R ,

$$M_R = \begin{pmatrix} e & d & 0 \\ d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R, \quad (12)$$

where by the flavor charge assignments, one Higgs singlet inducing a $\Delta L = 2$ transition contributes to the 12, 21 and 33 elements, while a second Higgs singlet also breaks lepton number but contributes only to the 11 matrix element. For simplicity we keep both d and e real, since the leptonic CP phase is inaccessible to measurement for Δm_{21}^2 values in the LOW region.

For either the LMA or LOW version, M_ν is then obtained by the seesaw formula [5], $M_\nu = N^T M_R^{-1} N$. With M_ν complex symmetric, both $M_\nu^\dagger M_\nu$ and M_ν itself can be diagonalized by the same unitary transformation, U_ν , where in the latter case we find

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_1, -m_2, m_3). \quad (13)$$

With real light neutrino masses, U_ν cannot be arbitrarily phase transformed and is uniquely specified up to sign changes on its column eigenvectors [7]. Hence U_{MNS} is found by applying phase transformations on $U_L^\dagger U_\nu$ to bring $U_L^\dagger U_\nu$ into the parametric form of Eq. (2) whereby the $e1$, $e2$, $\mu3$ and $\tau3$ elements are real and positive, the real parts of the $\mu2$ and $\tau1$ elements are positive, while the real parts of the $\mu1$ and $\tau2$ elements are negative. The inverse phase transformation of that applied on the right can then be identified with the Majorana phase matrix, Φ_M of Eq. (2). The evolution of the predicted values between the GUT scale and the low scales can be safely ignored [14], since $\tan \beta \simeq 5$ is moderately low and the neutrino mass spectrum is hierarchical with the opposite CP parities present in Eq. (13).

We can now examine the viable region of GUT model parameter space that is consistent with either the LMA or LOW solar neutrino solution, and explore the predicted relationships among the observables $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{13}$, δ_{CP} , Δm_{32}^2 , and Δm_{21}^2 . We shall emphasize here the simpler cases in which there are, in effect, only two additional real dimensionless GUT model parameters, a and b in the LMA version or d and e in the LOW version. In either version, the third parameter Λ_R sets the scale of Δm_{32}^2 . The more general CP results obtained for the LMA solution with the presence of a complex parameter a in Eq. (11) have been explored in detail in [7].

The viable region of GUT model parameter space consistent with the LMA solar solution is shown in Fig. 1. Both parameters a and b are constrained by the data to be close to unity, with $1.0 \lesssim a \lesssim 2.4$ and $1.8 \lesssim b \lesssim 5.2$. Superimposed on the allowed region, Fig. 1(a) shows contours of constant $\sin^2 2\theta_{12}$ and contours of constant Δm_{21}^2 . Fig. 1(b) similarly displays the allowed region with contours of constant $\sin^2 2\theta_{12}$ and $\sin^2 2\theta_{13}$ superimposed. The nearly parallel nature of the contours of Δm_{21}^2 in (a) and $\sin^2 2\theta_{13}$ in (b) indicates a strong correlation between them. As the predicted Δm_{21}^2 increases, the predicted $\sin^2 2\theta_{13}$ decreases. Note that if the LMA solution is indeed the correct solution, KamLAND [15] is expected to provide measurements of Δm_{21}^2 and $\sin^2 2\theta_{12}$ to a precision of about 10% [16]. From these measurements the model parameters a and b can be determined from Fig. 1(a). Fig. 1(b) can then be used to give a prediction for $\sin^2 2\theta_{13}$ with a precision also of order 10%.

In Table 1 we have selected six points in the LMA allowed parameter region to illustrate the neutrino oscillation predictions of the GUT model. The correlations noted above are evident. It is also striking how nearly maximal are the values for the atmospheric mixing parameter, $\sin^2 2\theta_{23}$. This apparently arises not from some additionally imposed symmetry but rather from the fine tuning between the right-handed Majorana and Dirac neutrino mass matrices, cf. Eqs. (6) and (10). However, if an additional phase is incorporated into M_R for this LMA case as indicated in Eq. (11), the maximality of the atmospheric mixing is decreased to the lower bound in Eq. (3) as $|\delta_{\text{CP}}|$ approaches 50° . See [7] for more details.

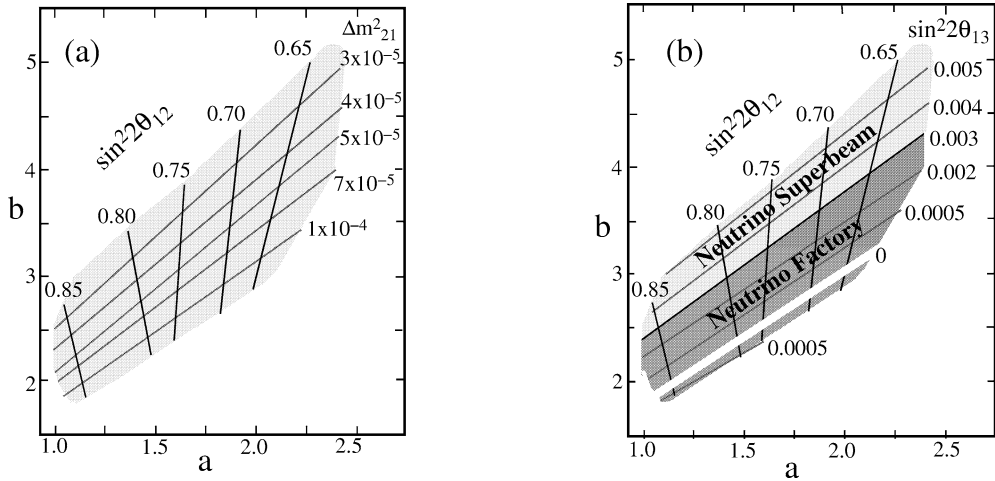


Fig. 1. The viable region of GUT parameter space consistent with the present bounds on the LMA MSW solution. Contours of constant $\sin^2 2\theta_{12}$ are shown together with (a) contours of constant Δm_{21}^2 and (b) contours of $\sin^2 2\theta_{13}$.

Table 1

List of six points selected in the LMA allowed parameter region to illustrate the neutrino oscillation parameter predictions of the GUT model. Here the CP phase δ_{CP} arises from ϕ in L alone, as no phase ϕ' has been introduced in M_R

a	b	Δm_{21}^2 (eV ²)	Δm_{32}^2 (eV ²)	$\tan^2 \theta_{12}$	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	δ_{CP}
1.0	2.0	6.5×10^{-5}	3.2×10^{-3}	0.49	0.88	0.994	0.0008	-4°
1.2	2.8	3.3×10^{-5}	3.2×10^{-3}	0.43	0.84	0.980	0.0038	-1°
1.6	2.9	6.1×10^{-5}	3.2×10^{-3}	0.35	0.77	0.998	0.0015	-3°
1.7	2.7	10.9×10^{-5}	3.2×10^{-3}	0.32	0.73	0.996	0.00008	-14°
1.7	3.4	4.0×10^{-5}	3.2×10^{-3}	0.33	0.75	0.992	0.0033	-2°
2.2	3.5	8.8×10^{-5}	3.2×10^{-3}	0.24	0.63	0.996	0.0008	-4°

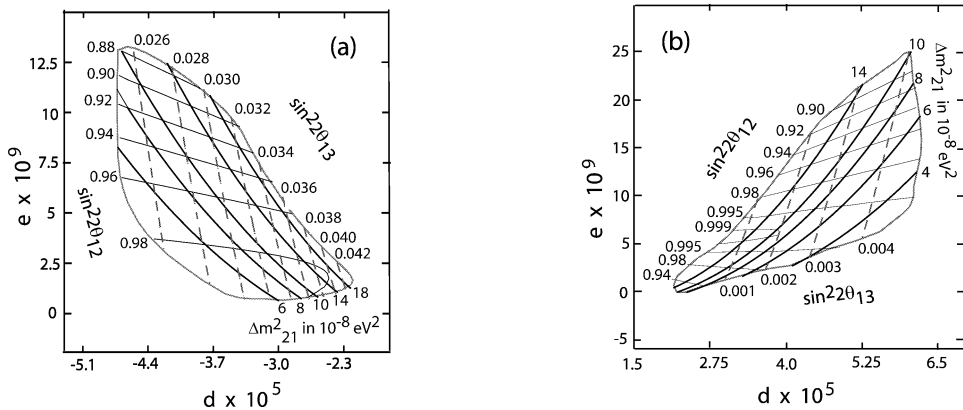


Fig. 2. The viable region of GUT parameter space consistent with the present bounds on the LOW MSW solution for (a) negative d and (b) positive d . Contours of constant $\sin^2 2\theta_{13}$, $\sin^2 2\theta_{12}$ and Δm_{21}^2 are shown.

Table 2

List of six points selected in the LOW allowed parameter region to illustrate the neutrino oscillation parameter predictions of the GUT model

d	e	Δm_{21}^2 (eV ²)	Δm_{32}^2 (eV ²)	$\tan^2 \theta_{12}$	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$
-4.2×10^{-5}	10.0×10^{-9}	1.20×10^{-7}	3.0×10^{-3}	0.56	0.906	0.911	0.028
-4.1×10^{-5}	4.0×10^{-9}	0.52×10^{-7}	3.0×10^{-3}	0.81	0.975	0.899	0.027
-3.6×10^{-5}	3.0×10^{-9}	0.64×10^{-7}	3.0×10^{-3}	0.86	0.980	0.898	0.030
3.6×10^{-5}	5.0×10^{-9}	0.98×10^{-7}	3.0×10^{-3}	1.00	0.999	0.914	0.0016
5.3×10^{-5}	10.0×10^{-9}	0.50×10^{-7}	3.0×10^{-3}	0.82	0.989	0.912	0.0039
5.0×10^{-5}	13.0×10^{-9}	0.85×10^{-7}	3.0×10^{-3}	0.70	0.966	0.918	0.0033

Turning now to the GUT model version for the LOW solution, we find that there are two parametric regions shown in Fig. 2 for the presently allowed solutions, corresponding to $-4.8 \lesssim d \times 10^5 \lesssim -2.2$, $0 \lesssim e \times 10^9 \lesssim 13$ and $2.0 \lesssim d \times 10^5 \lesssim 6.0$, $0 \lesssim e \times 10^9 \lesssim 25$. Here no dramatic correlation between $\sin^2 2\theta_{13}$ and Δm_{21}^2 exists, so we have plotted contours of Δm_{21}^2 , $\sin^2 2\theta_{12}$ and $\sin^2 2\theta_{13}$ on the same figures. If KamLAND fails to see a signal for the LMA region while Borexino [17], for example, identifies oscillations corresponding to the LOW region and determines $\sin^2 2\theta_{12}$ and Δm_{21}^2 with nearly 10% precision [18], $\sin^2 2\theta_{13}$ will be specified up to a two-fold ambiguity in the GUT model in question. A first measurement of $\sin^2 2\theta_{13}$ would resolve the ambiguity, and a precise measurement would test the model. For the negative d version, a SuperBeam facility capable of probing down to $\sin^2 2\theta_{13} \simeq 0.003$ will be able to test the model, while for the positive d version the complete parameter space can only be tested with a Neutrino Factory. Table 2 gives the relevant mixing solutions for a set of six points. In contrast to the LMA results with small CP phases, we see that the atmospheric mixing for the LOW solution is large but not nearly so maximal.

In conclusion, we have studied predictions for a particular but representative GUT model that can accommodate both the LMA and LOW solar neutrino solutions. We find that precise measurements of $\sin^2 2\theta_{12}$, Δm_{21}^2 and $\sin^2 2\theta_{13}$ are needed to test the theory. Given the observed near maximal value of $\sin^2 2\theta_{23}$ the LMA solution, which requires some fine tuning of the M_R matrix, is favored by the model. The model then predicts that the CP phase δ_{CP} is small and $\sin^2 2\theta_{13} \lesssim 0.006$. For the LOW solution which requires no fine tuning, $\sin^2 2\theta_{13}$ can be as small as this, or an order of magnitude larger, depending upon the sign of the d model parameter in M_R . Our work suggests progress on testing GUTs can be made with Neutrino Superbeams, but ultimately a Neutrino Factory will be needed to help identify the correct model.

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