PHYSICS LETTERS B

Critical remarks on Finsler modifications of gravity and cosmology by Zhe Chang and Xin Li

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ABSTRACT

I do not agree with the authors of papers arXiv:0806.2184 and arXiv:0901.1023v1 (published in [Zhe Chang, Xin Li, Phys. Lett. B 668 (2008) 453] and [Zhe Chang, Xin Li, Phys. Lett. B 676 (2009) 173], respectively). They consider that *"In Finsler manifold, there exists a unique linear connection – the Chern connection . . . It is torsion freeness and metric compatibility . . ."*. There are well-known results (for example, presented in monographs by H. Rund and R. Miron and M. Anastasiei) that in Finsler geometry there exist an infinite number of linear connections defined by the same metric structure and that the Chern and Berwald connections *are not metric compatible*. For instance, the Chern's one (being with zero torsion and "weak" compatibility on the base manifold of tangent bundle) is not generally compatible with the metric structure on total space. This results in a number of additional difficulties and sophistication in definition of Finsler spinors and Dirac operators and in additional problems with further generalizations for quantum gravity and noncommutative/string/brane/gauge theories. I conclude that standard physics theories can be generalized naturally by gravitational and matter field equations for the Cartan and/or any other Finsler metric compatible connections. This allows us to construct more realistic models of Finsler spacetimes, anisotropic field interactions and cosmology.

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There is a recent interest for a new physics beyond the Standard Model related to Finsler theories of curved spacetime and quantum gravity and possible applications in modern cosmology, see [1–4] and references therein. Such theories constructed on tangent bundles of spacetime manifolds are positively with local Lorentz violations which may be related to new directions in particle physics and dark energy and dark matter models in cosmology. It is very important that researches in particle physics and cosmology would became more familiar with some important methods and perspectives and possible applications of Finsler geometry in standard and nonstandard theories of physics (a survey and reviews of main ideas and results oriented to applications in high energy physics are presented respectively in Refs. [5–7]).

In this Letter I comment on some ambiguities existing in the above mentioned two papers by Zhe Chang and Xin Li, suggest possible solutions of the Chern–Finsler "nonmetricity" problem and speculate about "well-defined" Finsler gravity theories and cosmological models. The goal is also to present a brief review of the main concepts and results on Finsler gravity modifications of the

Einstein gravity theory. We emphasize the possibility to model (pseudo) Finsler configurations as exact solutions in general relativity and analyze the most important consequences for quantum gravity and applications in cosmology.

1. Problems with the Chern–Finsler nonmetricity

Just before formula (5) in [1], the authors wrote: "In Finsler manifold, there exist a unique linear connection – the Chern connection [22]. It is torsion freenes and metric compatibility . . .".[1] Perhaps, such a conclusion was drawn from formulas (5) and (9) in [1] stating that the metric compatibility and zero torsion conditions hold for the Chern connection on the base manifolds of tangent bundles. In general, Finsler spaces endowed with Chern and/or Berwald connections, and various their modifications, are with generic nonmetricity (when the metric and connection structures are not compatible). The geometry and physical properties of such Finsler-affine (and generalized Lagrange-affine) spaces and

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[1] We wrote [22] which means the corresponding citation in [1]; similar sentences are written also before formula (2) in [2].

nonholonomic metric-affine gravity theories were studied in details in Part I of book [7].

We emphasize that a Finsler geometry/gravity model can be defined completely only on the total space of a tangent bundle TM of a manifold M (see, for instance, [3,4,7–9] and, chronologically, some most important monographs on Finsler geometry and applications [10–15]). If a (pseudo) Riemannian geometry on M is determined only by one fundamental geometric object, the symmetric metric tensor $\mathbf{g}$, a (pseudo) Finsler geometry has to be constructed from three fundamental geometric objects on total space TM.[2] For some canonical Finsler space models, all three fundamental geometric objects are completely defined by a generating (fundamental) Finsler function $F(x, y)$ subjected to certain homogeneity and other conditions, with x denoting the set of local coordinates on M (or, alternatively, on $h\mathbf{V}$) and y denoting the set of "fiber" like coordinates in TM (or, alternatively, on $v\mathbf{V}$). Following well-defined conventions for Cartan/Berwald/Chern–Finsler spaces, one generates by F on TM: 1) a (Finsler, for instance, Sasaki type) metric ${}^F\mathbf{g} = (h^F\mathbf{g}, v^F\mathbf{g})$, 2) a nonlinear connection (N-connection) $\mathbf{N}$ (associated to a splitting $TTM = hTM \oplus vTM$), and 3) a distinguished connection, d-connection ${}^F\mathbf{D} = (h\mathbf{D}, v\mathbf{D})$ (which is a linear connection adapted to an N-connection $h–v$ splitting, i.e. preserving under parallelism such a Whitney sum $\oplus$ stated on corresponding tangent spaces). So, a Finsler geometry is completely defined by a corresponding set of data $(F: {}^F\mathbf{g}, \mathbf{N}, {}^F\mathbf{D})$.

The authors of [1,2] cited the monograph [15] and used the Chern d-connection [17], ${}^F\mathbf{D} = {}^{Ch}\mathbf{D}$, derived in a unique form to satisfy the conditions: 1) vanishing torsion, ${}^{Ch}\mathbf{T} = 0$, and "horizontal" metric compatibility, $h^{Ch}\mathbf{D}(h^F\mathbf{g}) = 0$. In brief, the Chern d-connection ${}^{Ch}\mathbf{D}$ on TM generalizes the Levi-Civita connection ∇ on M in such a manner that the torsion is "pumped" into a "vertical" nonmetricity $v\mathbf{Q} \neq 0$, when (in general, on TM) $\mathbf{Q} \equiv {}^{Ch}\mathbf{D}^F\mathbf{g} \neq 0$. Such a proof exists in details in [15]. So, the authors of [1,2] were not right stating that ${}^{Ch}\mathbf{D}$ is "metric compatible". A "weak" compatibility exists only on the h-subspace but not for a general geometric/physical model on TM. It is also not metric compatible the Berwald model of Finsler geometry with ${}^F\mathbf{D} = {}^B\mathbf{D}$ (label B is from "Berwald"), see details in Refs. [7–9,15].

Alternatively to ${}^{Ch}\mathbf{D}$ and/or ${}^B\mathbf{D}$, there is in Finsler geometry a canonical metric compatible d-connection (called the Cartan d-connection [10]), ${}^F\mathbf{D} = {}^c\mathbf{D}$ (historically, it was the first one introduced for a completely defined model of Finsler space). It satisfies the metricity condition on TM, ${}^c\mathbf{D}^F\mathbf{g} = 0$, but has a nontrivial torsion, ${}^c\mathbf{T} \neq 0$. The interesting thing is that ${}^c\mathbf{T}$ is induced by h- and v-components of ${}^F\mathbf{g}$ and the coefficients of the canonical Cartan N-connection, $\mathbf{N} = {}^c\mathbf{N}$ (with an associated canonical semi-spray configuration [7–9,15]). All mentioned canonical values are determined by a fundamental Finsler function F. Such a torsion ${}^c\mathbf{T}$ is very different from those in Einstein–Cartan/gauge/string gravity when certain additional field equations are used for determining the torsion components.

In general, there is an infinite number of metric compatible connections in Finsler and Lagrange geometries (Lagrange geometry is a "nonhomogeneous" generalization of Finsler geometry, with a nondegenerate fundamental Lagrangian); see a theorem by R. Miron in [8,9], we discuss the physical implications in [5]. So, the authors of [1,2] erred stating that the Chern connection is the "unique" one for Finsler gravity. Here we also emphasize that

the nonmetricity of Finsler gravity models with Chern/Berwald d-connection, and with any other nonmetric one, results in more sophisticate "very nonstandard" physical theories. Together with an unclear status of nonmetricity fields, the metric incompatibility make more difficult the definition of spinors and conservation laws in Finsler gravity (there is a series of our works in this direction [18–22]) and does not allow "simple" (super) string and noncommutative generalizations like we proposed [25,26,23,24].[3] We studied in details the so-called Finsler/Lagrange-affine spaces and gravity models, in general, with nontrivial torsion and nonmetricity, in Part I of monograph [7] and discussed possible connections of Finsler geometry and methods to standard theories of physics in [5,27].

2. Ambiguities with Finsler generalizations of the Einstein equations

Before formula (9) in [2], the authors wrote: "The Ricci tensor on Finsler manifold was first introduced by Akbar-Zadeh [21]" (we put in this Letter the reference [28] which in that paper is [21]). Perhaps that was the first attempt to define a Ricci tensor for Finsler spaces with constant sectional curvature for the Chern d-connection ${}^{Ch}\mathbf{D}$, but this is not correct for all Finsler geometry/gravity models. In general, there were considered various types of Ricci type tensors in Finsler geometries (see, for instance, [11,12,8,9,14,7,5]). There is, for instance, a discussion on Finsler and Lagrange gravity by S. Ikeda in Appendix to [8]. We also analyzed in details such generalized Finsler gravity theories, with zero and non-zero nonmetricity, for "standard" and "nonstandard" theories of particle physics and gravity, providing a number of exact solutions and applications (together with generalizations to noncommutative/string/grave gravity), see reviews [5,6,27,7].

We note that in monographs [8,9] (see also references therein), the Einstein equations were first time written in a self-consistent form on vector/tangent bundles for the so-called canonical d-connection/$h–v$-connection (in our works, for instance, [5,27] denoted $\widehat{\mathbf{D}}$; see there further details and local formulas). For canonical ${}^c\mathbf{N}$ and ${}^c\mathbf{D}$, they contain the Einstein equations for the Cartan–Finsler gravity model on TM. In brief, we can say that such a Finsler gravity theory is similar to the Einstein gravity but the field equations are for ${}^c\mathbf{D}$. The corresponding Ricci, $Ric(\widehat{\mathbf{D}})$, and Einstein, $\mathbf{E}(\widehat{\mathbf{D}})$, tensors are determined not by the Levi-Civita connection ∇ (which is not a d-connection because it is not adapted to the N-connection structure) but constructed for $\widehat{\mathbf{D}}$. In a more particular case, for Finsler and Lagrange spaces, we can consider $\widehat{\mathbf{D}} = {}^c\mathbf{D}$.

In Refs. [8,9], for gravity theories on (generalized) Lagrange–Finsler spaces, on TM, and Finsler like configurations modelled on nonholonomic manifolds (for instance, defined as exact solu-

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[2] Physicists use "pseudo" for metric structure with local signature $\pm$. Alternatively, a Finsler geometry/gravity model can be constructed on a nonholonomic manifold $\mathbf{V}$ endowed with a conventional "horizontal–vertical" ($h–v$), splitting, $T\mathbf{V} = h\mathbf{V} \oplus v\mathbf{V}$, i.e. with a nonholonomic distribution; we follow notations and results outlined in [5], see also details and bibliography on the geometry and applications of nonholonomic manifolds in [16,7].

[3] Briefly, we sketch the problem of definition of Clifford structures and spinors and of conservation laws in Finsler spaces: In general relativity, we have $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}$, for gamma matrices γ_μ, when the Levi-Civita connection ∇ satisfies the metricity conditions $\nabla_\alpha g_{\mu\nu} = 0$. This allows us to introduce the Dirac operator on (pseudo) Riemannian spaces, induced by ∇_α and a fixed tetradic basis and to define the Dirac equations for the Einstein gravity theory when $\nabla_\alpha T^{\mu\nu} = 0$, for any matter fields with energy–momentum tensor $T^{\mu\nu}$.

For a Finsler gravity model with nonmetricity, when ${}^F\mathbf{D}^F\mathbf{g} \neq 0$, it is a problem to define self-consistent analogs of gamma matrices and (Finsler–)Dirac operators. Working with metric compatible d-connections with nonholonomically induced torsions, the constructions become similar to those in general relativity. Here we also note that for nonholonomic configurations the Ricci tensor, in general, is not symmetric and ${}^F\mathbf{D}_\alpha T^{\mu\nu} \neq 0$. Both the Finsler–Dirac operators and generalized conservation laws on Finsler space with metric compatible d-connections, and corresponding nonintegrable constraints, were constructed/defined in the works to which this footnote refers.

tions in Einstein gravity),[4] there were considered gravitational field equations of type

$$\mathbf{E}(\widehat{\mathbf{D}}) = \mathbf{\Upsilon}, \qquad (1)$$

where the Einstein tensor $\mathbf{E}(\widehat{\mathbf{D}})$ and source $\mathbf{\Upsilon}$ are constructed from $\widehat{\mathbf{D}}$, $^F\mathbf{g}$ and Lagrangians for matter fields following the same principles as in general relativity theory but extended on corresponding vector/tangent bundles, or nonholonomic manifolds, in N-adapted form (following a corresponding tensor/form/spinor/variational calculus preserving the h-/v-splitting). We can consider such $\mathbf{\Upsilon}$ when for $\widehat{\mathbf{D}} \to \nabla$ Eqs. (1) transform into the usual Einstein equations (for different purposes, and generality, we have to work with arbitrary dimensions), see details in [5,27,24]. We emphasize that

$$\widehat{\mathbf{D}} = \nabla + \widehat{\mathbf{Z}}, \qquad (2)$$

when both the linear connections $\widehat{\mathbf{D}}$ and ∇ and the distortion tensor $\widehat{\mathbf{Z}}$ are defined by the same metric structure $\mathbf{g} = {}^F\mathbf{g}$ (we can introduce Finsler variables by certain convenient frame/coordinate transforms).

The most important property of Finsler like theories of gravity is that the "locally anisotropic" gravitational field equations are formulated not for the Levi-Civita connection ∇ but for a d-connection $\mathbf{D}$, or $^F\mathbf{D}$. In explicit form, a "physical" d-connection has to be chosen following certain theoretical arguments and compatibility with experimental data. If $\mathbf{D} = \widehat{\mathbf{D}}$, or $\mathbf{D} = {}^c\mathbf{D}$, we work with metric compatible geometries and more realistic physical models (admitting fermionic and gauge fields which can be introduced similarly as in standard particle physics). Even, in general,

$$\widehat{\mathbf{D}}\big(\mathbf{E}(\widehat{\mathbf{D}})\big) = \widehat{\mathbf{D}}\mathbf{\Upsilon} \neq 0, \qquad (3)$$

contrary to the general relativity theory when

$$\nabla E = \nabla T(\text{matter}) = 0, \qquad (4)$$

we can consider that relations of type (3) follow from (4) and usual Bianchi identities for (pseudo) Riemannian spaces; this is uniquely defined by nonholonomic distortions of type (2). For Cartan–Finsler spaces, such generalized conservation laws (3) are uniquely determined by a fundamental Finsler function $F(x, y)$ via corresponding fundamental data ($^F\mathbf{g}, \mathbf{N}, {}^F\mathbf{D}$).

The solutions for Randers–Finsler gravity and cosmology (with the Chern d-connection of approximate Berwald type, etc.) provided in Refs. [1,2] are supposed to solve certain important issues in modified Einstein/Newton gravity and dark energy and dark matter new physics. The gravitational field equations considered in those works are for d-connections which are metric noncompatible and resulting in a number of conceptual and theoretical problems in defining (for instance) conservation laws, spinors and Dirac equations, and a less clear status for nonmetricity fields. Introducing various types of nonmetricity fields we can "suit a lot of experimental data". Nevertheless, the main question is this: Why in Finsler gravity theories we should use metric noncompatible connections if there are various types of metric compatible ones with less problems for "standard" physics and without strong experimental constraints analyzed in [29]? Here we also emphasize that for the Finsler like configurations derived as exact solutions of Einstein equations in general relativity, i.e. on nonholonomic pseudo Riemannian manifolds, the local Lorentz symmetry can be

preserved, see [5,27]. Such constructions can be naturally extended on (co)tangent bundle theories even in such cases we cannot avoid models with broken local symmetries. Our conclusion is that we may construct more "standard" physical Finsler classical/quantum gravity theories for metric compatible connections like the Cartan d-connection.

3. On "well-defined" Finsler gravity theories and cosmological models

It seems that Finsler like gravity theories on (co)tangent bundles (with metrics and connections depending on velocity/momentum type variables) are natural consequences of all models of quantum gravity, see physical arguments and a review of recent results provided in [30]. A principle of general covariance coming from the classical version of the Einstein gravity theory results in very general quantum nonlinear dispersions of Finsler and non-Finsler type. Perhaps, it is not the case to postulate from the very beginning that such a generalized Finsler spacetime is of any special Randers/Berwald/Chern/...–Finsler type with fixed (in general, non-quadratic) line elements like that, for instance, taken for the Very Special Relativity, etc.

We argue that the quantum gravity theory is "almost sure" of generalized Finsler type on a correspondingly quantized (co)tangent bundle which in certain classical limits is described by nonholonomic gravity configurations on (pseudo) Riemannian/Finsler spacetimes and possible observable effects in modern cosmology and quantum physics. It can be approached following well-defined geometric and physical principles when the concepts of metric, connection and frame of reference are postulated to be the fundamental ones (even, in general, as certain quantized fields and/or possibly redefined as some almost Kähler/generalized Poisson structures, etc.) in any spacetime geometry and gravity theories:

1. For general nonlinear non-quadratic line elements, we can consider generating fundamental Finsler, or Lagrange (on cotangent bundles, respectively, Cartan, or Hamilton; in general, of higher order, see Refs. [26,19–21,31–34]) functions. Lifts of Sasaki type, or another ones, allow us to define canonical (Finsler type and generalizations) metric, $^F\mathbf{g}$, and N-connection, $^c\mathbf{N}$, structures.

2. From the class of infinite number of metric compatible and noncompatible linear (Finsler type, or generalized) connections, we can always choose/construct, following the so-called Kawaguchi and Miron processes [8,9,7,5], a canonical d-connection $\widehat{\mathbf{D}}$. In particular, we can introduce, for any (pseudo) Finsler geometry, the Cartan d-connection, $^c\mathbf{D}$, which is metric compatible and completely defined by $^F\mathbf{g}$ and $^c\mathbf{N}$. This way we eliminate possible difficulties/sophistication related to the nonmetricity geometry and fields and may consider, or derive in certain limits, various types of Finsler–Lagrange (super) string, gauge, nonholonomic Clifford/spinor, Finsler-affine and/or noncommutative gravity theories [7,18,19,22–26].

3. The Einstein equations for $\widehat{\mathbf{D}}$, or $^c\mathbf{D}$ (using nonholonomic constraints, we can include and theories with ∇), can be solved in very general forms for different models of Einstein and Finsler gravity, and various noncommutative/supersymmetric, etc., generalizations, following the anholonomic frame deformation method, see reviews of results in applications in [27, 24,7,5,35]. From various classes of very general generic off-diagonal solutions (with metrics which cannot be diagonalized by coordinate transforms), we can chose well-defined subclasses having certain physical importance (describing locally anisotropic black hole/ellipsoid/torus configurations, cosmolog-

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[4] For our purposes, a nonholonomic manifold is a usual (pseudo) Riemannian one with a prescribed nonholonomic distribution; a Finsler space can be considered as an example of nonholonomic tangent bundle with a corresponding nonintegrable (equivalently, nonholonomic/anholonomic) distribution determined by an N-connection structure, see details in [5,6,27,7,9,8].

ical inhomogeneous and locally anisotropic solutions, solitons, etc., see various examples and reviews in [36–38,5,7,27]).

4. For applications in modern cosmology, for instance, with the aim to elaborate realistic Finsler like inflation, dark energy and dark matter scenarios, it is important to elaborate Finsler generalizations (they are, in general, with inhomogeneous and anisotropic metrics) of the Friedman and Robertson–Walker (FRW) models. Such cosmological models should be grounded on solutions of the (Finsler) Einstein equations (1) for certain types of d-connections (with the above mentioned priorities for the metric compatible ones). If such models may propose certain important ideas and solutions in modern cosmology, they would serve as explicit criteria for choosing as the fundamental ones certain examples of Finsler like linear and nonlinear connections, and relevant anisotropic metric configurations. It is not the case to postulate from the very beginning any particular cases of metric ansatz and/or d-connections for Randers/Berwald/Chern/...–Finsler spacetime even they may be preferred as some more "simple" generalizations of the (pseudo) Riemannian spacetimes. Viable gravity theories are with nonlinear field equations and the exact solutions for cosmology and astrophysics, in general, are for generic off-diagonal metrics and generalized connections. A rigorous mathematical approach does not obey obligatory any original assumptions on parametrization of metrics and connections and conventional splitting into "holonomic" and "nonholonomic" variables.

5. There are also two another very important properties of the Cartan d-connection (which do not exist for the Chern/Berwald and other metric noncompatible d-connections):

- $^c\mathbf{D}$ and $^F\mathbf{g}$, for a fixed $^c\mathbf{N}$, define canonical almost Kähler models of Finsler–Lagrange, Hamilton–Cartan, Einstein gravity and various generalizations. Such models can be quantized applying a nonholonomically generalized Fedosov method [39–41], following the A-brane formalism [42], and developing a two-connection perturbative approach to the Einstein and gauge gravity theories [43,44]. Such quantum Einstein and/or Finsler–Lagrange gravity theories can be elaborated to have in certain quasi-classical limits different terms with locally violated Lorentz invariance, anomalies, formal renormalization properties, etc. It is also possible to construct models limiting locally relativistic and covariant theories.

- Finsler–Lagrange evolutions of geometry/gravity theories [45,46], and various generalizations with nonsymmetric metrics [47], noncommutative corrections [23], etc., appear naturally if Ricci flows of (pseudo) Riemannian metrics are subjected to nonholonomic constraints on evolution equations, see also possible applications in modern gravity, cosmology and astrophysics [48,49]. They are positively related via certain nonlinear renorm group flows to fundamental problems in quantum gravity and "early" stage of anisotropic quantum universes.

Finally we conclude: There are two general classes of Finsler type gravity, and geometric mechanics, theories with applications in modern physics and cosmology. The first class of locally anisotropic gravity theories originates from E. Cartan works on Finsler geometry, spinors and bundle spaces.[5] Here we cite the monograph [10] and further developments in [11–13,8,9,14,16,31,

32,25,26,24,7,5]. Even in the just mentioned monographs and review papers a number of geometric and physical constructions and Finsler geometry methods were considered for both types of metric compatible or noncompatible d-connections in Finsler spaces, the most related to "standard physics" constructions were elaborated for the Cartan and canonical d-connections which are metric compatible and follow the geometric and physical principles 1–5 mentioned above. Alternatively (the second class of theories), there are Finsler geometry and gravity models grounded on the Berwald and Chern d-connections, see details in [17,15,1,2], and a comparative review of standard and nonstandard physical theories and applications in Part I of [7] and in Refs. [5,6].

The key issues which should be solved both theoretically and experimentally are those if certain fundamental problems in quantum gravity and/or modern cosmology can be approached following Finsler theories with metric compatible, or not compatible, d-connections. The recent interest in new Finsler gravity physics and cosmology was in the bulk oriented to models both with local Lorentz violations and nonmetricity, like [1,2]. It would be a grave error if non-experts in Finsler geometry but physicists and mathematicians working in gravity and particle physics and/or cosmology would consider that the Chern d-connection is a "unique metric compatible and the best one" for Finsler like theories. The reality is that only following approaches with metric compatible connections, like the Cartan d-connection, we can elaborate physically viable models which are closely related to standard physics (as we emphasized in our works, see [39–42,44,45,49,35–37,23,24, 26] and reviewed in [5,27,7]).

Acknowledgements

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