Computational research on film cooling under rotating frame by an anisotropic turbulence model

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Abstract A new anisotropic $k$–$\omega$ turbulence model was proposed in this paper. This new model, with the standard $k$–$\omega$ model and the standard $k$–$e$ model was embedded in our three-dimensional (3-D) Navier–Stokes code to compute rotational film cooling. In addition, the theoretical and numerical analysis on the influence of the Coriolis and buoyancy forces induced by the rotation was discussed in detail. Major findings of this study are as follows: (1) The new anisotropic $k$–$\omega$ model preformed much better compared to its isotropic counterparts. (2) In the region of $6D$–$12D$ downstream the film hole, the numerical results of the new model were much closer to the experimental data than that in the region of $0$–$6D$. (3) The constant density term can be balanced by the pressure gradient and would not influence the velocity and temperature distributions. But the centrifugal buoyancy force and the Coriolis force would change the trajectory of cooling air and temperature distributions.

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1. Introduction

In an advanced gas turbine, one of the effective ways to achieve high thermal efficiency and high power output is to increase its inlet temperature that may be higher than the melting point of the turbine blade material. Therefore, the blades need to be cooled for its safe and long operation. Film cooling is probably the
most efficient blade-cooling technique employed in the design of modern turbine airfoils. For film cooling in general, it is well known that the film cooling performances are influenced by many parameters, such as injection angle, the mainstream Reynolds numbers and blowing ratio, blade surface curvature, momentum ratios, mainstream turbulence, etc. For rotating turbine blades, there is an additional and very important parameter to be accounted for, i.e. the rotation number (\(R_t\)), because the blades are rotating at high speed and thus additional forces are introduced and they will influence the already very complex cooling process. Therefore, searching for the mechanism of this complex interaction is essential and meaningful.

Many researches have been conducted to understand the coolant film behavior and the interaction of the cooling air with mainstream flow since 1970s. Comprehensive reviews were given by Goldstein [1], Garg and Gaugler [2–4]. Garg [5] computed the flow over the ACE rotor blade by using the \(k–\omega\) model, the \(q–\omega\) model and the zero-equation Baldwin–Lomax (B–L) model. The results show that the B–L model yields a better comparison with experimental data than other models, while the \(k–\omega\) model provided a better comparison on the pressure surface. It is obvious that none of the models employed is superior in the overall computational domain. Hassan and Yavuzkurt [6] concentrated on the comparison of four different two-equation turbulence models in predicting film cooling performance. The four turbulence models were: the standard \(k–\varepsilon\) model, the renormalization group (RNG) \(k–\varepsilon\) model, and the realizable \(k–\varepsilon\) model as well as the standard \(k–\omega\) model. The capabilities of the four two-equation turbulence models in predicting film cooling effectiveness were investigated. Walters and Leylek [7] performed numerical analyses with the standard \(k–\varepsilon\) model on a film-cooled flat surface.

From previous contributions it can be concluded that the isotropic turbulence modeling and near-wall treatment was not suitable to capture the jet lift-off and reattachment of the film cooling. Furthermore, the turbulence models which can accurately represent anisotropic turbulence will be needed to resolve downstream characteristics, particularly the lateral spreading of the coolant. Lakehal et al. [8] employed the standard \(k–\varepsilon\) model and the \(k–\varepsilon\) based two-layer model for predicting the film cooling. They founded that the two-layer turbulence model yields a noticeable improvement compared to the standard \(k–\varepsilon\) model with wall function. In Lakehal et al. [9], the lateral spreading of the temperature field is underpredicted by the standard \(k–\varepsilon\) model. The situation is improved by using the two-layer model (TLK), which yields effectiveness contours in reasonably good agreement with the measurements. Furthermore, they applied the new model in the calculations of a turbine blade with film cooling near the leading edge [10]. The model yields excellent agreement with the experiments for the isentropic Mach number distributions on the blade surface. Azzi and Lakehal [11] compared two classes of turbulence models with respect to their predictive performance in reproducing near-wall flow physics and heat transfer. Their results clearly showed that only the anisotropic eddy-viscosity model could correctly predict the spanwise spreading of the temperature field and reduce the strength of the secondary vortices. In Lakehal’s extension work [12], a new modeling strategy using near-wall variation of the turbulent Prandtl number as a function of the local Reynolds number was employed and compared with the results published in Ref. [11]. The comparison between the calculated and measured wall-temperature distributions showed that the anisotropic turbulence model with near-wall variation of the turbulent Prandtl number produced the best agreement. It appears that only stationary condition was investigated by the aforementioned literatures [8–12]. However, there may be a major difference between the fluid dynamics of film cooling in a rotating environment and film cooling under stationary condition. In our previous works [13,14], different kinds of turbulence models, including isotropic models and anisotropic model (TLVA), were employed to simulate film cooling under rotating frames. The computational results were compared with the corresponding experimental data. Our previous studies showed that the anisotropic turbulence model yielded a much better agreement with experimental data than the isotropic models. In present paper, a new anisotropic \(k–\omega\) turbulence model was proposed and embedded in our three-dimensional (3-D) Navier–Stokes code to compute rotational film cooling. In addition, this paper also focused on the theoretical and numerical analysis on the influence of the Coriolis and buoyancy forces induced by the rotation.
2. Computational details

2.1. Governing equation and solution method

The conservation form of the averaged transport equations governing unsteady-state, three-dimensional turbulent flows under rotating frames can be written as

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho \mathbf{u}_j) = 0
\]

(1)

\[
\frac{\partial (\rho \mathbf{u}_j)}{\partial t} + \frac{\partial}{\partial x_j} (\rho \mathbf{u}_j \mathbf{u}_j) = -\frac{\partial \rho}{\partial x_j} \mathbf{u}_j \mathbf{u}_j + \frac{\partial}{\partial x_j} \left[ \left( \frac{\mathbf{u}_i \mathbf{u}_i}{\rho} \right) - \rho \mathbf{u}_j \mathbf{u}_j \right] - 2 \rho \mathbf{w}_{ij} \mathbf{w}_{ij} \Omega_k \Omega_l X_m
\]

(2)

\[
\frac{\partial (\rho h)}{\partial t} + \frac{\partial}{\partial x_j} (\rho \mathbf{u}_j h) = \frac{\partial}{\partial x_j} \left( \frac{\mu}{\rho} \frac{\partial h}{\partial x_j} - \rho C_p \mathbf{u}_j T \right)
\]

(3)

where the \( \mathbf{u}_j \mathbf{u}_j \) is the Reynolds stress tensor and the \( \mathbf{u}_j T \) is the turbulence heat flux.

A three-dimensional Navier–Stokes code named Hitrans was coded for the solution of the above governing equations using the arbitrary Lagrangian–Eulerian method. The forward difference, which advances the flow solution in time from initial guess to the steady state, is employed for the temporal discretization. The spatial difference is formed on a finite-volume mesh that subdivides the computational region into a number of arbitrary hexahedrons. A staggered grid arrangement is adopted, for the velocity vector is stored at cell centers, while other scalar variables like density and temperature, are stored at cell centers. In each time step, the discrete equations are solved by the conjugate residual method. A quasi-second-order upwind scheme is used for convection calculation. Iterations were completed when the differences between the predicted and corrected fluid parameters were converged to 1 \( \times 10^{-4} \).

2.2. Turbulence model

Previous film-cooling and jet-in-cross-flow calculations have revealed consistently that the isotropic turbulence models could not deliver satisfactory predictions as they failed in the capture of the jet lift-off and reattachment and also they predicted jets too narrow compared to the experimental observations. Researches revealed that in the vicinity of film injections the lateral turbulent fluctuations \( \overline{w^2} \) are generally larger than the normal fluctuations \( \overline{v^2} \). Thus the isotropic turbulence models will weaken the lateral spreading and will accordingly weaken the lateral heat transfer process.

To overcome the shortcoming mentioned above, Azzi and Lakehal proposed a two-layer anisotropic \( k-\omega \) model (named TLVA model), which was used in our previous work [14]. In this model, a rate of turbulence anisotropy \( \gamma \) was employed, which is the key factor for solving the problem of under-predicting lateral spreading.

\[
\gamma = \max \left[ \frac{10^3 \left( \gamma^+ \right)^{0.42}}{2.682 \left( \gamma^+ \right)^2 - 5.463}, 4.25 \right]
\]

(4)

In this paper, we introduced the rate \( \gamma \) into the standard \( k-\omega \) model and proposed a new anisotropic \( k-\omega \) model, hoping that the numerical simulation results would be more accurate. Assuming that the \( x \)-axis is perpendicular to the wall, the \( y \)-axis is along the lateral direction, and the \( z \)-axis is the along the direction of mainstream. In this model, we still adopt the Boussinesq approximation to calculate Reynolds stresses. But the eddy viscosity is no longer a scalar, but a tensor, as shown in Eqs. (5) and (6).

\[
-\rho u_i u_j = \mu_{ij} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij}
\]

where

\[
\mu_{ij} = \begin{bmatrix}
\mu_t & \gamma \mu_t & \mu_t \\
\gamma \mu_t & \mu_t & \mu_t \\
\mu_t & \mu_t & \mu_t \\
\end{bmatrix}
\]

(5)

\[
\mu_t = \frac{k}{\omega}
\]

(6)

\[
-\overline{wT} = \mu_t \frac{\partial T}{\partial x}
\]

(7)

\[
-\overline{\nu T} = \mu_t \frac{\partial T}{\partial y}
\]

(8)

\[
-\overline{\nu T} = \mu_t \frac{\partial T}{\partial z}
\]

(9)

\[
-\overline{wT} = \mu_t \frac{\partial T}{\partial z}
\]

(10)

2.3. Computational model and mesh

For the validation of this new anisotropic \( k-\omega \) model, a three-dimensional Navier–Stokes code with the new model was used to simulate the experiment of Yang [15]. For comparison, the numerical study chose the same geometry of the experimental work as the computational domain, as shown in Figure 1. The \( x \)-axis is perpendicular to the wall, the \( y \)-axis is along the lateral direction (also along the centrifugal direction), and the \( z \)-axis is the along the direction of mainstream (also the direction of rotation axis). The cross section of mainstream was 70 mm \( \times \) 70 mm. The rotational radius of the film hole was 450 mm. There are four film holes in the model, and the diameter of film hole \( D \) is 4 mm. The film hole is 30° inclined. The lateral distance between adjacent holes is 2D.
Prior to the actual numerical simulation, a grid independence study for the condition of \( M=0.5, \ R_t=0 \) was performed by using the different grids with 105,532, 159,340, 181,760, 226,600 and 249,020 cells. As shown in Figure 2, a compromise between computation accuracy and computing capability led to the use of 181,760 cells (as shown in Figure 3). It is to be noted that the first grid point adjacent to all the bounding wall surfaces was spaced within \( y^+ \) less than 2 in order to meet the requirements for the \( k-\omega \) turbulence model.

2.4. Boundary condition

For mainstream, the Reynolds number based on the mainstream velocity and hydraulic diameter of the mainstream channel was 4682.8. The temperature of mainstream was 319.15 K, while the temperature of coolant was 307.15 K. The flow rates \( M \) are 0.3 and 0.5 respectively. 5% turbulence intensity was assumed for both mainstream and coolant. No slip and adiabatic boundary condition were applied on the wall boundary. At the outlet, the static pressure was 1 atm (101,325 Pa). Four different rotating speeds were 0, 300, 600, 900 rpm (revolutions per minute) and corresponding \( R_t \) number are 0, 0.00615, 0.0123 and 0.0183 respectively. For \( \Omega > 0 \), the wall is pressure surface, while for \( \Omega < 0 \), the wall is suction surface.

In the study, the lateral average adiabatic film cooling effectiveness (\( \eta \)) was selected as the key parameter in representing the film cooling performance, it was defined as

\[
\eta = \frac{\sum_{i=1}^{n} \eta_i A_i}{\sum_{i=1}^{n} A_i}
\]

(11)

\( \eta \) is the average adiabatic film cooling effectiveness within \( \pm 3.5D \) in lateral direction. In Eq. (11), \( n \) presents the number of grids located in \( \pm 3.5D \) at a certain position in flow direction.
3. Results and discussion

3.1. The anisotropic $k$–$\omega$ model

In order to evaluate the new anisotropic $k$–$\omega$ turbulence model proposed in this paper, different turbulence models were employed to simulate certain experimental conditions and their numerical results were compared with corresponding data consequently. Figure 4 shows that the lateral average adiabatic cooling effectiveness $\bar{\eta}$ predicted by different models at the condition of $M=0.3$, $Rt=0.0123$ and the comparison with experimental data [15]. As can be obviously seen from Figure 4, the anisotropic $k$–$\omega$ model yields a much better performance than the standard $k$–$\omega$ and $k$–$\varepsilon$ models due to introducing the anisotropic factor $\gamma$. In the range of 0–6$D$, the numerical results by the standard $k$–$\omega$ and $k$–$\varepsilon$ models both under-predicted the average cooling effectiveness $\bar{\eta}$ compared to the experimental data. Although the numerical results provided by the anisotropic $k$–$\omega$ model still are lower than the experimental data either, they are much closer to the data compared to the results of other two models. In the range of 6$D$–12$D$, the results of the anisotropic $k$–$\omega$ model and the standard $k$–$\omega$ model are very close to the experimental data, which those of the standard $k$–$\varepsilon$ model are still lower than the experimental data.

In authors’ opinion, the primary reason for the anisotropic $k$–$\omega$ model performing much more accurately is the introduction of the turbulence anisotropic rate $\gamma$, which enhanced the lateral spreading of cooling air. Figure 5(a)–(f) are the lateral average cooling effectiveness $\bar{\eta}$ predicted by the anisotropic $k$–$\omega$ model at $M=0.3$ under different rotating speeds. On the whole, the numerical results yield a better agreement with the experimental data, except in the range of 0–6$D$. In the vicinity downstream the film hole, the Coriolis force played a major influence factor on the cooling air. That may be the reason for discrepancy happened in the range of 0–6$D$.

The effect of the Coriolis force can be more clearly reflected in Figures 6 and 7. In Figures 6 and 7, the secondary vortices were plotted at stream wise distance $Z/D=1$ for different rotating speeds at $M=0.3$. For the

![Figure 4](image-url) Figure 4  Lateral average cooling effectiveness $\bar{\eta}$ predicted by different models. (a) Pressure surface and (b) suction surface.
stationary case ($R_t=0$), the Coriolis force does not exist and the secondary vortices are symmetrical. However, with the increased rotating speeds, the secondary vortices become asymmetrical due to the increased effect of the Coriolis force. For the pressure surface, the cooling air perpendicular to the wall was deflected centripetally by the Coriolis force. On the contrary, the cooling air perpendicular to the wall was deflected centrifugally by the Coriolis force for the suction surface.

3.2. The effect of the centrifugal force

There may be a major difference between the fluid dynamics of film cooling in a rotating environment and film cooling under stationary condition. The additional forces induced by rotation, Coriolis and centrifugal forces, will influence the flow field and make the interaction between main flow and cooling air more complicated. Hence, the mechanism of film cooling under rotating frame by an anisotropic turbulence model.

Figure 5 Lateral average cooling effectiveness $\bar{\eta}$ predicted by the anisotropic $k-\omega$ model at $M=0.3$ under different rotating speeds. (a) $R_t=0.00615$, pressure surface, (b) $R_t=0.00615$, suction surface, (c) $R_t=0.0123$, pressure surface, (d) $R_t=0.0123$, suction surface, (e) $R_t=0.0183$, pressure surface and (f) $R_t=0.0183$, suction surface.
cooling under rotating fame needs to be further investigated. However, in experiments, the two kinds of additional force could not be studied separately. While numerical simulation could realize that by adding the Coriolis force and centrifugal force in momentum equation.

According to Boussinesq approximation, the steady vector form of momentum equation including centrifugal force can be written as

$$\rho_0 (V \cdot \nabla) V = -\nabla P_{eff} + (\mu + \mu_t) \nabla^2 V + \rho_0 \beta \Delta T \Omega \times \Omega \times r$$  

(12)
\[ P_{\text{eff}} = P - \rho_0 \frac{Q^2 r^2}{2} \quad \beta = 1/T \]

In Eq. (12), the centrifugal force was divided into two terms: constant density term \((\rho_0 \mathbf{Q} \times \mathbf{Q} \times r)\) and centrifugal buoyancy force term \((\rho_0 \beta \Delta T \mathbf{Q} \times \mathbf{Q} \times r)\). The constant density term is conservative force and can be merged into the pressure gradient term. The constant density term can be balanced by the pressure gradient and would not influence the velocity and temperature distributions. That means only the centrifugal buoyancy force would change the velocity and temperature distributions.

Figure 8 is the contour of film cooling effectiveness at \(M=0.5, \quad Rt=0.0366\), only adding the centrifugal force into the momentum equation. The temperature of mainstream is 319.15 K, and the temperature of coolant is 307.15 K. The rotating speed is 1800 rpm. As can be clearly seen from Figure 8, the cooling air hardly deflects, which means the centrifugal buoyancy was very small. This can be further verified by Figure 9, in which the secondary vortices are symmetrical, similar to that in stationary case.

In order to further investigate the effect of centrifugal buoyancy force, rotating speed and the difference in temperature between mainstream and cooling air are both increased. Figure 10 is the contour of film cooling effectiveness at \(M=0.5, \quad Rt=0.0615\), only adding the centrifugal force into the momentum equation. In this case, the temperature of mainstream is 357.15 K, and the temperature of coolant is 307.15 K. The rotating speed is 3000 rpm. As can be seen from Figure 10, for the density of cooling air is larger than that of mainstream, the former is deflected centrifugally due to the effect of the centrifugal buoyancy force. Figure 11 is the formation of secondary vortices at \(Z/D=1\). Different from Figure 9, the symmetrical formation of secondary vortices is seriously destroyed by the buoyancy force. The cooling air next to the wall, for its density is larger, is deflected in higher-radius direction. The mainstream away from the wall, for its density is smaller compared to the cooling air, is deflected in lower-radius direction.

**Figure 9** Formation of secondary vortices at stream wise \(Z/D=1\) at \(M=0.5, \quad Rt=0.0366\) (only add centrifugal force).

**Figure 10** Contour of film cooling effectiveness at \(M=0.5, \quad Rt=0.0615\) (only add centrifugal force, 50 K difference in temperature).

**Figure 11** Formation of secondary vortices at stream wise \(Z/D=1\) at \(M=0.5, \quad Rt=0.0615\) (only add centrifugal force, 50 K difference in temperature).
Those are all caused by the effect of the centrifugal buoyancy force.

3.3. The effect of the Coriolis force

The steady vector form of momentum equation only including the Coriolis force can be written as

$$\rho(V \cdot \nabla)V = -\nabla P + (\mu + \mu_t)V^2 - 2\rho\Omega \times V$$  \hspace{0.5cm} (13)

The Coriolis force is not conservative force, hence it could not be included in the pressure gradient term. Compared to the centrifugal force, the effect of the Coriolis force would be more obvious. Figure 12 is the contour of film cooling effectiveness on the suction surface and the pressure surface at $M=0.5$, $Rt=0.0366$, only adding the Coriolis force into the momentum equation. The temperature of mainstream is $319.15$ K, and the temperature of coolant is $307.15$ K. The rotating speed is 1800 rpm. On the pressure surface, according to the right-hand principle, the cooling air is deflected centrifugally due to the Coriolis force. On the contrary, the cooling air is deflected centripetally on the suction surface. According to Eq. (13), the value of Coriolis force on the pressure surface is equal to that on the suction surface, but in a reversal direction. Figure 13 shows the formations of secondary vortices at $Z/D=1$ on the pressure surface and the suction surface. As can be clearly seen from Figure 13, the symmetrical formation of secondary vortices is seriously destroyed by the increased Coriolis force.

![Figure 12](image1.png)  \hspace{1cm} ![Figure 13](image2.png)

**Figure 12** Contour of film cooling effectiveness at $M=0.5$, $Rt=0.0366$ (only add Coriolis force). (a) Pressure surface and (b) suction surface.

**Figure 13** Formation of secondary vortices at $Z/D=1$ at $M=0.5$, $Rt=0.0366$ (only add Coriolis force). (a) Pressure surface and (b) suction surface.
4. Conclusions

This paper proposed a new anisotropic $k-\omega$ turbulence model. For the validation of this new model, we used a three-dimensional Navier–Stokes code with this new model and other isotropic models (the standard $k-\omega$ model and the standard $k-\varepsilon$ model) to simulate our rotational film cooling experiments. The calculation results of three models had been compared with the experimental data. In addition, the theoretical and numerical analysis on the influence of the Coriolis and buoyancy forces induced by the rotation was discussed in detail. Major findings of this study are as follows:

(1) The new anisotropic $k-\omega$ model preformed much better compared to its isotropic counterparts and produced the closest lateral average cooling effectiveness $\eta$ to the experimental results.

(2) In the region of $6D-12D$ downstream the film hole, the numerical results obtained by the new model were much closer to the experimental data than that in the region of $0-6D$. The reason for those phenomena is that the Coriolis force and centrifugal force played main function in the vicinity downstream the film hole.

(3) The centrifugal force could be divided into two terms: constant density term and centrifugal buoyancy force term. The constant density term can be balanced by the pressure gradient and would not influence the velocity and temperature distributions. But the centrifugal buoyancy force and the Coriolis force would change the trajectory of cooling air and temperature distributions.

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