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The processes $e^+e^- \to J/\Psi\chi_{c0}, \Psi(2S)\chi_{c0}$ at $\sqrt{s} = 10.6$ GeV in the framework of light cone formalism

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Abstract

In this Letter we have analyzed the production of pair charmonium mesons in the reactions $e^+e^- \to J/\Psi\chi_{c0}$, $e^+e^- \to \Psi(2S)\chi_{c0}$ at energy $\sqrt{s} = 10.6$ GeV in the framework of the light cone formalism. In comparison with NRQCD the numerical results for the cross sections are in better agreement with experiment.

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1. Introduction

Exclusive double charmonium meson production in e^+e^- annihilation at energy $\sqrt{s} = 10.6$ GeV remains very interesting problem for theoretical investigations. NRQCD that is often used to predict the cross section of such processes [1] fails to archive agreement with Belle and BaBar experiments. The predictions for the cross sections obtained in the framework of NRQCD [2] are by an order of magnitude less than the experimental results [3,4].

An interesting approach to the problem was proposed in paper [5], where the process $e^+e^- \to \psi\eta_c$ at energy $\sqrt{s} = 10.6$ GeV was considered in the framework of light cone formalism. Using physical model for the light cone wave functions of ψ, η_c mesons the authors received the prediction for the cross section of this process that agrees with experimental result obtained at Belle. In addition to the agreement between the theoretical prediction and the experimental results light cone approach allows one to understand that charmonium mesons wave functions are two wide to be considered in the framework of NRQCD in such processes.

Further progress in the understanding of exclusive double charmonium mesons production is connected with paper [6]. In this paper it was shown that contrary to NRQCD the study of the processes $e^+e^- \to \Psi(1S)\eta_c(2S), \Psi(2S)\eta_c(1S), \Psi(2S)\eta_c(2S)$ in the framework of light cone formalism leads to a good agreement with Belle and BaBar results [3,4].

So light cone formalism better than NRQCD predicts the cross sections $e^+e^- \to J/\Psi\eta_c(1S), J/\Psi\eta_c(2S), \Psi(2S)\eta_c(1S), \Psi(2S)\eta_c(2S)$. In addition to these reactions Belle and BaBar have measured the cross section of the processes $e^+e^- \to J/\Psi\chi_{c0}, \Psi(2S)\chi_{c0}$. The agreement of NRQCD prediction and experimental results is also poor for this reactions. The aim of this Letter is the application of light cone formalism to these reactions.

This Letter is organized as follows. In Section 2 light cone wave function of J/Ψ, $\Psi(2S)$ and χ_{c0} mesons will be considered. In Section 3 the expression for the amplitudes of the processes under consideration will be derived. Numerical analysis of the cross sections $e^+e^- \to J/\Psi\chi_{c0}, \Psi(2S)\chi_{c0}$ will be presented in Section 4. Finally the results of the calculation will be discussed in Section 5.

2. Light cone wave functions of J/Ψ, $\Psi(2S)$ and χ_{c0} mesons

To calculate the amplitude of the process $e^+e^- \to V\chi_{c0}$ [$V = J/\Psi, \Psi(2S)$] one needs to know light cone wave func-

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tions of the final charmonium mesons. The functions for J/Ψ and $\Psi(2S)$ mesons are defined as follows [5]

$$\langle V_\lambda(p)|\bar{Q}_\beta(z)Q_\alpha(-z)|0\rangle_\mu = \frac{f_V M_V}{4}\int_0^1 dx_1\, e^{i(pz)(x_1-x_2)}\left\{\hat{p}\frac{(e_\lambda z)}{(pz)}V_L(x) + \left(\hat{e}_\lambda - \hat{p}\frac{(e_\lambda z)}{(pz)}\right)V_\perp(x) + \frac{f_v^t(\mu)}{M_V}\left(\sigma_{\mu\nu}e_\lambda^\mu p^\nu\right)V_T(x) + f_v^a(\mu)\left(\epsilon_{\mu\nu\alpha\beta}\gamma_\mu\gamma_5 e_\lambda^\nu p^\alpha z^\beta\right)V_A(x)\right\}_{\alpha\beta}. \tag{1}$$

The dependence of the light cone wave functions on the scale μ is very slow and it will not be considered in the full form. Only renormalization factors of the corresponding local currents will be regarded. The constants f_v^t, f_v^a can be determined from QCD equations of motion

$$f_v^t(\mu) = \frac{2\bar{M}_Q}{M_\Psi}Z_t,\qquad f_v^a(\mu) = \frac{1}{2}\left(1 - Z_t Z_m\frac{4\bar{M}_Q^2}{M_\Psi^2}\right), \tag{2}$$

where $\bar{M}_Q = M_Q^{\overline{\rm MS}}$ $(\mu = M_Q^{\overline{\rm MS}})$. The factors Z_p, Z_t, Z_m can be written in the form

$$Z_p = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\bar{M}_Q^2)}\right]^{\frac{-3C_F}{b_0}},\qquad Z_t = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\bar{M}_Q^2)}\right]^{\frac{C_F}{b_0}},\qquad Z_m = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\bar{M}_Q^2)}\right]^{\frac{3C_F}{b_0}}, \tag{3}$$

where $C_F = 4/3$, $b_0 = 25/3$.

For the light cone wave functions of J/Ψ meson the model proposed in [5] will be used

$$\phi_i\left(x, v^2\right) = c_i\left(v^2\right)\phi_i^a(x)\left\{\frac{x_1x_2}{[1-4x_1x_2(1-v^2)]}\right\}^{1-v^2}. \tag{4}$$

For the light cone wave functions of $\Psi(2S)$ meson the model proposed in [6] will be used

$$\phi_i\left(x, v^2\right) = c_i\left(v^2\right)\phi_i^a(x)\left(1 - 8v^2\beta\frac{(1-v^2)x_1x_2}{[1-4x_1x_2(1-v^2)]}\right) \times\left\{\frac{x_1x_2}{[1-4x_1x_2(1-v^2)]}\right\}^{1-v^2}. \tag{5}$$

Here v is a characteristic speed of quark–antiquark pair in meson, c_i is the coefficient fixed by the wave function normalization condition $\int dx\,\phi_i(x, v^2) = 1$. The constant β is fixed by the condition that zero of the wave function of $2S$ state meson [6] must coincide with zero obtained from the solution of Schrödinger equation with Buchmüller–Tye potential [7]. ϕ_i^a are the asymptotic expressions of the wave functions

$$V_A(x) = V_L(x) = V_T(x) = \phi_{\rm asy}(x) = 6x_1x_2, \tag{6}$$

$$V_\perp(x) = \frac{3}{4}\left[1 + (x_1 - x_2)^2\right]. \tag{7}$$

It should be noted here that the expressions for light cone wave functions (4), (5) link different limits: quark–antiquark pair in meson being at rest $v \to 0$ and very light quark $v \to 1$. In the former limit one obviously gets $\sim \delta(x - 1/2)$, the later one leads to the asymptotic function $\sim \phi^a$.

Let us consider the light cone wave functions of χ_{c0} meson:

$$\langle\chi_{c0}(p)|\bar{Q}(z)Q(-z)|0\rangle = f_S\int dy\, e^{ipz(y_1-y_2)}S_S(y),$$
$$\langle\chi_{c0}(p)|\bar{Q}(z)\gamma_\mu Q(-z)|0\rangle = f_V^{(1)}p_\mu\int dy\, e^{ipz(y_1-y_2)}S_V(y) + f_V^{(2)}z_\mu\int dy\, e^{ipz(y_1-y_2)}S_V^{(2)}(y),$$
$$\langle\chi_{c0}(p)|\bar{Q}(z)\sigma_{\mu\nu}Q(-z)|0\rangle = f_T(p_\mu z_\nu - p_\nu z_\mu)\int dy\, e^{ipz(y_1-y_2)}S_T(y). \tag{8}$$

The functions $\phi_i = S_S(y), S_V^{(2)}(y), S_T(y)$ are normalized as follow $\int dy\,\phi_i = 1$, the normalization condition for $S_V(y)$ is $\int dy\,(y_1 - y_2)S_V(y) = 1$. Using QCD equation of motion the constants $f_S, f_T, f_V^{(2)}$ can be related to $f_V^{(1)}$:

$$f_S = -3\frac{f_V^{(1)}M_\chi^2}{2\bar{M}_Q},\qquad f_V^{(2)} = -iM_\chi^2 f_V^{(1)},\qquad f_T = i\frac{f_V^{(1)}M_\chi^2}{2\bar{M}_Q}. \tag{9}$$

The constant f_S and consequently $f_V^{(1)}$ can be expressed through matrix element $\langle\chi_{c0}(p)|\bar{Q}(0)Q(0)|0\rangle$ found in [8]. As the result we get

$$f_V^{(1)} = \sqrt{\frac{3R'(0)^2}{2\pi m_Q^3}} = \sqrt{\frac{\langle O_1\rangle_P}{3m_Q^3}}, \tag{10}$$

where $R'(0)$ is derivative of the wave function of heavy quarkonium at the origin, m_Q is the mass of c quark in potential model, $\langle O_1\rangle_P$ is well known from NRQCD matrix element [9]. It should be noted that formulae (9) were found in paper [10]. The expression (10) for the constant $f_V^{(1)}$ differs from paper [10] and agrees with paper [11].

Although there are four light cone χ_{c0} meson wave functions only $S_S(y)$ and $S_V(y)$ contribute to the amplitude of the processes at accuracy considered in our Letter, the others give power correction to the result. Below we will consider only $S_S(y)$ and $S_V(y)$.

As in the case of vector mesons we are not going to regard full μ-dependence of light cone wave functions. Only overall renormalization factor of corresponding operator will be considered. The renormalization factor for light cone wave function $S_S(y)$ equals to $Z_p(\mu)$. To find renormalization factor for the function $S_V(y)$ it is worth noting that the corresponding operator $\langle\chi_{c0}(p)|\bar{Q}(0)\gamma_\mu Q(0)|0\rangle$ equals zero. So the renormalization factor for the function $S_V(y)$ equals the renormalization factor of the first nonvanishing multiplicatively renormalized local operator $\langle \chi_{c0}(p)|\bar{Q}\gamma_\mu C_1^{3/2}(zD/z\partial)Q|0\rangle$. It equals to [10]

$$Z_v = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\bar{M}_Q^2)}\right]^{\frac{8C_F}{9b_0}}. \tag{11}$$

In order to get the expressions for the light cone wave functions $S_V(y)$ and $S_S(y)$ we will use the same procedure that was used in paper [5] to get expressions (4).

First let us consider the leading twist wave function of χ_{c0} meson $S_V(y)$. There are three wave functions of $2P$ meson with different orbital momentum projection L_z in the direction of meson motion($m = \pm 1, 0$). But only $m = 0$ projection gives contribution to the leading twist light cone wave function [10]. This wave function can be approximated by Coulomb wave function of $2P$ state

$$\Psi_c \sim \cos\theta \frac{k}{(k^2 + m_c^2 v^2)^3}, \tag{12}$$

where v is the characteristic relative velocity. Substituting [12]

$$\vec{k}_\perp \to \vec{k}_\perp, \qquad k_z \to (y_1 - y_2)\frac{M_0}{2}, \quad M_0^2 = \frac{m_c^2 + \vec{k}_\perp^2}{y_1 y_2} \tag{13}$$

and than carrying out the integration

$$\phi \sim \int d^2k_\perp \, \Psi_c(y, \vec{k}_\perp) \tag{14}$$

one gets the expression for the light cone wave function

$$\phi \sim S_V^{\mathrm{as}} \left\{ y_1 y_2 \frac{1 - 2y_1 y_2(1 - v^2)}{(1 - 4y_1 y_2(1 - v^2))^2} \right\}. \tag{15}$$

In last formula the expression $S_V^{\mathrm{as}} = y_1 y_2 (y_1 - y_2)$ is asymptotic form of the light cone wave function $S_V(y)$. To get final expression for the wave function we will modify (15) similar to wave functions of $1S$ and $2S$ mesons (4), (5)

$$S_V(y) = c_V(v^2) S_V^{\mathrm{as}}(y) \left\{ y_1 y_2 \frac{1 - 2y_1 y_2(1 - v^2)}{(1 - 4y_1 y_2(1 - v^2))^2} \right\}^{1 - v^2}, \tag{16}$$

where $c_V(v^2)$ is the coefficient which is fixed by the wave function normalization $\int dy\, S_V(y)(y_1 - y_2) = 1$. For the $S_S(y)$ wave function the following expression will be taken

$$S_S(y) = c_S(v^2) S_S^{\mathrm{as}}(y) \left\{ y_1 y_2 \frac{1 - 2y_1 y_2(1 - v^2)}{(1 - 4y_1 y_2(1 - v^2))^2} \right\}^{1 - v^2}, \tag{17}$$

where asymptotic form of the wave function $S_S^{\mathrm{as}}(y) = 1$, $c_S(v^2)$ is the coefficient which is fixed by the wave function normalization $\int dy\, S_S(y) = 1$.

3. The calculation of the amplitude of the processes $e^+e^- \to V(p_1, \epsilon_1)\chi_{c0}(p_2)$

Leading asymptotic behavior of the matrix element $\langle V(p_1, \epsilon_1), \chi_{c0}(p_2)|J_\mu^{\mathrm{el}}|0\rangle$ can be derived from formula [13]

$$\langle M(p_1, \lambda_1) M(p_2, \lambda_2)\big| J_\mu^{\mathrm{el}}|0\rangle \sim \left(\frac{1}{\sqrt{s}}\right)^{|\lambda_1 + \lambda_2| + 1}, \tag{18}$$

where J_μ^{el} is the electromagnetic current. For the processes under consideration we have $M(p_1, \lambda_1) = V(p_1, \lambda_1)$, $M(p_2, \lambda_2) = \chi_{c0}(p_2)$. Obviously the helicity λ_2 equals zero. As to the vector meson $V(p_1, \lambda_1)$ the leading contribution is given by the helicity $\lambda_1 = 0$. So the asymptotic behavior of the amplitude is

$$\langle V(p_1, \lambda_1 = 0), S(p_2)\big| J_\mu^{\mathrm{el}}|0\rangle \sim \frac{1}{\sqrt{s}} \tag{19}$$

and the asymptotic behavior of the cross section $\sigma(e^+e^- \to VS)$ is $\sim 1/s^3$. Unfortunately one can show that it is not possible to disregard NLO contribution in $1/s$ expansion. To see this let us consider NRQCD result for the cross section of the process $e^+e^- \to J/\psi(1S)\chi_{c0}$ obtained in paper [2]. Cross section of this process can be written as

$$\sigma = \frac{\pi^3}{3^5 s}\alpha^2 \alpha_s^2 q_c^2 F_0 r^2 \sqrt{1 - r^2}\frac{f_V^2 f_S^2}{m_c^4}, \tag{20}$$

where $r^2 = 16m_c^2/s$ and $F_0 = 2(18r^2 - 7r^4)^2 + r^2(4 + 10r^2 - 3r^4)^2$. Let us substitute $s \to 10.6^2\xi$ and expand the above formula in $1/\xi$ series (numerical inputs are the same as in [2]). We get

$$\sigma = \frac{0.23}{\xi^3} + \frac{2.91}{\xi^4} - \frac{0.89}{\xi^5} + O(1/\xi^6). \tag{21}$$

Thus one sees that in the framework of NRQCD NLO correction at energy $\sqrt{s} = 10.6$ GeV is by an order of magnitude larger than the leading one. So one can suppose that NLO term in $1/s$ expansion gives considerable contribution and must be regarded in our analysis. The same is true for the process $e^+e^- \to \psi(2S)\chi_{c0}$.

Two diagrams that contribute to the processes $e^+e^- \to V(p_1, \lambda_1)\chi_{c0}(p_2)$ are shown in Fig. 1 and the other two can be obtained from them by charge conjugation. Having the expressions for the light cone wave functions (4), (5), (16), (17) it is not difficult to obtain the matrix element $\langle V(p_1, \lambda_1), \chi_{c0}(p_2)|J_\mu|0\rangle$:

$$\begin{aligned}&\langle V(p_1, \lambda_1)\chi_{c0}(p_2)\big| J^\mu|0\rangle \\ &\quad = g_1\big(p_1^\mu - p_2^\mu\big)(e_{\lambda_1}p_2) + g_2\big((e_{\lambda_1}p_2)p_1^\mu - e_{\lambda_1}^\mu(p_1p_2)\big),\end{aligned} \tag{22}$$

where the formfactors g_1 and g_2 are

$$\begin{aligned}g_1 = {}& 2\frac{\pi}{9}\frac{f_V^{(1)} f_V}{s^2} M_V M_\chi \int_0^1 dx_1 \int_0^1 dy_1\, \alpha_s(k) \\ &\times \left(-16 Z_v(k)\frac{S_V(y)V_L(x)}{M_\chi}\frac{1}{d(x, y)}\right),\end{aligned} \tag{23}$$

$$\begin{aligned}g_2 = {}& 2\frac{\pi}{9}\frac{f_V^{(1)} f_V}{s^2} M_V M_\chi \int_0^1 dx_1 \int_0^1 dy_1\, \alpha_s(k) \\ &\times \left(2 Z_v(k)\frac{S_V(y)V_A(x)}{M_\chi}\left(1 - Z_m(k)Z_t(k)\frac{4\bar{M}_Q^2}{M_V^2}\right)\right.\end{aligned}$$

$$\times \frac{1+y_1}{d(x,y)^2} + 16 Z_t(k) Z_v(k) Z_m(k) \frac{S_V(y) V_T(x)}{s(x) d(x,y)} \frac{\bar{M}_Q^2}{M_V^2 M_\chi} - 8 Z_v(k) \frac{S_V(y)}{M_\chi d(x,y)} \left(V_L(x) - 2V_\perp(x) - \frac{V_\perp(x)}{s(y)} \right) + 32 Z_p(k) Z_t(k) \frac{S_S(y) V_T(x)}{s(x) d(x,y)} \frac{\bar{M}_Q}{M_V^2} f \Bigg), \tag{24}$$

where $d(x,y), s(x), s(y)$ are dimensionless quark and gluon propagators defined as follows:

$$d(x,y) = \frac{k^2}{q_0^2} = \left(x_1 + \frac{\delta}{y_1}\right)\left(y_1 + \frac{\delta}{x_1}\right), \quad \delta = \frac{(Z_m(k)\bar{M}_Q)^2}{s}, \tag{25}$$

$$s(x) = \left(x_1 + \frac{(Z_m(\sigma)\bar{M}_Q)^2}{y_1 y_2 s}\right), \quad s(y) = \left(y_1 + \frac{(Z_m(\sigma)\bar{M}_Q)^2}{x_1 x_2 s}\right), \tag{26}$$

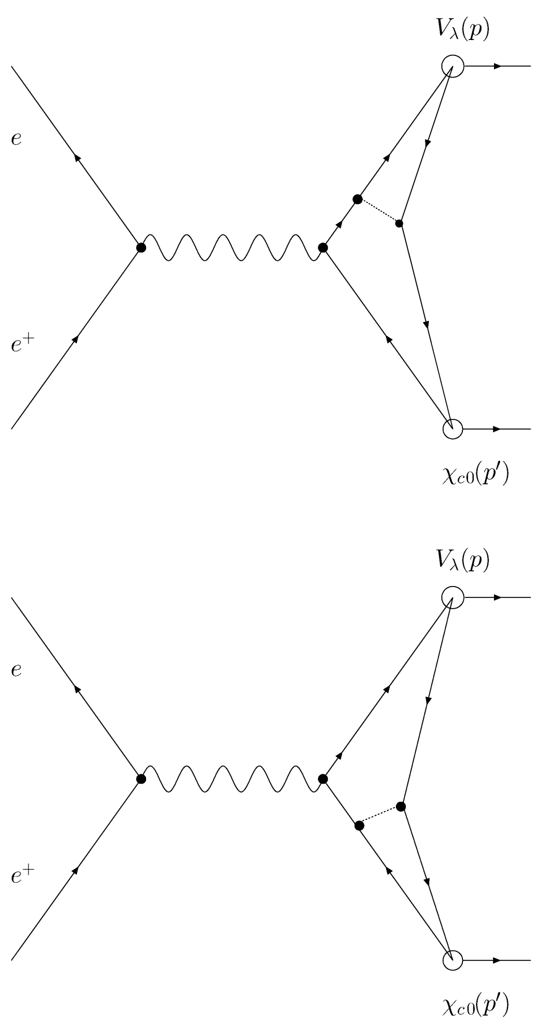

Fig. 1. The diagrams that contribute to the processes $e^+e^- \to J/\Psi \chi_{c0}$, $\Psi(2S)\chi_{c0}$.

where k is a momentum of virtual gluon, σ is a characteristic momentum of virtual quark in the diagrams shown in Fig. 1, the constant $f = -3M_{\chi_{c0}}/2\bar{M}_Q$.

It is interesting to note that there are two factors in the expressions (23), (24) for the formfactors g_1 and g_2. The first is NRQCD result for the formfactors that does not regard internal motion of quark–antiquark pair inside mesons and it is proportional to $\sim f_V^{(1)} f_V \sim R(0)R'(0)$. The second factor regards internal motion and it is proportional to the integrals $\int dx_1\, dy_1$ in expressions (23), (24).

In the limit $v \to 0$ the mesons V and χ_{c0} have equal masses $M = M_V = M_\chi$ and formulae (23), (24) must reproduce NRQCD result for the process $e^+e^- \to J/\Psi \chi_{c0}$:

$$g_1 = \frac{128\pi}{9} \frac{f_V^{(1)} f_V}{s^2} M \left(1 - 4\frac{M^2}{s}\right), \quad g_2 = -\frac{128\pi}{9} \frac{f_V^{(1)} f_V}{s^2} M \left(9 - 14\frac{M^2}{s}\right). \tag{27}$$

Really if one takes the limit $v \to 0$ in expressions (23), (24) one gets

$$g_1 = \frac{128\pi}{9} \frac{f_V^{(1)} f_V}{s^2} M \left(1 + O\left(\frac{M^2}{s}\right)\right), \quad g_2 = -\frac{128\pi}{9} \frac{f_V^{(1)} f_V}{s^2} M \left(9 + O\left(\frac{M^2}{s}\right)\right). \tag{28}$$

So one sees that at accuracy considered in our Letter the expressions (27) and (28) coincide.

The cross section of the processes $e^+e^- \to V\chi_{c0}$ is given by the formula

$$\sigma = \frac{\pi \alpha_{\rm el}^2}{s^3} q_c^2 \left(\frac{|\mathbf{p}_1|}{\sqrt{s}}\right) F, \tag{29}$$

where q_c is the charge of c-quark, $\mathbf{p}_1$ is the momentum of vector meson in final mesons' center mass frame, for the electromagnetic current of the form (22) the function F is given by the formula

$$\begin{aligned} F = {} & \frac{g_1^2 q_0^8}{6M_V^2} + \frac{g_2^2 q_0^6}{3} - \frac{1}{3} g_1 g_2 q_0^6 + \frac{1}{2} g_2^2 M_V^2 q_0^4 - \frac{4}{3} g_1^2 M_\chi^2 q_0^4 \\ & + \frac{1}{3} g_2^2 M_\chi^2 q_0^4 - \frac{2}{3} g_1 g_2 M_\chi^2 q_0^4 + \frac{2}{3} g_2^2 M_V^2 M_\chi^2 q_0^2 \\ & + \frac{4}{3} g_1 g_2 M_V^2 M_\chi^2 q_0^2 + \frac{8}{3} g_1^2 M_V^2 M_\chi^4 + \frac{2}{3} g_2^2 M_V^2 M_\chi^4 \\ & + \frac{8}{3} g_1 g_2 M_V^2 M_\chi^4 \\ = {} & \frac{g_1^2 s^4}{6M_V^2} - \frac{2g_1^2 s^3}{3} + \frac{g_2^2 s^3}{3} - \frac{2g_1^2 M_\chi^2 s^3}{3M_V^2} - \frac{1}{3} g_1 g_2 s^3 \\ & + O(1/s^2), \end{aligned} \tag{30}$$

where $q_0^2 = s - M_V^2 - M_\chi^2$.

4. Numerical calculation

In the numerical analysis the following parameters will be used:

$$\bar{M}_c = 1.2 \text{ GeV},$$
$$\psi(1S), \quad f_V = 0.41 \text{ GeV},$$
$$\psi(2S), \quad f_V = 0.28 \text{ GeV}. \tag{31}$$

The values f_V were obtained from decay width

$$\Gamma\big(V \to e^+e^-\big) = \frac{16\pi\alpha^2}{27}\frac{|f_V|^2}{M_V}. \tag{32}$$

As to the value of the constant $f_V^{(1)}$ it is related to the value of the constant $f_S = \langle\chi_{c0}(p)|\bar{Q}(0)Q(0)|0\rangle$ found in [8] using QCD sum rules

$$f_V^{(1)} = 0.084 \text{ GeV}. \tag{33}$$

For $\alpha_s(\mu)$ one loop result will be used

$$\alpha_s(\mu) = \frac{4\pi}{b_0 \log(\mu^2/\Lambda^2)}, \tag{34}$$

with $\Lambda = 200$ MeV. The other parameters needed for the numerical analysis is the width of wave function v^2. It will be taken from potential model [7]

$$J/\Psi \quad v^2 = 0.23,$$
$$\chi_{c0} \quad v^2 = 0.25,$$
$$\psi(2S) \quad v^2 = 0.29. \tag{35}$$

Now let us consider the formula of the cross section (29). It is seen from (30) that leading order contribution (LO) to the cross section is given by the first term $\sim g_1^2$ and the rest of the formula (30) is the power correction to the LO. In our Letter the power correction to the formfactor g_1 is beyond the accuracy of our calculation. But it is well seen from (30) that in order to get the cross section up to $O(1/s^5)$ one must know $1/s$ correction to the formfactor g_1. Fortunately LO term $\sim g_1^2$ gives negligible contribution to the cross section and varying g_1^2 in a reasonable region the cross section is changed by few percent. Sure we do not pretend to the accuracy about few percent in our calculation.

In our Letter we use the model for the light cone wave function defined by Eqs. (4), (5), (16) and (17) with the widths (35). To estimate the size of this uncertainty results from this model in addition to the widths (35) the calculation of the cross sections with shifted widths (about 10%) is done. The results of the calculation is presented in Table 1. The second and the third columns contain experimental results measured at BaBar and Belle experiments. In the fourth column the results of this Letter are presented. Central values of the cross sections correspond to the light cone wave functions with widths (35). The upper values of the cross section correspond to the widths $0.26(J/\Psi), 0.32(\Psi(2S)), 0.28(\chi_{c0})$. The lower values of the cross section correspond to the widths $0.2(J/\Psi), 0.26(\Psi(2S)), 0.22(\chi_{c0})$. In order to compare the result with NRQCD predictions for the processes under consideration the fifth column contains the predictions in the framework of this model.

From Table 1 one sees that the predictions of the cross section of the processes $e^+e^- \to J/\Psi\chi_{c0}, \Psi(2S)\chi_{c0}$ in the framework of light cone are larger than NRQCD predictions and the agreement with the experimental results is better. As it was noted in [6] the difference of the NRQCD and light cone prediction for the cross sections can be attributed to the fact that at leading approximation NRQCD does not regard the motion inside final mesons. So NRQCD predictions for the cross section of the processes under consideration are unreliable.

In addition to the uncertainties described above and uncertainty due to the unknown size of radiative QCD correction one can suppose that there is very important $1/s$ correction. The size of this correction can be estimated from formula (21) for NRQCD result of the cross section of the process $e^+e^- \to J/\Psi\chi_{c0}$. It is seen from (21) that in the framework of NRQCD $O(1/s^5)$ contribution changes the value of the cross section by 30%. Moreover $O(1/s^5)$ correction diminish the value of the cross section. It was noted above that to get light cone result one must multiply NRQCD by a factor regarding internal mesons' motion. If one suppose further that these factors for $O(1/s^4)$ and $O(1/s^5)$ contribution are of the same order than one can claim that in the framework of light cone formalism $O(1/s^5)$ contribution diminish the size of the cross section by 30%. After including this correction the value of the cross sections can be estimated as $\sigma(e^+e^- \to J/\Psi\chi_{c0}) \sim 9$ fb and $\sigma(e^+e^- \to \Psi(2S)\chi_{c0}) \sim 5$ fb.

It should be noted here that light cone prediction for the cross section of the process $e^+e^- \to \Psi(2S)\chi_{c0}$ is almost twice less than Belle result. One can attribute the difference to any source of uncertainty described above. But another source the disagreement can arise from the higher Fock state of χ_{c0} meson. Really it is known from NRQCD that color octet contribution of χ_{c0} meson is of the same order in relative velocity expansion as color singlet. Moreover it is known that color octet state gives NLO contribution to the amplitude, i.e., it changes main contribution to the cross section. So it would be interesting to estimate the size of this correction.

5. Discussion

In this Letter the calculation of the cross sections of the processes $e^+e^- \to J/\Psi\chi_{c0}$ and $e^+e^- \to \Psi(2S)\chi_{c0}$ at energy $\sqrt{s} = 10.6$ GeV in the framework of light cone formalism has been carried out. It is shown that regarding the internal motion of mesons in the hard part of the amplitude in the framework of light cone results to the considerable enhancement of the cross

Table 1
The second and third column contain experimental result. The results of our calculation is presented in the forth column. The last column contains NRQCD results

$H_1 H_2$	$\sigma_{\text{BaBar}} \times Br_{H_2\to\text{charged}>2}$ (fb) [4]	$\sigma_{\text{Belle}} \times Br_{H_2\to\text{charged}>2}$ (fb) [3]	σ (fb)	σ_{NRQCD} (fb) [2]
$\psi(1S)\chi_{c0}$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$6.4 \pm 1.7 \pm 1.0$	$14.4^{15.5}_{13.3}$	2.3
$\psi(2S)\chi_{c0}$	–	$12.5 \pm 3.8 \pm 3.1$	$7.8^{8.3}_{7.3}$	1.0

Table 2
The second and third column contain experimental result. The results of paper [6] are presented in the forth column. The last column contains NRQCD results

$H_1 H_2$	$\sigma_{\text{BaBar}} \times Br_{H_2 \to \text{charged}>2}$ (fb) [4]	$\sigma_{\text{Belle}} \times Br_{H_2 \to \text{charged}>2}$ (fb) [3]	σ_{LO} (fb) [6]	σ_{NRQCD} (fb) [2]
$\psi(1S)\eta_c(1S)$	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$25.6 \pm 2.8 \pm 3.4$	26.7	2.31
$\psi(2S)\eta_c(1S)$	–	$16.3 \pm 4.6 \pm 3.9$	16.3	0.96
$\psi(1S)\eta_c(2S)$	$16.4 \pm 3.7^{+2.4}_{-3.0}$	$16.5 \pm 3.0 \pm 2.4$	26.6	0.96
$\psi(2S)\eta_c(2S)$	–	$16.0 \pm 5.1 \pm 3.8$	14.5	0.40

sections in comparison to the NRQCD where internal motion is disregarded. So NRQCD is unreliable for the calculation of these cross sections.

In addition to the processes $e^+e^- \to J/\Psi \chi_{c0}, \Psi(2S)\chi_{c0}$ Belle and BaBar experiments have measured the cross sections $e^+e^- \to J/\Psi \eta_c, \Psi(2S)\eta_c, J/\Psi \eta_c(2S), \Psi(2S)\eta_c(2S)$. In the framework of light cone formalism these reactions were considered in paper [6]. The results obtained in this Letter are presented in Table 2. Comparing the results for the cross sections measured in Belle and BaBar with light cone and NRQCD predictions one can claim that despite a number of uncertainties the results obtained in the framework of light cone are in better agreement with Belle and BaBar results than NRQCD predictions.

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