



Cosmological two-stream instability

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ABSTRACT

Two-stream instability requires, essentially, two things to operate: a relative flow between two fluids and some type of interaction between them. In this Letter we provide the first demonstration that this mechanism may be active in a cosmological context. Building on a recently developed formalism for cosmological models with two, interpenetrating fluids with a relative flow between them, we show that two-stream instability may be triggered during the transition from one fluid domination to the other. We also demonstrate that the cosmological expansion eventually shuts down the instability by driving to zero the relative flow and the coupling between the two fluids.

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Two-stream instability of two, interpenetrating plasmas is a well-established phenomenon [11,7]. It is expected also to play a role in two, interpenetrating fluids, such as superfluid helium [4], gravitational-wave instabilities in neutron stars [1], and glitches in pulsars (due to the interior superfluid components) [3,4]. It is not surprising that the instability shows up in diverse settings, since the basic requirements for it to operate are fairly generic: there must be a relative flow and some type of coupling between the two fluids. A window of instability may be opened when a perturbation of the fluids (hereafter, “mode”) appears to be “left-moving”, say, with respect to one fluid, but “right-moving” with respect to the other. The mechanism may be well-known, yet it was considered in a relativistic context only recently. Samuelsson et al. [13] demonstrated quite generally the existence of (local) two-stream instability for a system of two, general relativistic fluids (using only causality – mode-speed less than one, in geometric units – and absolute stability – modes are real for the fluids at rest – to constrain the fluid properties). In this Letter we consider a new application by demonstrating that two-stream instability may be triggered in cosmological settings.

Although we use a specific example in this Letter to exhibit the possibility of the mechanism to operate, we expect it to apply in various situations such as the presence of many inflaton scalar fields: the scalar fields turn into Bose–Einstein condensates behaving, for example, like the superfluids in a glitching neutron star, thus leading to two flows that have no particular reason to be

aligned. Similarly, around the transition between dark matter and dark energy domination, the instability would provide a means of discriminating between a cosmological constant and a dark energy fluid.

Observations over the past few decades have provided a wealth of information that can be used to constrain cosmological models [12]. One of the more stringent is the leading-order observed homogeneity and isotropy of the Universe. Is it possible to have a relative flow in cosmologically important settings that would not have been detected yet? An ideal stage is during a cosmological transition between one phase of domination to another. The requirement of a transition stems from the fact that one should naturally arrive at the current state of the Universe. We have shown, in a companion paper [9], that when one fluid flux dominates over the other (*i.e.* before and after the transition), one recovers the usual Friedmann–Lemaître–Robertson–Walker (FLRW) behavior. However, during the transition the model goes through a Bianchi I phase. We are thus led to ask whether the two-stream instability can be triggered during the same epoch. As we will show, the answer is “yes”. As a proof-of-principle we consider a fairly simplified picture, leaving actual cosmological consequences and constraints on more elaborate models (that deserve further examination) for future work.

Our cosmological two-fluid model has its relative flow along one direction, which we take to be the z -axis, *cf.* [9]. Orthogonal to the flow we impose two, mutually orthogonal spacelike Killing vector fields: one along the x -axis and another along the y -axis. The Killing vector fields \mathcal{X}^μ and \mathcal{Y}^μ are thus $\mathcal{X}^\mu = (0, 1, 0, 0)$ and $\mathcal{Y}^\mu = (0, 0, 1, 0)$. These two symmetries, and the remaining freedom in the choice of coordinates, imply the metric and fluid

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variables are functions of z and time t . The metric can therefore be written

$$ds^2 = -dt^2 + A_x^2 dx^2 + A_y^2 dy^2 + A_z^2 dz^2, \quad (1)$$

where the (still arbitrary) dimensionless functions A_i ($i \in \{x, y, z\}$) can in principle depend on both t and z .

To demonstrate two-stream instability, it is sufficient to consider fixed “moments” of cosmological time (e.g. assume that the time-scale for the oscillations is much less than that of the cosmological expansion). For our mode analysis, this means any time derivatives of the background metric and the corresponding fluid flux will be ignored. We will also ignore the metric perturbations and relax the z -dependence in the background metric and fluid flow so that our modes propagate in a spacetime of the well-known Bianchi I type. General cosmological perturbations in such a spacetime can be found in [14].

As described in [9], we use the multi-fluid formalism originally due to Carter [8] (see [2] for a review). The fluid variables are two conserved four-currents, to be denoted $n^\mu = nu_n^\mu$ (with $g_{\mu\nu}u_n^\mu u_n^\nu = -1$) and $s^\mu = su_s^\mu$ (with also $g_{\mu\nu}u_s^\mu u_s^\nu = -1$). The fluid equations of motion are obtained from a Lagrangian $\Lambda(n, s)$, which we take to be of the form of a fluid with constituent mass m coupled to a fluid with zero constituent mass; i.e.

$$\Lambda(n, s) = -mn^\alpha - \tau_{ns}n^{\sigma_n}s^{\sigma_s} - \kappa_s s^\beta, \quad (2)$$

where m , α , σ_n , σ_s , τ_{ns} , κ_s and β are constants. The “bare” sound speeds [13] are given by

$$c_n^2 \equiv \frac{\partial \ln \mu}{\partial \ln n}, \quad c_s^2 \equiv \frac{\partial \ln T}{\partial \ln s}, \quad (3)$$

where $\mu \equiv -\partial \Lambda / \partial n$ and $T \equiv -\partial \Lambda / \partial s$ are the associated conjugate momenta¹ of the two fluids. The cross-constituent coupling reads

$$C_{ns} \equiv \frac{\partial \ln \mu}{\partial \ln s} = \frac{T s}{\mu n} C_{sn}. \quad (4)$$

Performing perturbations to linear order, the fluid densities and the z -component of the unit four-velocities take the form

$$\begin{aligned} \bar{u}_{n,s}^z(t, z) &= u_{n,s}^z(t) + \delta U_{n,s}^z e^{ik_\mu x^\mu}, \\ \bar{n}(t, z) &= n(t) + \delta \mathcal{N} e^{ik_\mu x^\mu}, \\ \bar{s}(t, z) &= s(t) + \delta \mathcal{S} e^{ik_\mu x^\mu}, \end{aligned} \quad (5)$$

where $k_\mu = (k_t, 0, 0, k_z)$ is the constant wave-vector for the modes and $\delta \mathcal{N}$, δU_n^z , etc. are the constant wave-amplitudes. Within the setting of Eq. (5), we see that the short-wavelength approximation for the modes is expressible as $k_t, k_z \gg H_i \equiv \dot{A}_i / A_i$ for all $i \in \{x, y, z\}$.

The results in Fig. 1, which displays the time evolution of the two flows $u_{n,s} = A_z u_{n,s}^z / u_{n,s}^t$ as well as the couplings (C_{ns}, C_{sn}), show that the relevant coefficients are driven to zero by cosmological expansion, meaning, as expected and anticipated, that any instability will only operate during a finite time. Note that long before and after the transition, one fluid dominates over the other, so spacetime is effectively FLRW; only during the finite Bianchi I phase do the two fluids have comparable contributions [9].

Given a Bianchi I background, we place the terms of Eq. (5) into the Einstein and two-fluid equations of [9], expand, and keep only terms linear in the perturbed quantities, to arrive at the dispersion relation

¹ We use notation that is reminiscent of the matter and entropy flows where n is the total matter number density, s the entropy density, μ the chemical potential, and T the temperature.

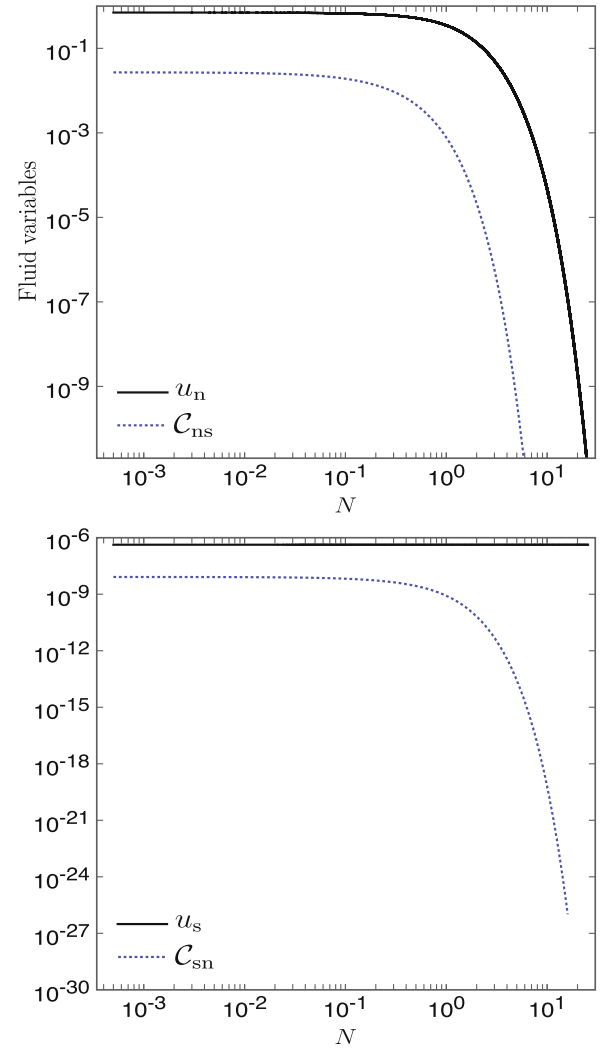


Fig. 1. Time evolution of the cosmological background quantities u_n and C_{ns} (upper), and u_s and C_{sn} (lower), as functions of the e-fold number N for $\alpha = 1$ and $\beta = 4/3$ (matter to radiation transition). With these values of α and β , the parameter κ_s is dimensionless while τ_{ns} has the dimensions of $m^{4-3(\sigma_n-\sigma_s)}$ (all functions and variables can be made dimensionless by means of a proper rescaling with some power of m). For all the graphs, we use $\kappa_s = 1$, $\tau_{ns} = 0.1m^{4-3(\sigma_n-\sigma_s)}$, $\sigma_n = 1.1$, and $\sigma_s = 1.1$. The initial values for the evolutions are $n(0) = 3.9 \times 10^{-7}$, $s(0) = 1$, $u_n(0) = 0.99$, $u_s(0) = -4.25 \times 10^{-7}$, $A_i(0) = 2$, and $H_i(0) = 2.89$ (for each $i \in \{x, y, z\}$).

$$\begin{aligned} &[(u_n \sigma_z - 1)^2 c_n^2 - (\sigma_z - u_n)^2][(u_s \sigma_z - 1)^2 c_s^2 - (\sigma_z - u_s)^2] \\ &- C_{ns} C_{sn} (u_n \sigma_z - 1)^2 (u_s \sigma_z - 1)^2 = 0 \end{aligned} \quad (6)$$

for the mode speed $\sigma_z = -A_z k_t / k_z$. This relation is of the exact same mathematical form as the dispersion relation obtained by Samuelsson et al. [13] [cf. their Eq. (69)]. It is thus immediately clear that our simplified model possesses all the ingredients for two-stream instability.

The presence of the instability is shown in Figs. 2 and 3. These figures provide graphs of the real and imaginary components of σ_z , versus the “e-folding” factor N defined as

$$N = \ln(A_x A_y A_z)^{1/3}. \quad (7)$$

Fig. 2 clearly shows four modes – denoted $\sigma_{z(i)}$ with $i = 1, \dots, 4$ – all less than one, as should be the case. These modes evolve asymptotically towards constants corresponding to the “sound” speeds expected for both fluids. The overall asymmetry of the

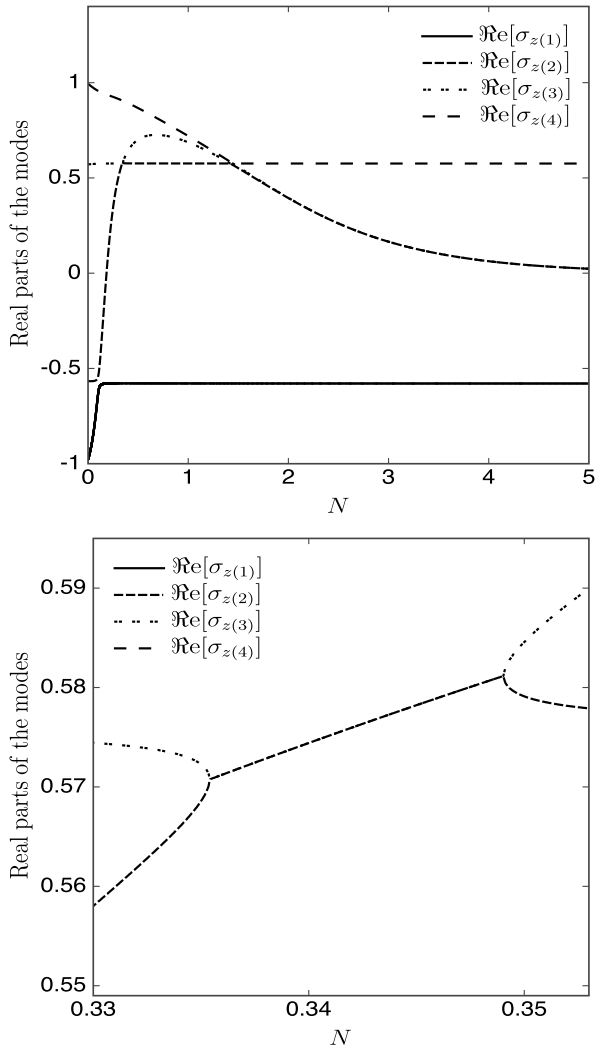


Fig. 2. Real parts of the perturbation mode speed σ_z solutions of Eq. (6), as a function of N throughout the transition (upper) for the same parameter values as in Fig. 1. The lower figure is a magnification of the region where two of the modes merge (to be distinguished from the points where lines simply cross, since they have zero imaginary parts, cf. Fig. 3). In the limit of large N , one recovers asymptotically the four modes of constant speeds $\pm 1/\sqrt{3}$ and 0, as expected for radiation and matter with zero relative velocity. The two solutions that merge (the lower panel) pick up imaginary components (cf. Fig. 3) and thus correspond to the epoch in which the instability initiates, develops, and ends.

modes is due to the background relative flow. In the lower left corner of the figure, it is interesting to note the presence of a so-called avoided crossing, at which two of the modes “exchange” identity.

The instability window is best seen in Fig. 3 where the imaginary parts of σ_z are graphed. Consistent with this is the “mode-merger” of the real parts that occurs in Fig. 2. The results show that the window opens and closes during the same epoch as the transition takes place. Of course, the window closes because of the behavior shown in Fig. 1; e.g. the relative velocity is quenched by the cosmological expansion.

It should be clear that the mechanism discussed here is robust and should apply equally well, for example, to a system of coupled condensates. As stressed at the beginning, any system with two or more fluids, and a (non-gravitational) coupling between them, could be subject to two-stream instability. What is perhaps unique in the cosmological context is that the instability shuts down au-

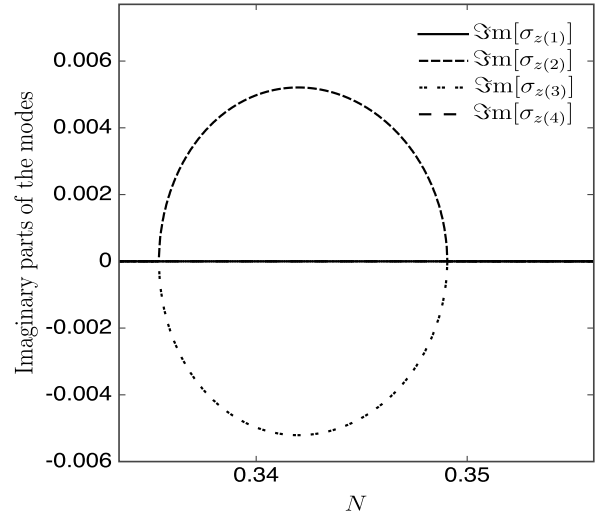


Fig. 3. Imaginary parts of the perturbation mode speed σ_z solutions of Eq. (6), for the same parameter values as in Fig. 2, as a function of N throughout the epoch where the modes are unstable (ignoring the rest of the time evolution since all the modes are real). This corresponds to the region in Fig. 2 (lower panel) where the real parts become degenerate: the number of parameters needed to describe the perturbations is therefore constant during the transition.

tomatically, without any fine-tuning, so long as there is overall expansion.

The cosmological expansion thus provides a means of both initiating and ending the instability. This is mostly due to the cosmological principle, which states that the Universe should be, at almost all times, well described by a FLRW model. This implies that only finite epochs can exhibit a different behavior, in the case at hand that of a non-isotropic Bianchi phase. There remains to address the question of the origin of the relative flow. One might argue, for instance, that the inflation era, almost by construction, drives any primordial anisotropy to zero. During many-field inflation itself, instabilities of this sort could be spontaneously initiated, even for a limited amount of time, and lead to new constraints.

In the companion paper [9], we suggested several scenarios that lead to a non-vanishing relative flow, some being also related to the question of the cosmologically coherent magnetic fields which are thought to exist [5]. These may actually reverse the question: it is conceivable that any model aimed at producing primordial magnetic fields on sufficiently large scales will also induce coherent anisotropies on these scales (the metric we used here is thus meant to describe only a cosmologically small region of space). The mechanism producing these magnetic fields should thus be tested against the instability suggested here. This question is quite natural given that two-stream instability is well-established for plasmas, and there is no reason why it should matter that these are placed in a cosmological setting.

Based on the results presented here, we suggest that cosmological two-stream instability should be taken into account in further studies of all the transitions that could lead to its occurrence. In particular, the supposedly latest such transition, that ended the matter-domination era to the ongoing accelerated phase, could lead to drastically different observational predictions if the latter was driven by a mere cosmological constant or by a cosmic fluid [6]: this new fluid would have no particular reason to be aligned along the matter flow, and hence an instability could develop, producing a characteristic anisotropy whose features still have to be investigated, at the typical scale corresponding to the transition. Some have argued that such an anisotropy has already

been measured or that it could be using forthcoming Planck data [10].

The instability demonstrated in this Letter offers a new avenue for understanding cosmological data, in the sense of new constraints, as well as a potential mechanism for generating anisotropies at specific scales and increase the tensor mode contribution and non-Gaussianities. In this regard, much work obviously remains to be done before two-stream instability might be viewed as a viable, cosmological mechanism: we need to consider flows at arbitrary angles, include dissipation, how relative flows may develop, back-reaction of instabilities on the whole system, and so on.

Finally, we note that two-stream instability is just one example of how multi-fluid dynamics differs from that of a single fluid. We have only considered a simple two-fluid model, with many features left out, but it still illustrates well the possibilities. Perhaps the key point is that the two-stream mechanism cannot operate in the various one-fluid systems that are sometimes called “multi-fluids”. Although these models have several different constituents, they do not account for relative flows. The example considered here clearly demonstrates why relative flows may have interesting consequences, and motivates further studies of the implications. The Cosmological Principle demands a frame in which all constituents are at rest, but we believe strict adherence is too severe, may limit progress, and prevent new insights into the structure and evolution of the Universe.

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