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A new method for non-Fourier thermal response in a single layer skin tissue

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ABSTRACT

The non-Fourier and Fourier thermal responses in one-dimensional single layer skin tissue under selective boundary conditions are investigated by applying the Laplace Transformation method (LTM). The present method accurately describes the deviation of the temperature response of the non-Fourier model from the Fourier model and roles of important physiological parameters. A systematic exact analytical study on the discrepancies between the thermal wave model and the Pennes' bioheat model shows that the present method is an alternate reliable technique to describe the complicated bioheat problems under different boundary conditions. Three cases, namely constant skin temperature, constant and variable heat flux conditions at the skin surface have been taken to determine the tissue temperature whereas an insulated condition at the core has always been satisfied. From the result, it can be highlighted that cosine heat flux at the skin surface amplifies non-Fourier's response of temperature as well.

1. Introduction

Due to the inherent mathematical difficulties associated with biological heat transfer problems, obtaining exact solutions to them is an important thrust area. As skin plays a significant role due to its 'interfacing' between the outside materials and human inside body, analytical methods like exact Laplace transformation to solve time-dependent problems on skin bioheat transfer finds wide importance in military and space research dealing with extreme environmental conditions.

The parabolic natured bioheat transfer equation was first introduced by Pennes in 1948 [1]. Arkin et al. [2] argued that the Pennes' interpretation of the vascular contribution to heat transfer in perfused tissues fails to account for the actual thermal equilibration process between the flowing blood and the surrounding tissue. Wulff [3] also argued the same based on physical arguments and numerical results.

Based on finite propagation of thermal wave, Cattaneo [4] and Vernott [5] independently proposed hyperbolic nature conduction. This hyperbolic or phase-lag behaviour in the thermal wave may be justified if the layer has very small thickness or the time scale of the problem is very short [6].

Literature review suggests that most of the earlier analytical works were carried out on parabolic type bioheat problems. Like, Shih et al. [7] analytically studied the parabolic Pennes' bioheat equation subjected to an oscillatory heat flux boundary condition at the skin by employing Laplace transformation method. Liu [8] derived an analytical solution to the Pennes' bioheat transfer equation in three-dimensional geometry with practical hyperthermia boundary conditions and random

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Nomenclature

C	thermal wave speed in the medium (m s^{-1})
c_b	specific heat of blood (J kg^{-1} °C^{-1})
c_t	specific heat of tissue (J kg^{-1} °C^{-1})
k	thermal conductivity of tissue (W m^{-1} °C^{-1})
L	skin thickness (m)
m	parameter defined in Eq. (12) (m^{-1})
M	parameter defined in Eq. (13) (°C m^{-2})
n	positive integer number
$\bar{q}$	heat flux vector (W m^{-2})
q_0	surface heat flux applied to skin surface for a constant heating (W m^{-2})
q_c	constant surface heat flux, see Eq. (23) (W m^{-2})
q_{ext}	heat generated (W m^{-3})
q_m	heat flux parameter, q_w/k (m^{-2} °C)
q_{met}	metabolic heat generation (W m^{-3})
q_w	amplitude of transient heat flux (W m^{-2})
Res	residue at pole
s	Laplace parameter (s)
$S1_n$, $S2_n$	poles
S_p	notation defined in Eqs. (30), (33) and (36)
t	time (s)
T	tissue temperature (°C)
T_a	blood temperature (°C)
$T_i(x, 0)$	initial tissue temperature (°C)
T_s	constant skin surface temperature (°C)
T_{si}	initial skin surface temperature (°C)
W_b	notation used in Eq. (9) (s^{-1})
x	distance perpendicular to the surface (m)

Greek Symbols

α	thermal diffusivity (m^2 s^{-1})
β	notation defined in Eq. (6) (m^{-1})
γ_1, γ_2	notations defined in Eqs. (7) and (8), respectively (m^{-2} s^{-1}, m^{-2})
λ_n	Eigen value
ρ_b	density of blood (kg m^{-3})
ρ_t	density of skin tissue (kg m^{-3})
τ	thermal relaxation time (s)
ω_b	perfusion rate of blood (m^3 kg^{-1} s^{-1})
ω	frequency of surface heating (s^{-1})
θ	elevation temperature with respect to steady, $T - T_i$ (°C)
θ_0	initial elevation temperature (°C)
$\tilde{\theta}$	temperature in Laplace domain (°C)
ψ	heat addition parameter defined in Eqs. (13) and (16)

heating. Durkee et. al. [9] presented exact solutions to the classical unsteady Pennes' bioheat equation in one-dimensional multi-regional Cartesian and spherical geometries with the constant physiological parameters. In their paper, Mitra et al. [10] made experimental study on processed meat with different boundary conditions and observed wave-like phenomena in conduction heat transfer and demonstrated that the hyperbolic heat conduction model is an accurate representation, on a macroscopic level, of the heat conduction process in such biological material. A good account of numerical works in the relevant field is also reported. Xu et al. [11] reviewed previous researches and obtained numerical solutions for multi layer skin model using finite difference method. They observed large discrepancies among the predictions of the two bioheat models. Liu [12] discussed the non-Fourier heat transfer in a multi-layer skin tissue through the thermal wave model with the skin surface subjected to a step, pulse, linear, exponential and an oscillatory heating by the finite difference method. Ozen et. al. [13] studied the temperature variations in the skin when exposed to microwave for the thermal wave model of bio-heat transfer by employing finite difference method. Most of the non-Fourier bioheat problems were solved numerically and thus necessitates the need for exact solutions.

For the first time, Liu et al. [14] solved the thermal wave model of bioheat transfer (TWMBT) in a finite medium using separation of variables and the results showed distinctive wave behaviours of bioheat transfer in skin subjected to instantaneous heating. Fazlali and Ahmadikia [15] studied the non-Fourier skin-bioheat transfer under arbitrary periodic surface temperature at skin surface with the assumption of initially constant temperature throughout the domain but practically this assumption needs modification.

Liu [16] investigated the non-Fourier thermal behaviour in a living tissue by employing a modified discretization scheme based on numerical inversion of Laplace transform and expressed the findings in terms of an elevated temperature, but a representation in terms of dimensional temperature always gives a better appreciation of temperature response. A comprehensive study on flow and heat transfer of nanofluid has been investigated by many researchers [17–21].

In the present work, the accuracy of the Laplace Transformation technique is studied on a single layer model having finite thickness for the exact analysis of heat transfer in skin, where the skin is treated as a homogeneous medium with uniform, isotropic properties. A survey of the literatures suggests that most works related to non-Fourier bioheat transfer have been carried out by numerical analysis. Here, analytical solutions are obtained for the temperature response in the skin layer for both Fourier and non-Fourier heat transfer mechanisms where three boundary conditions; namely constant temperature surface heating (Case 1), constant heat flux (Case 2) and transient heat flux (Case 3) applied at skin surface are discussed. For a better practical representation, an initial temperature distribution in the domain is taken into account which is not considered in most of the relevant previous works. It is shown that the present technique is an alternate reliable analytical method to accurately describe the complicated non-linear hyperbolic bioheat problems by having a comparative study on the discrepancies between the thermal wave model and the Pennes' model. This difference is significant at initial times and for the higher value of relaxation value. Thus, the exact Laplace transform method can be a very useful tool to check the accuracy of various numerical schemes employed to solve relevant problems in bioheat transfer research.

2. Mathematical formulation

The Pennes equation used for modelling the skin tissue heat transfer is expressed as

$$-\nabla\cdot\vec{q} + \rho_b\omega_b c_b(T_a - T) + q_{met} + q_{ext} = \rho_t c_t \partial T/\partial t. \tag{1}$$

Considering the concept of finite heat propagation velocity, a general form of the thermal wave model of bioheat transfer (TWMBT) in living tissues is expressed by [12]

$$\rho_t c_t \tau\frac{\partial^2 T}{\partial t^2} + (\rho_t c_t + \tau\rho_b\omega_b c_b)\frac{\partial T}{\partial t} + \rho_b\omega_b c_b(T - T_a) = \nabla\cdot(k\nabla T) + \left(q_{met} + \tau\frac{\partial q_{met}}{\partial t} + q_{ext} + \tau\frac{\partial q_{ext}}{\partial t}\right) \tag{2}$$

It is clear that the above equation boils down to Eq. (1) if $\tau = \alpha/C^2 = 0$. This paper performs selective one dimensional case study where the heat propagation takes place in the direction perpendicular to the skin surface with q_{met} = constant and $q_{ext} = 0$. If $T_i(x, 0)$ is the initial steady state temperature distribution, then Eq. (2) can be written in the form

$$\rho_t c_t \tau\frac{\partial^2 \theta}{\partial t^2} + (\rho_t c_t + \tau\rho_b\omega_b c_b)\frac{\partial \theta}{\partial t} + \rho_b\omega_b c_b\theta - k\frac{\partial^2 \theta}{\partial x^2} = 0 \tag{3}$$

where the elevation temperature, $\theta = (T - T_i)$. The initial conditions are to be consistent throughout the case studies as

$$\theta(x,\ 0) = 0 \ \text{ and } \ \partial\theta(x, 0)/\partial t = 0. \tag{4}$$

2.1. Exact analysis under steady condition

Based on the initial conditions of Eq. (4), Eq. (3) is transformed into a steady form in Laplace domain by applying Laplace transformation method

$$\frac{d^2\tilde{\theta}}{dx^2} - \beta^2\tilde{\theta} = 0 \tag{5}$$

where

$$\beta = \sqrt{\tau\gamma_1 s^2 + (\gamma_1 + \tau\gamma_2)s + \gamma_2} \tag{6}$$

$$\gamma_1 = \rho_t c_t/k \tag{7}$$

$$\gamma_2 = W_b c_b/k \tag{8}$$

$$W_b = \rho_b\omega_b \tag{9}$$

The solution of $T_i(x, 0)$ is obtained by considering at T_{si} = 32.5 °C steady state condition and the assumption that heat flux approaches zero in deep into the tissue

$$T(0,\ 0) = T_{si} \ \text{ and } \ dT(L, 0)/dx = 0. \tag{10}$$

Based on Eq. (10), the initial temperature distribution is obtained as

$$T_i(x,\ 0) = \left(\frac{m^2 T_{si} - M}{m^2}\right)\frac{\cosh\left[m(L - x)\right]}{\cosh\left(mL\right)} + \frac{M}{m^2} \tag{11}$$

where

$$m = \sqrt{W_b c_b/k} \tag{12}$$

and

$$M = \left(W_b c_b T_b + q_{met}\right)/k \tag{13}$$

2.2. Exact solution of thermal wave model of bioheat transfer (TWMBT) equation

Three different types of boundary conditions: constant surface temperature heating, constant heat flux, and sinusoidal surface heating at skin surface are used in this work. The solutions of Eq. (5) subjected to respective boundary conditions transformed in the Laplace domain are obtained by their inversion to t-domain by employing inverse theorem for Laplace transformation. The residues at poles are calculated from Cauchy's residue theorem [22].

2.2.1. Constant surface temperature heating on a finite domain skin surface for thermal wave model

This type of situation arises when the surface temperature of the skin is kept constant as the skin contacts with a large steel plate at a high temperature. The boundary conditions are given by

$$T(0,t) = T_s, \quad \partial T(L,t)/\partial x = 0. \tag{14}$$

By calculating the residues at these poles, the solution is obtained as

$$\begin{aligned}\theta(x,t) &= \mathrm{Re}\, s(0) + \mathrm{Re}\, s(S1_n) + \mathrm{Re}\, s(S2_n)\\ &= \frac{\theta_0 \cosh\left[\sqrt{\gamma_2}(L-x)\right]}{\cosh\left(\sqrt{\gamma_2}L\right)} + \sum_{n=1}^{\infty}\frac{2\theta_0 e^{S1_n t}\lambda_n \cos[\lambda_n(L-x)/L]}{L^2 S1_n\left(2\tau\gamma_1 S1_n + \gamma_1 + \tau\gamma_2\right)\sin(\lambda_n)} + \sum_{n=1}^{\infty}\frac{2\theta_0\, e^{S2_n t}\lambda_n \cos[\lambda_n(L-x)/L]}{L^2 S2_n\left(2\tau\gamma_1 S2_n + \gamma_1 + \tau\gamma_2\right)\sin(\lambda_n)}\end{aligned} \tag{15}$$

where

$$(S1_n,\ S2_n) = \frac{1}{2\tau\gamma_1}\left[-(\gamma_1+\tau\gamma_2) \pm \sqrt{(\gamma_1+\tau\gamma_2)^2 - 4\tau\gamma_1\left\{\gamma_2 + \left(\frac{\lambda_n}{L}\right)^2\right\}}\right] \tag{16}$$

$$\lambda_n = \left(\frac{2n-1}{2}\right)\pi, \quad n = 1, 2, \ldots, \infty. \tag{17}$$

2.2.2. Constant heat flux applied on a finite domain skin surface for thermal wave model

In this case, the boundary conditions are

$$-k\partial T(0,t)/\partial x = q_0, \quad \partial T(L,t)/\partial x = 0 \tag{18}$$

By taking the Laplace transformation of Eq. (3), Eq. (5) was obtained. The exact solution of Eq. (3) subjected to boundary conditions of Eq. (10) are obtained by calculating the residues at these poles as

$$\begin{aligned}\theta(x,t) &= \mathrm{Re}\, s(0) + \mathrm{Re}\, s(S1_n) + \mathrm{Re}\, s(S2_n) + \mathrm{Re}\, s(-1/\tau) + \mathrm{Re}\, s\left(-\gamma_2/\gamma_1\right)\\ &= \frac{\psi \cosh\left[\sqrt{\gamma_2}(L-x)\right]}{\sqrt{\gamma_2}\sinh\left(\sqrt{\gamma_2}L\right)} + \sum_{n=1}^{\infty}\frac{2\psi \exp(S1_n t)\cos[\lambda_n(L-x)/L]}{S1_n L\left(2\tau\gamma_1 S1_n + \gamma_1 + \tau\gamma_2\right)\cos\lambda_n} + \sum_{n=1}^{\infty}\frac{2\psi \exp(S2_n t)\cos[\lambda_n(L-x)/L]}{S2_n L\left(2\tau\gamma_1 S2_n + \gamma_1 + \tau\gamma_2\right)\cos\lambda_n}\\ &\quad - \frac{\tau\psi \exp(-1/\tau)}{L\left(\tau\gamma_2 - \gamma_1\right)} + \frac{\gamma_1\psi \exp\left(-\gamma_2/\gamma_1\right)}{\gamma_2 L\left(\tau\gamma_2 - \gamma_1\right)}\end{aligned} \tag{19}$$

where

$$(S1_n,\ S2_n) = \frac{1}{2\tau\gamma_1}\left[-(\gamma_1+\tau\gamma_2) \pm \sqrt{(\gamma_1+\tau\gamma_2)^2 - 4\tau\gamma_1\left[\gamma_2 + (\lambda_n/L)^2\right]}\right] \tag{20}$$

$$\lambda_n = (2n-1)\pi/2 \tag{21}$$

$$\psi = q_0/k + \left(M - m^2 T_{si}\right)\frac{\tanh(mL)}{m} \tag{22}$$

m and M are given in Eqs. (12) and (13), respectively.

2.2.3. Transient heat flux applied on a finite domain skin surface for thermal wave model

This type of boundary condition comes into play when there is repeated irradiation from a laser source causing this kind of heating. The boundary conditions can be written by

$$-k\partial T(0,t)/\partial x = q_c + q_w\cos(\omega\, t), \quad \partial T(L,t)/\partial x = 0 \tag{23}$$

where, q_c and q_w are the constant and amplitude of the transient surface heating. By calculating the residues at these poles, the function $\theta(x, t)$ is obtained as

$$\theta(x, t) = \mathrm{Re}\, s(0) + \mathrm{Re}\, s(S1_n) + \mathrm{Re}\, s(S2_n) + \mathrm{Re}\, s(i\omega) + \mathrm{Re}\, s(-i\omega) + \mathrm{Re}\, s(-1/\tau) + \mathrm{Re}\, s\left(-\gamma_2/\gamma_1\right) \tag{24}$$

After calculating residues of Eq. (24), the temperature can be expressed as follows:

$$\begin{aligned}\theta(x, t) = {} & \frac{\psi \cosh\left\{\sqrt{\gamma_2}(L-x)\right\}}{\sqrt{\gamma_2}\sinh\left(\sqrt{\gamma_2}L\right)} + \sum_{n=1}^{\infty} \frac{2e^{S1_n t}\left\{\psi\left(S1_n^2 + \omega^2\right) + q_m S1_n^2\right\}\cos\left[\lambda_n(L-x)/L\right]}{S1_n L\left(S1_n{}^2 + \omega^2\right)\left(2\tau\gamma_1 S1_n + \gamma_1 + \tau\gamma_2\right)\cos\lambda_n} \\ & + \sum_{n=1}^{\infty} \frac{2e^{S2_n t}\left\{\psi\left(S2_n^2 + \omega^2\right) + q_m S2_n^2\right\}\cos\left[\lambda_n(L-x)/L\right]}{S2_n L\left(S2_n^2 + \omega^2\right)\left(2\tau\gamma_1 S2_n + \gamma_1 + \tau\gamma_2\right)\cos\lambda_n} + \frac{\gamma_1\left\{\psi\left(\gamma_2^2 + \omega^2\gamma_1^2\right) + q_m\gamma_2^2\right\}\exp\left(-\gamma_2/\gamma_1\right)}{\gamma_2 L\left(\gamma_2^2 + \omega^2\gamma_1^2\right)} \\ & + \frac{q_m \exp(i\omega t)\cosh\left[\sqrt{\left(\gamma_2 + i\omega\gamma_1\right)(1 + i\omega\tau)}(L-x)\right]}{2\sqrt{\left(\gamma_2 + i\omega\gamma_1\right)(1 + i\omega\tau)}\sinh\left[\sqrt{\left(\gamma_2 + i\omega\gamma_1\right)(1 + i\omega\tau)}L\right]} - \frac{\tau\left\{\psi\left(1 + \omega^2\tau^2\right) + q_m\right\}\exp(-1/\tau)}{L\left(1 + \omega^2\tau^2\right)\left(\tau\gamma_2 - \gamma_1\right)} \\ & + \frac{q_m \exp(-i\omega t)\cosh\left[\sqrt{\left(\gamma_2 - i\omega\gamma_1\right)(1 - i\omega\tau)}(L-x)\right]}{2\sqrt{\left(\gamma_2 - i\omega\gamma_1\right)(1 - i\omega\tau)}\sinh\left[\sqrt{\left(\gamma_2 - i\omega\gamma_1\right)(1 - i\omega\tau)}L\right]}\end{aligned} \tag{25}$$

where

$$\left(S1_n, S2_n\right) = \frac{1}{2\tau\gamma_1}\left\{-\left(\gamma_1 + \tau\gamma_2\right) \pm \sqrt{\left(\gamma_1 + \tau\gamma_2\right)^2 - 4\tau\gamma_1\left[\gamma_2 + \left(\lambda_n/L\right)^2\right]}\right\} \tag{26}$$

$$\lambda_n = n\pi; \quad q_m = \frac{q_w}{k}; \quad \psi = \frac{q_c}{k} + \left(M - m^2 T_{si}\right)\frac{\tanh(mL)}{m} \tag{27}$$

2.3. Exact solution of Pennes bioheat transfer equation

In a similar fashion, the analytical solutions of parabolic Pennes' equation can be obtained based on the selective boundary conditions. The final expressions of temperature response for the Fourier based Pennes' model is given in the next step. Fig. 1

2.3.1. Constant surface temperature

$$\theta(x, t) = \mathrm{Re}\, s(0) + \mathrm{Re}\, s\left(S_p\right) \tag{28}$$

Eq. (28) yields,

$$\theta(x, t) = \frac{\theta_0 \cosh\left\{\sqrt{\gamma_2}(L-x)\right\}}{\cosh\left(\sqrt{\gamma_2}L\right)} + \sum_{n=1}^{\infty} \frac{2\theta_0\lambda_n \exp\left(S_p t\right)\cos\left[\lambda_n(L-x)/L\right]}{L^2 S_p\left(2\tau\gamma_1 S_p + \gamma_1 + \tau\gamma_2\right)\cos\left(\lambda_n\right)} \tag{29}$$

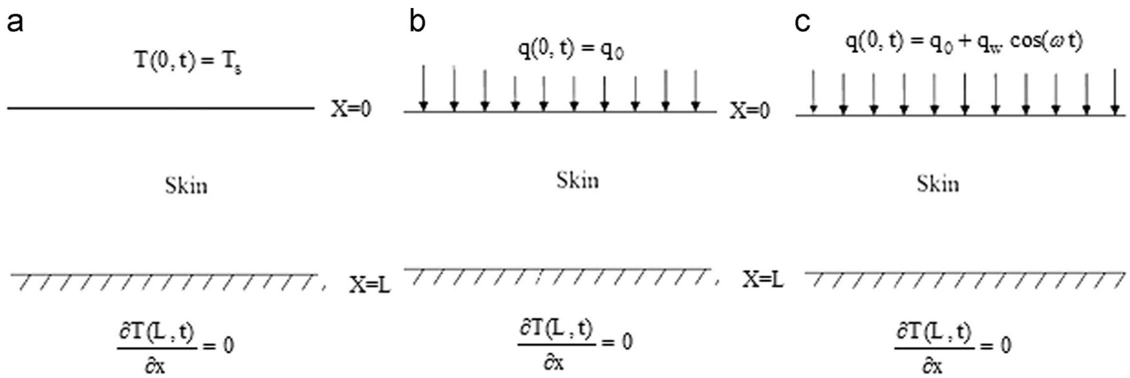

Fig. 1. The geometrical problems of Finite domain of skin tissue with three types of boundary conditions.

where

$$S_p = -\left[\gamma_2 + (\lambda_n/L)^2\right]/\gamma_1; \quad \lambda_n = (2n-1)\pi/2 \tag{30}$$

2.3.2. Constant heat flux at the skin

$$\theta(x,t) = \mathrm{Re}\, s(0) + \mathrm{Re}\, s\left(S_p\right) + \mathrm{Re}\, s\left(-\gamma_2/\gamma_1\right) \tag{31}$$

From Eq. (31), temperature can finally be expressed as

$$\theta(x,t) = \frac{\psi \cosh\left\{\sqrt{\gamma_2}(L-x)\right\}}{\sqrt{\gamma_2}\sinh\left(\sqrt{\gamma_2}L\right)} + \sum_{n=1}^{\infty}\left(\frac{2\psi \exp\left(S_p t\right)\cos\left[\lambda_n(L-x)/L\right]}{S_p L\gamma_1 \cos\left(\lambda_n\right)}\right) - \frac{\psi \exp\left(-\gamma_2/\gamma_1\right)}{\gamma_2 .\, L} \tag{32}$$

where

$$S_p = -\left[\gamma_2 + \left(\lambda_n/\gamma_1\right)^2\right]/\gamma_1; \quad \lambda_n = n\pi \tag{33}$$

2.3.3. Transient heat flux at the skin

$$\theta(x,t) = \mathrm{Re}\, s(0) + \mathrm{Re}\, s\left(S_p\right) + \mathrm{Re}\, s(+i\omega) + \mathrm{Re}\, s(-i\omega) + \mathrm{Re}\, s\left(-\gamma_2/\gamma_1\right) \tag{34}$$

Eq. (34) can be expanded as follows:

$$\begin{aligned}\theta(x,t) = {} & \frac{\psi \cosh\left\{\sqrt{\gamma_2}(L-x)\right\}}{\sqrt{\gamma_2}\sinh\left(\sqrt{\gamma_2}L\right)} + \sum_{n=1}^{\infty}\frac{2\exp\left(S_p t\right)\left\{\psi\left(S_p^2+\omega^2\right) + q_m S_p^2\right\}\cos\left\{\lambda_n(L-x)/L\right\}}{S_p L\gamma_1\left(S_p^2+\omega^2\right)\cos\lambda_n} \\ & + \frac{q_m \exp(\mathrm{i}\omega t)\cosh\left[\sqrt{\left(\gamma_2 + i\omega\gamma_1\right)}(L-x)\right]}{2\sqrt{\left(\gamma_2 + i\omega\gamma_1\right)}\sinh\left[\sqrt{\left(\gamma_2 + i\omega\gamma_1\right)}L\right]} + \frac{q_m \exp(-\mathrm{i}\omega t)\cosh\left[\sqrt{\left(\gamma_2 - i\omega\gamma_1\right)}(L-x)\right]}{2\sqrt{\left(\gamma_2 - i\omega\gamma_1\right)}\sinh\left[\sqrt{\left(\gamma_2 - i\omega\gamma_1\right)}L\right]} \\ & - \frac{\gamma_1\left\{\psi\left(\gamma_2^2 + \omega^2\gamma_1^2\right) + q_m\gamma_2^2\right\}\exp\left(-\gamma_2/\gamma_1\right)}{\gamma_2 L\left(\gamma_2^2 + \omega^2\gamma_1^2\right)}\end{aligned} \tag{35}$$

where

$$S_p = -\frac{1}{\gamma_1}\left\{\gamma_2 + (\lambda_n/L)^2\right\}; \quad \lambda_n = n\pi \tag{36}$$

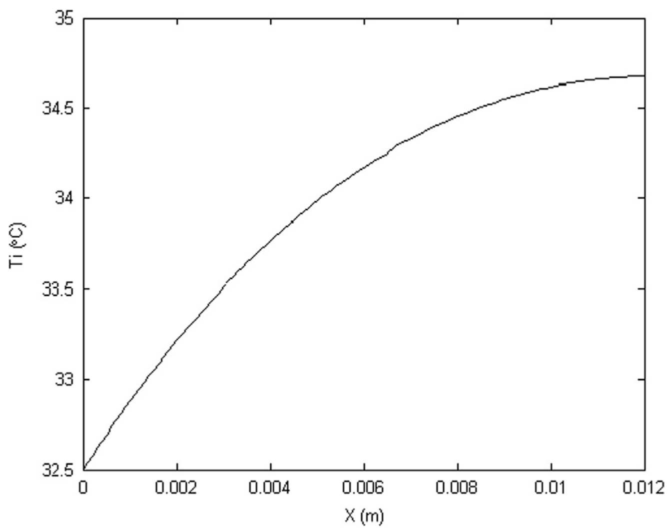

Fig. 2. Initial temperature distribution in the skin.

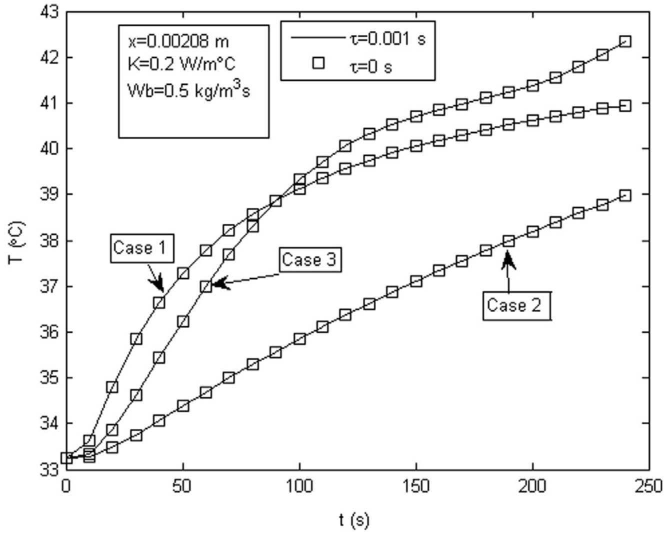

Fig. 3. Skin temperature as predicted by the proposed method.

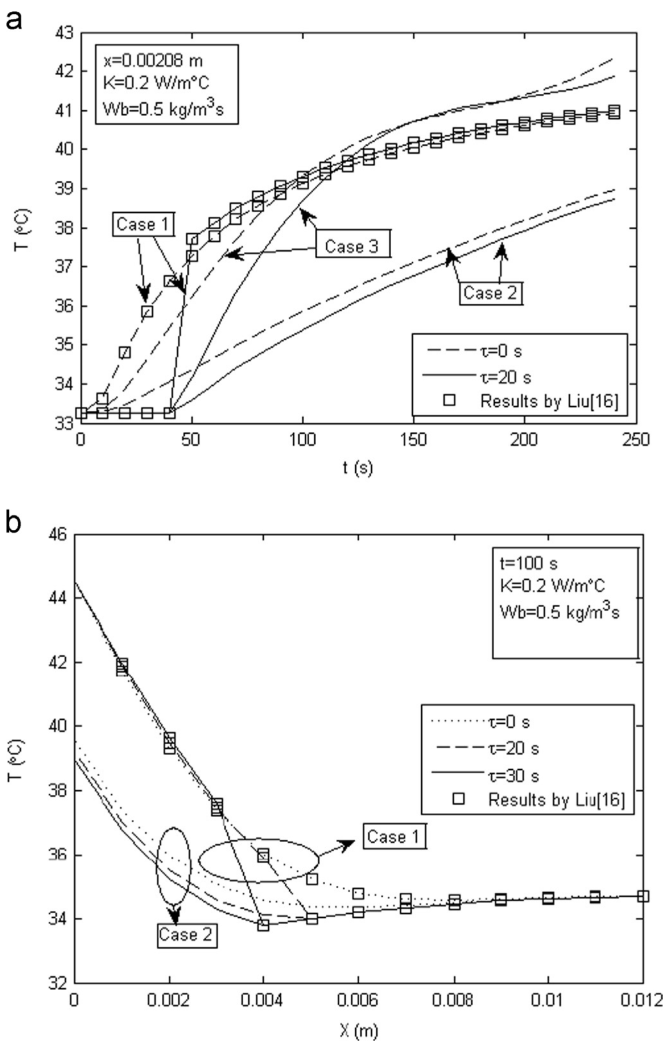

Fig. 4. Skin temperature distribution as a function of relaxation time for the selected boundary conditions: (a) temperature as a function of time, and (b) temperature as a function of space.

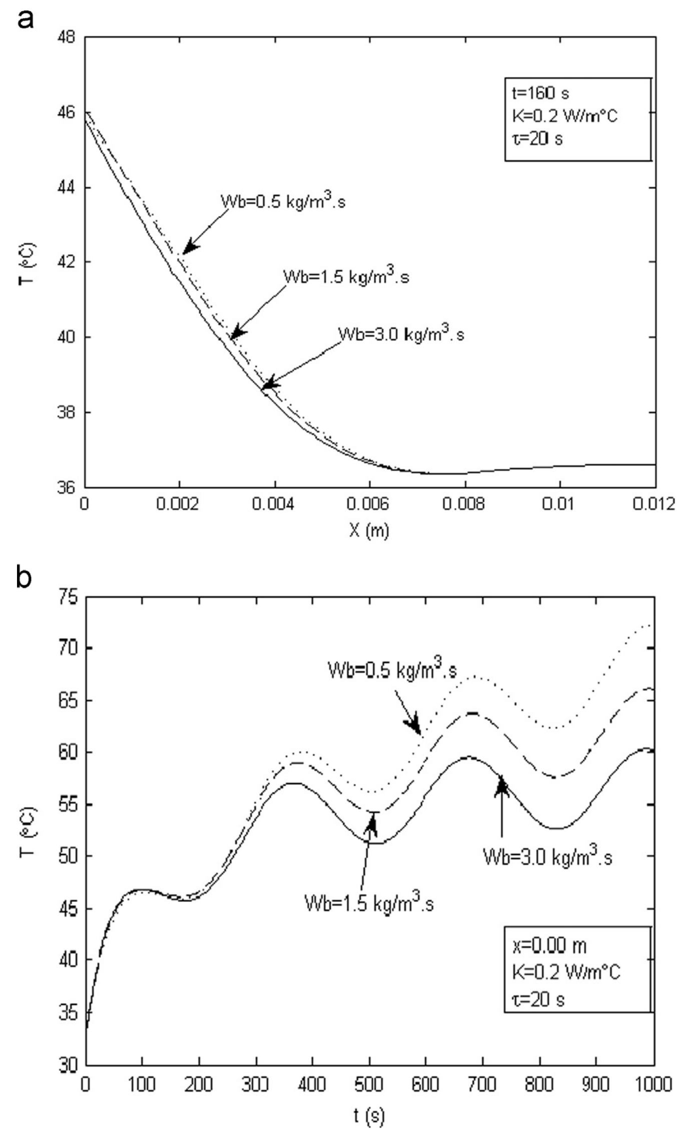

Fig. 5. Skin temperature response as a function of blood perfusion rate for the transient heat flux applied at skin surface: (a) temperature variation in a spatial coordinate, and (b) temperature variation with respect to time.

3. Results and discussion

The present study employs Laplace transformation technique for the exact analysis of Fourier and non-Fourier temperature response during heat conduction in a finite domain skin tissue. An initial temperature distribution as shown in Fig. 2 not considered in most of the early works has been included in the present calculation to obtain a better perception of the results.

In the present analysis of non-Fourier heat transfer, adopting a limiting value of τ to zero to be analyzing Fourier heat transfer may not be possible because of τ's presence in the denominator of some mathematical expressions [23,24]. Thus, in this study, Fourier and non-Fourier analysis are completed separately. However, selecting a very small value of τ may produce the same result obtained from the Fourier analysis. This verification was done and is depicted in the plot shown in Fig. 3. Assuming $\tau = 0.001$ s, the non-Fourier temperature is identical to the Fourier temperature as shown in this figure and it was determined from the present analytical study. Therefore, it can be concluded that both models have been formulated correctly. The physical parameters used in this paper are taken from Ref. [16]. The difference of temperature response for the three cases has been found due to difference in skin surface boundary conditions. The temperature response in a tissue gradually for every case increases gradually with time until they reach to a steady condition.

In this study, $\theta_0 = 12^{\circ}$ C, $q_0 = 500\ \mathrm{W\ m^{-3}}$, and $q = 1000 + 500\cos(0.02t)$ are taken. Fig. 4 shows skin temperature response as a function of relaxation time for the selected boundary conditions. The results obtained by the present method for

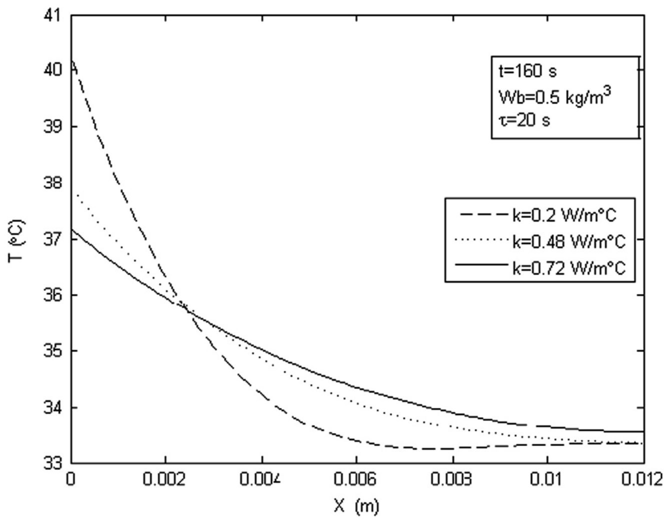

Fig. 6. Skin temperature response as a function of thermal conductivity for the constant heat flux condition at skin surface.

the case of constant temperature surface heating is in excellent agreement with the results of Liu [16] where the results were shown in terms of the elevated temperature. It is noted that the results obtained by Liu [16] were based on a modified discretization method and numerical inverse Laplace technique which is much more rigorous to its solution and an exact solution is always a first choice for calculation whenever possible. The present results are in terms of true dimensional temperature considering initial temperature distribution in the domain i.e. the results for the first case is in perfect match with the results by Liu but the present scale used is only different. Fig. 4a shows that the length over which the temperature variation is insignificant and is also dependent on τ value. It is clear from the plots that Fourier and non-Fourier models are the same for higher values of time. The analysis by the present exact Laplace transform method also shows the fact that the thermal wave propagation is prominent when τ is large with a smaller value of $t$as shown in Fig. 4b. The penetration depth of the thermal signal is inversely proportional to the value of τ. The analysis by the present technique indicates that constant temperature heating has the most sharp step change of deviation in temperature profile when compared to other cases.

To further check the accuracy of the present method, the temperature response with increasing blood perfusion rate for the third case i.e. transient heat flux at skin surface is shown in Fig. 5. The blood perfusion develops a cooling function, since the skin temperature is higher than the arterial temperature. The heat energy taken away by the blood is proportional to the perfusion rate. Clearly, skin temperature decreases with increasing perfusion rate due to carry out more heat with high blood flow. Thus, it is found that the present method interprets blood perfusion correctly as found in Fig. 5.

Fig. 6 plots the temperature response with increasing values of thermal conductivity for the second case i.e. constant heat flux at the skin surface. The heat propagation velocity is proportional to the thermal diffusivity α and is inversely proportional to the relaxation time τ from the definition of finite heat propagation velocity, $V=\sqrt{\alpha/\tau}$ [25,26]. Now, $\alpha = k/\rho_t c_t$. Hence, the heat propagation velocity is also proportional to thermal conductivity of skin tissue. It is found that the temperature distribution curve becomes much smoother with increasing k, i.e. the heat propagation velocity is raised by the larger value of k, and thus the energy transferred from the skin surface at $x=0.00$ m due to heating transfers through the skin tissue by conduction in a shorter time period.

4. Conclusions

The exact Laplace transformation method correctly describes the hyperbolic and parabolic bioheat problems mentioned above. It is to note that though a finite domain skin tissue is considered for this analysis, the condition of finite geometry in orthogonal directions is not required when the Laplace transformation employed [22]. Moreover, advantages of using the Laplace transformation method increase with further complexity of the problem [24]. The present study shows that the deviation in the temperature response of the non-Fourier model from the Fourier one is correctly described by this method as it is also supported from the literature. It is observed that the deviation is not only dependent on the time and τ value, but is also related to the nature of boundary conditions. For the same time and τ value, the non-Fourier effect dominates in the case of constant temperature surface heating condition. A lower value of time and higher value of τ always create significant non-Fourier temperature responses. The dimensional temperature variation plotted in the study gives better perception about the thermal response in the domain along with the consideration of an initial temperature distribution in the skin whereas most previous researchers considered an initially constant temperature throughout the domain. The analytical results given by the exact analysis by Laplace transformation method correctly highlights the effects of the thermal conductivity, blood perfusion on the temperature distribution which can be valuable in the construction of temperature field for thermal diagnosis and treatment. Solutions obtained from all the cases show that the blood perfusion rate has most pronounced effect which strongly influences the temperature distribution in living tissue. Thus it can be concluded that the exact Laplace transformation method can also be used as a reliable alternate technique to solve the complicated non-linear bioheat problems.

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