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# Focus point gauge mediation with incomplete adjoint messengers and gauge coupling unification

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## ABSTRACT

As the mass limits on supersymmetric particles are gradually pushed to higher values due to their continuing non-observation at the CERN LHC, looking for focus point regions in the supersymmetric parameter space, which shows considerably reduced fine-tuning, is increasingly more important than ever. We explore this in the context of gauge mediated supersymmetry breaking with messengers transforming in the adjoint representation of the gauge group, namely, octet of color SU(3) and triplet of weak SU(2). A distinctive feature of this scenario is that the focus point is achieved by fixing a single combination of parameters in the messenger sector, which is invariant under the renormalization group evolution. Because of this invariance, the focus point behavior is well under control once the relevant parameters are fixed by a more fundamental theory. The observed Higgs boson mass is explained with a relatively mild fine-tuning  $\Delta = 60\text{--}150$ . Interestingly, even in the presence of incomplete messenger multiplets of the SU(5) GUT group, the gauge couplings still unify perfectly, but at a scale which is one or two orders of magnitude above the conventional GUT scale. Because of this larger unification scale, the colored Higgs multiplets become too heavy to trigger proton decay at a rate larger than the experimentally allowed limit.

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## 1. Introduction

Though still elusive, Supersymmetry (SUSY), as a class of models, continues to be the leading candidate for physics beyond the Standard Model (SM). In addition to showing the virtue of gauge coupling unification, supersymmetry provides a dynamical origin of the *negative* mass-square of a neutral scalar that triggers electroweak symmetry breaking (EWSB). As we know by now, the origin of EWSB is completely explained if the scalar top (stop) mass is around the weak scale. However, the continuing absence of SUSY signals at the CERN Large Hadron Collider (LHC) has pushed up the gluino and squark masses to larger than about 1.2–1.6 TeV [1]. Additionally, the observed Higgs boson mass around 125 GeV [2] in the SUSY framework requires large radiative corrections from the stops [3]. This in turn necessitates the average stop mass to be at least 3–5 TeV [4], which is significantly larger than the weak scale. Consequently, settling the EWSB scale at the correct value requires a large fine-tuning of the Higgs potential in general.

Under these circumstances, the focus point SUSY [5] (see also [6] for recent discussions) deserves special attention. In this class of scenarios, one or more fixed ratios among soft SUSY breaking masses are introduced, which lessens the fine-tuning of the Higgs potential lending more credibility to the natural explanation of the EWSB scale even if the SUSY particles turn out to be very heavy.

Among focus point SUSY scenarios [7–10], the scenarios based on gauge mediation [11]<sup>1</sup> have the advantage of suppressing the FCNC processes.<sup>2</sup> In the context of gauge mediation, the issue of focus point has been addressed in Ref. [13], where the numbers of the weakly and strongly coupled messenger multiplets ( $N_2$  and  $N_3$ , respectively) are different from each other. Thanks to sizable cancellation between soft mass parameters for particular choices of  $N_2$  and  $N_3$  during the renormalization group running, the EWSB scale is realized with milder fine-tuning. However, owing to the pres-

<sup>1</sup> For early attempts, see also Ref. [12].

<sup>2</sup> The focus point SUSY models based on gaugino mediation [8] and Higgs-gaugino mediation [9] also do not suffer from the SUSY FCNC problem. Moreover, the latter model can easily explain the muon  $g - 2$  anomaly [10].

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ence of these large number of incomplete multiplets of the grand unified theory (GUT) group, the gauge couplings do not unify.

The gauge coupling unification may be achieved non-trivially in a framework where the messenger particles of gauge mediation transform in the adjoint representation of the GUT group. First, in Ref. [14], it was shown that the presence of adjoint matter multiplets with mass around  $10^{13}$ – $10^{14}$  GeV can lead to gauge coupling unification around the string scale [15], which is one or two orders of magnitude above the conventional GUT scale, *even if* the adjoint matters do not form complete GUT multiplets. Subsequently, it was noticed that these adjoint multiplets can be employed as messenger superfields [16] for gauge mediation that would generate soft SUSY breaking masses. Such adjoint gauge mediation scenarios naturally lead to mass splitting among colored and uncolored particles right at the messenger scale [16,17].

We note at this point that in the context of SU(5) GUT, working with adjoint messengers, namely, SU(3) octet and SU(2) triplet, which are incomplete multiplets of SU(5), has a certain advantage over using messengers of complete multiplets, e.g. SU(3) triplet and SU(2) doublet. In the latter case, the requirement of precise gauge coupling unification demands that some colored Higgs multiplets weigh around  $10^{15}$ – $10^{16}$  GeV [18], which would prompt unacceptably large proton decay. On the other hand, gauge coupling unification with adjoint messengers would necessitate the colored Higgs multiplets to hover around  $10^{17}$ – $10^{18}$  GeV due to the larger unification scale, which is consistent with proton lifetime [19,20].

In this Letter, we exhibit how the fine-tuning of the EWSB can be reduced by utilizing the mass splitting of the adjoint representation messengers, more specifically, between the SU(3) octet and SU(2) triplet messengers. The focus point behavior is controlled by fixing one single combination of the superpotential parameters. Remarkably, this combination is invariant under the renormalization group evolution, i.e. it is stable against radiative corrections. Thus the focus point behavior in adjoint messenger gauge mediation model is more robust (assuming that the value of this combination is fixed by some more fundamental physics) than other SUSY breaking scenarios in the general class of minimal supersymmetric standard model (MSSM). In the latter scenarios, to reach the focus point region, various relations among soft SUSY breaking and/or preserving (like  $\mu$ ) parameters need to be assumed which are neither invariant under renormalization group evolution nor independent of the SUSY breaking scale. This lends a substantial credibility to the attainment of focus point in adjoint messenger gauge mediation models.

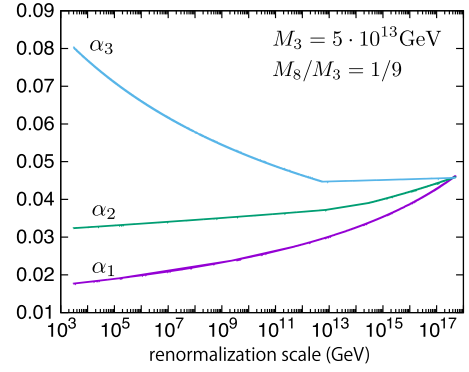
## 2. Adjoint messenger gauge mediation (AMGMSB)

In the present scenario SUSY breaking is accomplished by gauge mediation with messengers transforming in the adjoint representation of the gauge group [16,17,21]. These messengers transform as  $(\mathbf{8}, \mathbf{1})$  and  $(\mathbf{1}, \mathbf{3})$  under  $SU(3)_C \times SU(2)_L$  gauge group, and may have originated from the non-Goldstone modes of the  $\mathbf{24}$  dimensional Higgs multiplet in the SU(5) GUT gauge group. The resultant soft masses of weakly and strongly interacting supersymmetric particles, which are significantly different from those in minimal GMSB, allow for a significant reduction of fine-tuning [21]. The superpotential in the messenger sector is:

$$W_{\text{mess}} = (M_8 + \lambda_8 Z) \text{Tr}(\Sigma_8^2) + (M_3 + \lambda_3 Z) \text{Tr}(\Sigma_3^2), \quad (1)$$

where  $Z$  is a spurion field whose  $F$ -term vacuum expectation value (VEV)  $F_Z$  breaks supersymmetry, whose effects are transmitted to the observable sector via messenger loops.

Even though the messenger multiplets in our model are incomplete SU(5) multiplets, the gauge coupling unification is still



**Fig. 1.** Unification of the three gauge couplings at the two-loop level with SU(3)<sub>C</sub> octet and SU(2)<sub>L</sub> triplet messengers with their masses around  $10^{13}$  GeV. Here,  $\alpha_s(M_Z) = 0.1185$  and  $m_{\text{SUSY}} = 3$  TeV.

achieved for  $M_3 \sim M_8 \sim 10^{13}$ – $10^{14}$  GeV at a scale somewhat higher than the conventional  $M_{\text{GUT}} \simeq 10^{16}$  GeV [14], being around  $M_{\text{str}} \approx 5 \cdot 10^{17}$  GeV, which we call the string scale [15]. It is expected that at this scale the gauge and gravitational couplings are unified.

For illustration of gauge unification, we display the one-loop beta-functions of the gauge couplings. The gauge couplings at  $M_{\text{str}}$  are given by

$$\begin{aligned} \alpha_1^{-1}(M_{\text{str}}) &= \alpha_1^{-1}(m_{\text{SUSY}}) - \frac{b_1}{2\pi} \ln \frac{M_{\text{str}}}{m_{\text{SUSY}}}, \\ \alpha_2^{-1}(M_{\text{str}}) &= \alpha_2^{-1}(m_{\text{SUSY}}) - \frac{b_2 + 2}{2\pi} \ln \frac{M_{\text{str}}}{m_{\text{SUSY}}} + \frac{2}{2\pi} \ln \frac{M_3}{m_{\text{SUSY}}}, \\ \alpha_3^{-1}(M_{\text{str}}) &= \alpha_3^{-1}(m_{\text{SUSY}}) - \frac{b_3 + 3}{2\pi} \ln \frac{M_{\text{str}}}{m_{\text{SUSY}}} + \frac{3}{2\pi} \ln \frac{M_8}{m_{\text{SUSY}}}, \end{aligned} \quad (2)$$

where  $b_i = (33/5, 1, -3)$  is the coefficient of one-loop beta-function for the gauge coupling  $g_i$ , and  $m_{\text{SUSY}}$  is the typical mass scale of strongly interacting SUSY particles, defined here more specifically as the stop mass scale  $m_{\text{SUSY}} \equiv (m_{Q_3} m_{\bar{U}_3})^{1/2}$ . It turns out that  $\alpha_{1,2,3}^{-1} \simeq (57, 31, 13)$  at  $m_{\text{SUSY}} = 3$  TeV.

From Eq. (2), we can write (following the discussion in Ref. [22])

$$\begin{aligned} (5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_{\text{SUSY}}) &= \frac{6}{\pi} \ln \left[ \left( \frac{M_{\text{mess}}}{m_{\text{SUSY}}} \right) \left( \frac{M_{\text{str}}}{m_{\text{SUSY}}} \right)^2 \right] \\ (\alpha_1^{-1} - 3\alpha_2^{-1} + 2\alpha_3^{-1})(m_{\text{SUSY}}) &= -\frac{6}{5\pi} \ln \left( \frac{M_{\text{str}}}{m_{\text{SUSY}}} \right) + \frac{3}{\pi} \ln \frac{M_3}{M_8}. \end{aligned} \quad (3)$$

Using the above it is straightforward to obtain  $M_{\text{str}}^2 M_{\text{mess}} \simeq M_{\text{GUT}}^3$ , where  $M_{\text{mess}} \equiv (M_3 M_8)^{1/2}$ . Requiring  $M_{\text{str}} \lesssim 10^{18}$  GeV, it follows that  $M_{\text{mess}} \gtrsim 10^{12}$  GeV. From the second equation of Eq. (3), we see that the larger  $M_{\text{str}}$ , or equivalently smaller  $M_{\text{mess}}$ , requires a larger ratio of  $M_3/M_8$  for the gauge coupling unification. For instance, for  $M_{\text{str}} = 10^{17}$  ( $10^{18}$ ) GeV, one requires  $M_3/M_8 \simeq 7(18)$  at the one-loop level. We, however, employ two-loop renormalization group equations (RGE) for the running of the gauge couplings, which is displayed in Fig. 1.

It is appropriate at this stage to highlight an advantage of using adjoint messengers for gauge mediation in GUT framework, more specifically, with SU(5) as the GUT group. The high scale spectra invariably contain colored Higgs multiplets, namely,  $H_C$  and  $\bar{H}_C$ , which belong to  $\mathbf{5}_H (= (H_u, H_C))$  and  $\bar{\mathbf{5}}_H (= (H_d, \bar{H}_C))$  of SU(5), where  $H_u$  ( $H_d$ ) denotes the up-type (down-type) weak doublet Higgs multiplets. The mass of the colored Higgs multiplet  $M_{H_C}$  is

predicted to be around the unification scale.<sup>3</sup> Adjoint messenger gauge mediation has the distinct advantage of pushing the unification scale beyond the conventional GUT scale to  $M_{\text{str}} (\sim M_{H_c}) = 10^{17} - 10^{18}$  GeV, which can easily accommodate the experimental constraints from the proton lifetime [20]. This is because the proton decay rate ( $p \rightarrow K^+ \bar{\nu}$ ) is suppressed by  $1/M_{H_c}^2$  [19].

On the contrary, if the messengers are complete multiplets of SU(5), the unification scale is  $M_{\text{GUT}} \sim 10^{16}$  GeV (the conventional scale), and then  $M_{H_c} \sim M_{\text{GUT}}$ . Moreover, the precise gauge coupling unification requires  $M_{H_c}$  to be  $10^{15} - 10^{16}$  GeV. This necessitates inclusion of threshold corrections to the gauge couplings, namely,  $-(1/5\pi) \ln(M_{\text{GUT}}/M_{H_c})$  to  $\alpha_1^{-1}$  and  $-(1/2\pi) \times \ln(M_{\text{GUT}}/M_{H_c})$  to  $\alpha_3^{-1}$ . Then the proton decay rate would overshoot the experimental limit for (sub-)TeV scale SUSY [18].

With these messenger multiplets, the gaugino masses from the messenger loops at the scale  $M_{\text{mess}}$  are

$$M_{\tilde{B}} \simeq 0, \quad M_{\tilde{W}} \simeq \frac{g_2^2}{16\pi^2} (2\Lambda_3), \quad M_{\tilde{g}} \simeq \frac{g_3^2}{16\pi^2} (3\Lambda_8), \quad (4)$$

where  $\Lambda_3 \equiv \lambda_3 \langle F_Z \rangle / M_3$  and  $\Lambda_8 \equiv \lambda_8 \langle F_Z \rangle / M_8$ , provided that  $\lambda_3 \langle Z \rangle$  and  $\lambda_8 \langle Z \rangle$  are much smaller than  $M_3$  and  $M_8$ , respectively. The sfermion masses at  $M_{\text{mess}}$  are given by

$$\begin{aligned} m_Q^2 &\simeq \frac{2}{(16\pi^2)^2} \left[ \frac{4}{3} g_3^4 (3\Lambda_8^2) + \frac{3}{4} g_2^4 (2\Lambda_3^2) \right], \\ m_D^2 = m_U^2 &\simeq \frac{2}{(16\pi^2)^2} \frac{4}{3} g_3^4 (3\Lambda_8^2), \\ m_L^2 = m_{H_u}^2 = m_{H_d}^2 &\simeq \frac{2}{(16\pi^2)^2} \frac{3}{4} g_2^4 (2\Lambda_3^2), \quad m_E^2 \simeq 0. \end{aligned} \quad (5)$$

One can see that the bino and right-handed sleptons are massless, since there is no messenger field charged under the  $U(1)_Y$  gauge group. In order to give masses to the right-handed sleptons, we consider the minimal Kähler for the MSSM matter multiplets and the spurion  $Z$ . Then the MSSM matter fields receive a common mass  $m_0$  from the supergravity scalar potential, which is equal to the gravitino mass  $m_{3/2} = \langle F_Z \rangle / (\sqrt{3} M_P)$ .

In this setup, the gluino mass at the soft SUSY breaking mass scale ( $\sim$  TeV) is

$$\begin{aligned} M_{\tilde{g}}(m_{\text{SUSY}}) &= \frac{\alpha_3(m_{\text{SUSY}})}{4\pi} (3\Lambda_8) \\ &\simeq 4.0 \text{ TeV} \cdot \left( \frac{\lambda_8}{0.001} \right) \left( \frac{m_{3/2}}{500 \text{ GeV}} \right) \left( \frac{M_8}{10^{13} \text{ GeV}} \right)^{-1}. \end{aligned} \quad (6)$$

The bino can get a mass from the gauge kinetic function:

$$\mathcal{L} \ni \frac{1}{4g_1^2} \int d^2\theta \left( 1 - \frac{2kZ}{M_P} \right) W_\alpha^1 W^{\alpha 1} + \text{h.c.} \quad (7)$$

Then,

$$M_{\tilde{B}}(M_{\text{str}}) = \frac{k \langle F_Z \rangle}{M_P} = \sqrt{3} k m_{3/2}. \quad (8)$$

Alternatively, one can consider the sequestered form of the Kähler potential, which ensures the absence of FCNC. In this case, the right-handed slepton masses are generated by the bino loop, which is nothing but the gaugino mediation mechanism. Another

option is to introduce a pair of  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  messengers to generate the bino and right-handed slepton masses, which would also contribute to other masses. In this Letter, for simplicity, we work with only SU(3) octet and SU(2) triplet adjoint messengers and stick to the case of the minimal Kähler potential, as mentioned above.

### 3. Focus point in the AMGMSB

We consider the fine-tuning of the EWSB scale with soft masses generated from these adjoint messengers. The EWSB conditions are given by

$$\begin{aligned} \frac{g_1^2 + g_2^2}{4} v^2 &= \left[ -\mu^2 - \frac{(m_{H_u}^2 + \frac{1}{2v_u} \frac{\partial \Delta V}{\partial v_u}) \tan^2 \beta}{\tan^2 \beta - 1} \right. \\ &\quad \left. + \frac{m_{H_d}^2 + \frac{1}{2v_d} \frac{\partial \Delta V}{\partial v_d}}{\tan^2 \beta - 1} \right]_{m_{\text{SUSY}}}, \\ \frac{\tan^2 \beta + 1}{\tan \beta} &= \left[ \frac{1}{B\mu} \left( m_{H_u}^2 + \frac{1}{2v_u} \frac{\partial \Delta V}{\partial v_u} + m_{H_d}^2 \right. \right. \\ &\quad \left. \left. + \frac{1}{2v_d} \frac{\partial \Delta V}{\partial v_d} + 2\mu^2 \right) \right]_{m_{\text{SUSY}}}, \end{aligned} \quad (9)$$

where  $\Delta V$  denotes a one-loop correction to the Higgs potential, and  $m_{H_u}^2$  and  $m_{H_d}^2$  are the soft masses for the up-type and down-type Higgs, respectively. The Higgsino mass parameter is denoted by  $\mu$ , and  $B\mu$  is a soft SUSY breaking parameter of the Higgs bilinear term. The above equations tell us that the EWSB scale is determined dominantly by  $\mu^2$  and  $[m_{H_u}^2 + 1/(2v_u)(\partial \Delta V / \partial v_u)]$  for large  $\tan \beta$  ( $\equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$ ).

Now we consider renormalization group running of  $m_{H_u}^2$  from the high scale to weak scale. To understand the behavior of this running intuitively, we first demonstrate it using approximate analytic solutions of RGEs. The Higgs soft masses receive negative contributions from stop and gluino loops and positive contributions from the wino loop. The dominant negative contributions induced by the top-Yukawa coupling  $y_t$  are<sup>4</sup>

$$\begin{aligned} (m_{H_u}^2(Q_r))_{\text{neg}} &\simeq \frac{k-1}{2} \left[ m_{Q_3}^2(M_{\text{mess}}) + m_{U_3}^2(M_{\text{mess}}) \right] \\ &\quad - k_{\tilde{g}} M_{\tilde{g}}^2(M_{\text{mess}}), \end{aligned} \quad (11)$$

where  $Q_r$  is the renormalization scale taken to be the stop mass scale and

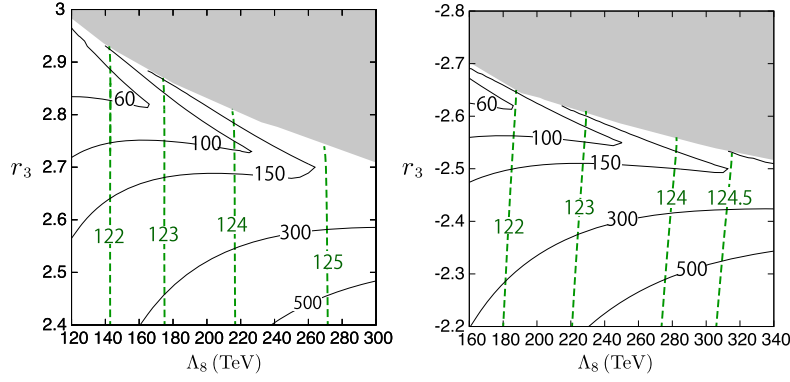
$$\begin{aligned} k &= \exp \left[ \int_0^t \left( \frac{3y_t^2(t')}{4\pi^2} \right) dt' \right], \quad \text{with } t = \ln(Q_r/M_{\text{mess}}), \\ k_{\tilde{g}} &= \int_0^t dt' \frac{y_t^2(t') g_3^2(M_{\text{mess}})}{2\pi^4} t' \left[ \frac{1 - \eta_3 t'/2}{(1 - \eta_3 t')^2} \right] \\ &\quad - \int_0^t dt' \frac{y_t^2(t') g_3^4(M_{\text{mess}})}{6\pi^6} \frac{t'^2}{(1 - \eta_3 t')^2}, \quad \text{with} \end{aligned}$$

<sup>4</sup> The difference of the coefficients in front of  $m_{Q_3}^2$  and  $m_{U_3}^2$  between Eq. (11) and Eq. (15) arises from  $U(1)_Y$  contributions:

$$(16\pi^2) \frac{dm_{H_u}^2}{dt} \ni \frac{3}{5} g_1^2 [\text{Tr}(m_{Q_3}^2 - 2m_{U_3}^2) + \dots]. \quad (10)$$

However, the above  $U(1)_Y$  contributions are eventually canceled out in the most of the gauge mediation models when their effects on each individual soft masses are summed up.

<sup>3</sup> The contributions of these colored Higgs states to gauge coupling evolution cannot be ignored if  $M_{H_c}$  is smaller than  $M_{\text{str}}$ . To account for their contributions, one must add  $-(1/5\pi) \ln(M_{\text{str}}/M_{H_c})$  to  $\alpha_1^{-1}$  and  $-(1/2\pi) \ln(M_{\text{str}}/M_{H_c})$  to  $\alpha_3^{-1}$  in Eq. (2).



**Fig. 2.** Contours of  $\Delta$  (solid) and  $m_h/\text{GeV}$  (dashed). In the gray region, the EWSB does not occur. The messenger scale is taken as  $M_{\text{mess}} = 10^{13}$  GeV. Here,  $\tan\beta = 15$ ,  $m_t(\text{pole}) = 173.34$  GeV and  $\alpha_s(M_Z) = 0.1185$ .

$$\eta_3 = -3 \frac{g_3^2(M_{\text{mess}})}{8\pi^2}. \quad (12)$$

In addition to the above negative contributions, there are positive contributions arising from the wino loop and tree-level Higgs soft mass:

$$(m_{H_u}^2(Q_r))_{\text{pos}} \simeq \frac{k+1}{2} m_{H_u}^2(M_{\text{mess}}) + k_{\tilde{W}} M_{\tilde{W}}^2(M_{\text{mess}}), \quad (13)$$

where we show the only dominant contributions and

$$k_{\tilde{W}} = -\frac{3g_2^2(M_{\text{mess}})}{8\pi^2} t \left[ \frac{1 - \eta_2 t/2}{(1 - \eta_2 t)^2} \right] - \int_0^t dt' \frac{9y_t^2(t') g_2^2(M_{\text{mess}})}{32\pi^4} t' \left[ \frac{1 - \eta_2 t'/2}{(1 - \eta_2 t')^2} \right], \quad \text{with} \\ \eta_2 = \frac{g_2^2(M_{\text{mess}})}{8\pi^2}. \quad (14)$$

The sizes of the coefficients are  $k \sim 0.4$ ,  $k_{\tilde{g}} \sim 0.7$  and  $k_{\tilde{W}} \sim 0.2$ . Therefore in the case of the minimal GMSB with **5** and  $\bar{\mathbf{5}}$  messengers, the negative contributions substantially dominate over the positive contributions, leading to only a small cancellation. One thus needs larger  $M_{\tilde{W}}(M_{\text{mess}})$  and/or  $m_{H_u}^2(M_{\text{mess}})$  to obtain a sizable cancellation leading to small  $m_{H_u}^2$  at the soft mass scale. We will see below how it is achieved in our scenario.

Now we evaluate the value of  $m_{H_u}^2$  at  $m_{\text{SUSY}}$  more precisely by numerically solving two-loop RGEs [23]. By taking  $M_{\text{mess}} = 10^{13}$  GeV,  $\tan\beta = 15$ ,  $m_t(\text{pole}) = 173.34$  GeV and  $\alpha_s(m_Z) = 0.1185$ , we obtain

$$m_{H_u}^2(3 \text{ TeV}) = 0.704m_{H_u}^2 + 0.019m_{H_d}^2 \\ - 0.336m_Q^2 - 0.167m_U^2 - 0.056m_E^2 \\ + 0.055m_L^2 - 0.054m_D^2 \\ + 0.011M_B^2 + 0.192M_W^2 - 0.727M_{\tilde{g}}^2 \\ - 0.003M_{\tilde{B}}M_{\tilde{W}} - 0.062M_{\tilde{W}}M_{\tilde{g}} \\ - 0.010M_{\tilde{B}}M_{\tilde{g}}, \quad (15)$$

where soft SUSY breaking mass parameters in the right-hand side of Eq. (15) are defined at  $M_{\text{mess}}$ . By using Eqs. (4) and (5), we obtain ( $r_3 \equiv \Lambda_3/\Lambda_8$ )

$$m_{H_u}^2(3 \text{ TeV}) \simeq [0.165r_3^2 - 0.035r_3 - 1.222]M_{\tilde{g}}^2. \quad (16)$$

Note that  $m_{H_u}^2(3 \text{ TeV})$  nearly vanishes for  $r_3 \simeq 2.8, -2.6$ , i.e. we reach a focus point region. Here, we have neglected the contribution from the universal scalar mass. In fact, this contribution is rather small as  $m_{H_u}^2(3 \text{ TeV}) \ni 0.164m_0^2$ . Here we make a crucial observation that the ratio  $r_3 \simeq \lambda_3 M_8/(\lambda_8 M_3)$  is RGE invariant:

$$\lambda_{(3,8)}(t) = \lambda_{(3,8)}(t_0) \exp \left[ \int_{t_0}^t dt' (\gamma_Z + 2\gamma_{\Sigma(3,8)}) \right], \\ M_{(3,8)}(t) = M_{(3,8)}(t_0) \exp \left[ \int_{t_0}^t dt' (2\gamma_{\Sigma(3,8)}) \right], \quad (17)$$

where  $\gamma_i$  is the anomalous dimension of the field  $i$ . It immediately follows that

$$\frac{\lambda_3(t)M_8(t)}{\lambda_8(t)M_3(t)} = \frac{\lambda_3(t_0)M_8(t_0)}{\lambda_8(t_0)M_3(t_0)}. \quad (18)$$

Once the ratio  $r_3$  is fixed by some fundamental physics, it is stable against radiative corrections, i.e. invariant under RGE running. This unique property lends significant reliability and robustness to our scenario over other competitive focus point SUSY models.

Now, we estimate the fine-tuning of the EWSB scale using the following measure [24]:

$$\Delta = \max\{|\Delta_a|\}, \\ \Delta_a = \left[ \frac{\partial \ln v}{\partial \ln |F_Z|}, \frac{\partial \ln v}{\partial \ln \mu}, \frac{\partial \ln v}{\partial \ln B_0}, \frac{\partial \ln v}{\partial \ln M_1}, \frac{\partial \ln v}{\partial \ln m_0} \right]_{v=v_{\text{obs}}}, \quad (19)$$

where  $v_{\text{obs}} \simeq 174.1$  GeV and  $B_0$  is the scalar potential  $B$ -term at the messenger scale, which may, for example, be generated by the Giudice–Masiero mechanism [25] or from a constant term in the superpotential.

In Fig. 2, we show the contours of  $\Delta$  and  $m_h$ . The Higgs boson mass is calculated using FeynHiggs 2.10.3 [26], and  $\Delta$  is evaluated utilizing SOFTSUSY 3.6.1 [27]. To avoid the tachyonic stau, we take the universal scalar masses at  $M_{\text{mess}}$  as<sup>5</sup>

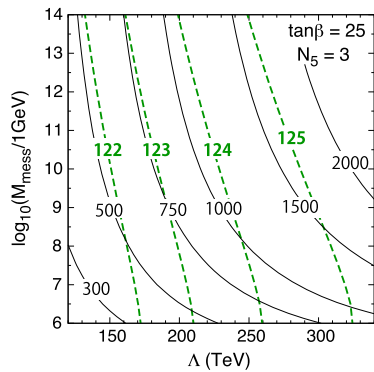
$$m_0(M_{\text{mess}}) = \left( \frac{\Lambda_8}{180 \text{ TeV}} \right) 500 \text{ GeV}. \quad (20)$$

Also, the bino mass is regarded as an input parameter at  $M_{\text{mess}}$ , and taken as  $M_{\tilde{B}}(M_{\text{mess}}) = 250$  GeV. The sign of the  $\mu$ -parameter is taken to be positive. In the gray region with large  $|r_3|$ , the

<sup>5</sup> Strictly speaking, the universal scalar masses should be taken at  $M_{\text{str}}$ . However, it makes only a small difference.

**Table 1**  
Sample mass spectra. We take  $M_{\text{mess}} = 10^{13}$  GeV.

P1		P2		P3	
$\Lambda_8$	180 TeV	$\Lambda_8$	280 TeV	$\Lambda_8$	230 TeV
$r_3$	2.8	$r_3$	8/3	$r_3$	−2.55
$\tan \beta$	15	$\tan \beta$	15	$\tan \beta$	15
$M_1(M_{\text{mess}})$	250 GeV	$M_1(M_{\text{mess}})$	250 GeV	$M_1(M_{\text{mess}})$	250 GeV
$m_0(M_{\text{mess}})$	450 GeV	$m_0(M_{\text{mess}})$	700 GeV	$m_0(M_{\text{mess}})$	600 GeV
$m_h$	123.1 GeV	$m_h$	125.1 GeV	$m_h$	123.0 GeV
$\Delta$	69	$\Delta$	156	$\Delta$	91
$ \Delta_\mu $	61	$ \Delta_\mu $	156	$ \Delta_\mu $	91
$\mu$	538 GeV	$\mu$	850 GeV	$\mu$	652 GeV
$m_{\text{gluino}}$	3.6 TeV	$m_{\text{gluino}}$	5.4 TeV	$m_{\text{gluino}}$	4.5 TeV
$m_{\text{squark}}$	3.4–4.5 TeV	$m_{\text{squark}}$	5.1–6.7 TeV	$m_{\text{squark}}$	4.2–5.5 TeV
$m_{\text{stop}}$	2.2, 4.1 TeV	$m_{\text{stop}}$	3.4, 6.2 TeV	$m_{\text{stop}}$	3.1, 5.1 TeV
$m_{\tilde{e}_L}(m_{\tilde{\mu}_L})$	3.1 TeV	$m_{\tilde{e}_L}(m_{\tilde{\mu}_L})$	4.5 TeV	$m_{\tilde{e}_L}(m_{\tilde{\mu}_L})$	3.6 TeV
$m_{\tilde{e}_R}(m_{\tilde{\mu}_R})$	473 GeV	$m_{\tilde{e}_R}(m_{\tilde{\mu}_R})$	727 GeV	$m_{\tilde{e}_R}(m_{\tilde{\mu}_R})$	618 GeV
$m_{\tilde{\tau}_1}$	221 GeV	$m_{\tilde{\tau}_1}$	399 GeV	$m_{\tilde{\tau}_1}$	394 GeV
$m_{\chi_1^0}$	128 GeV	$m_{\chi_1^0}$	124 GeV	$m_{\chi_1^0}$	131 GeV
$m_{\chi_1^\pm}$	550 GeV	$m_{\chi_1^\pm}$	870 GeV	$m_{\chi_1^\pm}$	670 GeV
$m_{\chi_2^\pm}$	2.6 TeV	$m_{\chi_2^\pm}$	3.8 TeV	$m_{\chi_2^\pm}$	3.1 TeV



**Fig. 3.** Contours of  $\Delta$  (solid) and  $m_h/\text{GeV}$  (dashed) in minimal GMSB (i.e. with 5 and  $\bar{5}$  messengers of SU(5) GUT). We set  $\tan \beta = 25$  and  $N_5 = 3$ . Other parameters are same as in Fig. 2.

EWSB does not occur. One can see the observed Higgs boson mass is explained with  $\Delta = 60\text{--}150$  for  $r_3 \sim 2.8$  (Fig. 2, left panel). When  $r_3$  is negative (Fig. 2, right-panel), the required fine-tuning to reach the correct Higgs boson mass is slightly larger than the positive  $r_3$  case. These results can be compared to the minimal GMSB case (with only 5 and  $\bar{5}$  messengers), shown in Fig. 3. Demanding  $m_h > 123$  GeV, the required  $\Delta$  is around 750–1500 for  $M_{\text{mess}} \gtrsim 10^9$  GeV. For this plot we have taken the number of 5 and  $\bar{5}$  pairs to be  $N_5 = 3$ , though the required  $\Delta$  does not significantly depend on this choice. Comparing Fig. 3 with Fig. 2 it is clear that our adjoint messenger model in the focus point region for  $r_3 \sim 2.8$  is significantly less tuned than minimal GMSB.

Finally we show some sample spectrum in Table 1. One can see that the stau can be light as 200–400 GeV, which may be testable at the LHC depending on the bino and Higgsino masses. Admittedly, the bino-like lightest neutralino may give rise to too large relic density causing over-closure of the universe. This can be avoided by tuning on a tiny amount of  $R$ -parity violation. In this case, axion could become a potential dark matter candidate.

#### 4. Conclusions

We have considered a gauge mediated SUSY breaking scenario with messengers transforming in the adjoint representation of the gauge group as color octet and weak triplet. We have shown that focus point exists in this framework. The fine-tuning of the

EWSB scale is considerably reduced in the focus point region:  $\Delta = 60\text{--}150$ , while explaining the observed Higgs boson mass around 125 GeV. In fact, the fine-tuning is considerably reduced in our scenario compared to that in minimal gauge mediation. Two distinctive features attribute a substantial credibility to our scenario: (i) a single combination of messenger sector parameters, which is RGE invariant, controls the focus point. This means that the focus point behavior is stable once a more fundamental theory fixes that combination; (ii) the special feature of color octet and weak triplet adjoint messengers triggering late gauge unification renders consistency of the scenario with colored Higgs mediated proton decay constraints.

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