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# Simultaneous state and actuator fault estimation for satellite attitude control systems



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**Abstract** In this paper, a new nonlinear augmented observer is proposed and applied to satellite attitude control systems. The observer can estimate system state and actuator fault simultaneously. It can enhance the performances of rapidly-varying faults estimation. Only original system matrices are adopted in the parameter design. The considered faults can be unbounded, and the proposed augmented observer can estimate a large class of faults. Systems without disturbances and the fault whose finite times derivatives are zero piecewise are initially considered, followed by a discussion of a general situation where the system is subject to disturbances and the finite times derivatives of the faults are not null but bounded. For the considered nonlinear system, convergence conditions of the observer are provided and the stability analysis is performed using Lyapunov direct method. Then a feasible algorithm is explored to compute the observer parameters using linear matrix inequalities (LMIs). Finally, the effectiveness of the proposed approach is illustrated by considering an example of a closed-loop satellite attitude control system. The simulation results show satisfactory performance in estimating states and actuator faults. It also shows that multiple faults can be estimated successfully.

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## 1. Introduction

A satellite attitude control system is an essential subsystem for accomplishing successful space missions. Due to the increasing requirement for high safety and reliability, fault diagnosis for satellite attitude control systems has been an important

research topic. Fruitful results can be found in many researches.<sup>1–3</sup>

During the last two decades, model-based fault diagnosis techniques have been widely researched and applied in modern systems.<sup>4,5</sup> Generally speaking, model-based fault diagnosis strategy performs three essential tasks: fault detection, fault isolation and fault estimation.<sup>6,7</sup> Fault estimation is the superior lever of the three tasks. Accurate fault estimation implies that it not only detects and isolates the fault automatically, but also provides details of the fault, such as the size and time-varying behavior of the fault. Besides, once a fault is determined, fault tolerant control can be adopted to compensate for it, which requires a simultaneous state and fault estimation.<sup>8,9</sup> Thus, state estimation observers that can provide the

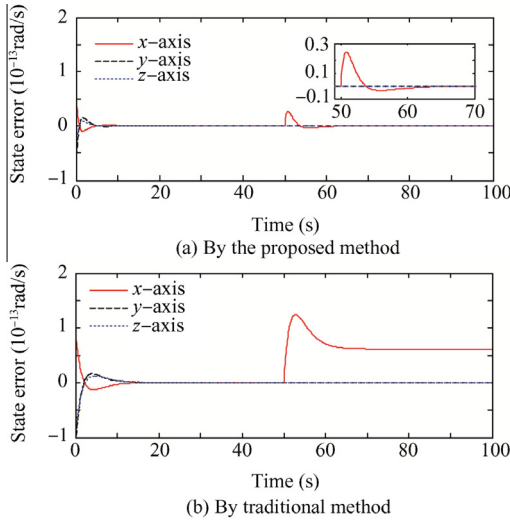
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**Fig. 1** State estimation errors in **Case 1** by different methods.

required state and fault information within one design have attracted a lot of attention.

Much research effort has been devoted in this area and fruitful results have been published. To mention a few, proportional multi-integral observers were designed in Refs.<sup>10,11</sup> to achieve fault estimation for linear and nonlinear descriptor system. In Refs.<sup>12,13</sup>, actuator fault estimation based on neural network was considered. In Refs.<sup>14,15</sup>, adaptive observer technique has been used to estimate fault. In Refs.<sup>16,17</sup>, fault estimation is investigated by sliding mode observers.

Among various approaches developed in the past, the augmented observer has attracted increasing attention due to its simplicity and the potential for simultaneously estimate system states and faults. The main idea of this kind of observer lies in addressing the faults as additional state variables. Accordingly, a variety of important results have been reported in the literature. For example, the actuator fault estimation based on augmented observer has been addressed in Refs.<sup>18,19</sup> for linear time invariant systems, and in Ref.<sup>20</sup> for linear parameter-varying systems. Fault diagnosis using augmented observer for rotor systems and satellite attitude control systems have been investigated in Refs.<sup>21,22</sup> and Refs.<sup>23,24</sup>, respectively. In Ref.<sup>25</sup>, a nonlinear augmented observer is designed and applied to a quadrotor aircraft. There are also much literature which can be viewed as the transformations of the augmented observers, such as Refs.<sup>26,27</sup>. However, the traditional augmented observer is conservative as the faults are assumed to be slowly-varying. In this situation, the constant fault estimation is guaranteed to be unbiased, but it fails to deal with the rapidly-varying fault. Besides, systematic and convenient approaches for the design of nonlinear augmented observers remain lacking in the available literature.

Inspired by the research problems above, in this paper, a nonlinear augmented observer is designed and applied to satellite attitude control systems. Unlike in Refs.<sup>23,24</sup>, the Takagi-Sugeno fuzzy model is used to linearise the satellite attitude dynamics or only slowly-varying fault is considered. The augmented observer proposed in this paper can handle the estimation problem for a large class of actuator faults. Moreover,

no equivalent transformations are needed for obtaining this observer. Our design uses only original coefficient matrices, thus the observer is convenient and reliable in computations.

In summary, the main contributions of this paper are as follows: (1) a new nonlinear augmented observer with a novel structure is proposed to estimate states and actuator faults for satellite attitude control systems; (2) the observer parameters can be computed directly using linear matrix inequalities (LMIs) with original coefficient matrices; (3) multiple rapidly-varying faults can be estimated within one design.

The rest of this paper is organized as follows. Section 2 briefly describes problem statement. In Sections 3 and 4, the design of the augmented observers is developed in detail for two cases, respectively. Section 3 concerns with the ideal case in which the finite times derivatives of the faults is assumed to be zero piecewise. Section 4 deals with the general case that the finite times derivatives of the faults is not null but bounded and disturbances cannot be neglected. Simulations are provided in Section 5 via an example of a satellite attitude control system. Conclusions are drawn in Section 6.

**Notation.** The notation used in the present paper is fairly standard.  $\mathbf{R}^n$  denotes the  $n$ -dimensional Euclidean space, and  $\mathbf{R}^{n \times m}$  is the set of all real matrices of dimension  $n \times m$ .  $\mathbf{P} > \mathbf{0}$  means that  $\mathbf{P}$  is real symmetric and positive definite.  $\|\cdot\|$  stands for the usual  $L_2$  norm.  $\lambda_{\max}(\mathbf{X})$  and  $\lambda_{\min}(\mathbf{X})$  denote the maximum and minimum eigenvalues of  $\mathbf{X}$ . The symmetric terms in a symmetric matrix are denoted by “\*”.

## 2. Problem formulation

Consider a nonlinear dynamic system with actuator fault as

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \boldsymbol{\Phi}(\mathbf{x}) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{d}(t) + \mathbf{L}\mathbf{f}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \end{cases} \quad (1)$$

where  $\mathbf{x}(t) \in \mathbf{R}^n$  is the system state vector;  $\mathbf{u}(t) \in \mathbf{R}^m$  and  $\mathbf{y}(t) \in \mathbf{R}^p$  are the input and the output vectors, respectively;  $\mathbf{d}(t) \in \mathbf{R}^l$  is the unknown disturbance vector and it is assumed to be  $L_2$  norm bounded;  $\mathbf{f}(t) \in \mathbf{R}^k$  is the unknown vector that represents all possible actuator faults;  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\mathbf{E}$  and  $\mathbf{L}$  are known constant real matrices of appropriate dimensions, and the pair  $(\mathbf{A}, \mathbf{C})$  is observable; the nonlinear vector function  $\boldsymbol{\Phi}(\mathbf{x})$  is assumed to be Lipschitz nonlinear with a Lipschitz constant  $\gamma$ , i.e.,

$$\|\boldsymbol{\Phi}(\mathbf{x}) - \boldsymbol{\Phi}(\hat{\mathbf{x}})\| \leq \gamma \|\mathbf{x} - \hat{\mathbf{x}}\| \quad (2)$$

where  $\hat{\mathbf{x}}$  is the estimation of  $\mathbf{x}$ .

In this paper, our goal is to develop a new augmented observer to estimate system states and fault simultaneously. And then an effective way to calculate the design parameters is given. First, Section 3 discusses an augmented observer for an ideal case in which system disturbances are neglected and  $\mathbf{f}(t)$  is assumed to be in a general form as follows:

$$\mathbf{f}(t) = \mathbf{F}_0 + \mathbf{F}_1 t + \mathbf{F}_2 t^2 + \dots + \mathbf{F}_{q-1} t^{q-1} \quad (3)$$

where  $\mathbf{F}_i$  ( $i = 0, 1, \dots, q-1$ ) are unknown constant vectors. One can see that the  $q$ th derivative of  $\mathbf{f}(t)$  with respect to time is zero (i.e.,  $\mathbf{f}^{(q)} = \mathbf{0}$ ). And then, Section 4 discusses a robust augmented observer for a more general case in which the system is subjected to disturbances and  $\mathbf{f}^{(q)}$  is not null but bounded. One can see that the fault considered in this paper may be unbounded.

It is worth noting that the fault in the form of Eq. (3) can describe a large class of faults.<sup>26,27</sup> For instance, constant faults correspond to Eq. (3) with  $q = 1$  and ramp-wise faults correspond to Eq. (3) with  $q = 2$ . Actually, since  $f^{(q)}$  is required to be bounded in Section 4, lots of faults can be described in the form of Eq. (3) using Taylor expansion. Thus, without loss of generality, we take Eq. (3) to express the considered fault.

### 3. Augmented observer design: the ideal case

Consider a nonlinear dynamic system without disturbance in the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + \Phi(x) + Bu(t) + Lf(t) \\ y(t) = Cx(t) \end{cases} \quad (4)$$

Letting

$$\xi_i = f^{(i)} \quad (i = 1, 2, \dots, q-1) \quad (5)$$

and using  $f^{(q)} = \mathbf{0}$ , an augmented system can be constructed as

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{\Phi}(x) + \bar{B}u(t) \\ y(t) = \bar{C}\bar{x}(t) \end{cases} \quad (6)$$

where

$$\begin{cases} \bar{x} = \begin{bmatrix} x^T, \xi_1^T, \dots, \xi_{q-1}^T \end{bmatrix} \in \mathbf{R}^{\bar{n}} \\ \bar{A} = \begin{bmatrix} A & L & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & I \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \in \mathbf{R}^{\bar{n} \times \bar{n}} \\ \bar{B} = [B^T, \mathbf{0}, \dots, \mathbf{0}]^T \in \mathbf{R}^{\bar{n} \times m} \\ \bar{C} = [C, \mathbf{0}, \dots, \mathbf{0}] \in \mathbf{R}^{p \times \bar{n}} \\ \bar{\Phi}(x) = [\Phi^T(x), \mathbf{0}, \dots, \mathbf{0}]^T \in \mathbf{R}^{\bar{n}} \end{cases}$$

and  $\bar{n} = n + kq$ .

According to the augmented system above, the observer can be constructed as

$$\begin{cases} \dot{\bar{z}}(t) = F\bar{z}(t) + T\bar{B}u(t) + T\bar{\Phi}(\hat{x}) + Gy(t) \\ \hat{x}(t) = \bar{z}(t) + Ny(t) \end{cases} \quad (7)$$

where  $\bar{z}(t) \in \mathbf{R}^{\bar{n}}$  is the state vector of the designed observer;  $\hat{x}(t) \in \mathbf{R}^{\bar{n}}$  is the estimation of the augmented system state vector;  $\hat{x}(t) = [I_n, \mathbf{0}_{n \times kq}]^T \hat{x}(t) \in \mathbf{R}^n$  is the estimation of the original system state;  $F$ ,  $T$ ,  $G$  and  $N$  are the matrices to be designed for the observer, which are required to satisfy Eqs. (8) and (9).

$$T = I - N\bar{C} \quad (8)$$

$$F = T\bar{A} - G\bar{C} \quad (9)$$

The augmented state estimation error can be defined as  $\bar{e}(t) = \bar{x}(t) - \hat{x}(t)$ . The following main concern is to design an observer such that  $\lim_{t \rightarrow \infty} \bar{e} = \mathbf{0}$ , that is,  $\lim_{t \rightarrow \infty} e_x = \mathbf{0}$  and  $\lim_{t \rightarrow \infty} e_f = \mathbf{0}$ , where  $e_x = x - \hat{x}$  and  $e_f = f - \hat{f}$  are the state estimation error and fault estimation error, respectively, with

$\hat{f}(t) = [\mathbf{0}_n, I_k, \mathbf{0}_{n \times k(q-1)}]^T \hat{x}(t) \in \mathbf{R}^k$  the estimation of fault vector.

**Theorem 1.** For the given constant  $\gamma$ , if there exist matrices  $P > \mathbf{0}$  and  $F$  such that the following condition holds:

$$A = F^T P + PF + \gamma^2 PTT^T P + I < \mathbf{0} \quad (10)$$

then the observer in the form of Eq. (7) is asymptotically stable and the estimated error of state and fault converges exponentially to zero.

**Proof.** According to the system Eq. (6) and observer Eq. (7), the dynamics of the augmented state error can be derived as

$$\begin{aligned} \dot{\bar{e}}(t) &= \dot{\bar{x}}(t) - \dot{\hat{x}}(t) = T\dot{\bar{x}}(t) - \dot{\bar{z}}(t) \\ &= F\bar{e}(t) + T(\bar{\Phi}(x) - \bar{\Phi}(\hat{x})) \end{aligned} \quad (11)$$

Choose the following Lyapunov function:

$$V_1(t) = \bar{e}^T(t) P \bar{e}(t) \quad (12)$$

The time derivative of it reads

$$\begin{aligned} \dot{V}_1(t) &= \dot{\bar{e}}^T(t) P \bar{e}(t) + \bar{e}^T(t) P \dot{\bar{e}}(t) \\ &= \bar{e}^T(t) (F^T P + PF) \bar{e}(t) + 2\bar{e}^T(t) P T (\bar{\Phi}(x) - \bar{\Phi}(\hat{x})) \end{aligned} \quad (13)$$

Considering that  $\|\bar{\Phi}(x) - \bar{\Phi}(\hat{x})\| \leq \gamma \|\bar{x} - \hat{x}\| = \gamma \|\bar{e}\|$ , we obtain

$$\begin{aligned} \dot{V}_1(t) &\leq \bar{e}^T(t) (F^T P + PF) \bar{e}(t) + \gamma^2 \bar{e}^T(t) P T T^T P \bar{e}(t) + \bar{e}^T(t) \bar{e}(t) \\ &= \bar{e}^T(t) A \bar{e}(t) \end{aligned} \quad (14)$$

It follows that

$$\dot{V}_1(t) \leq -\|\bar{e}\|^2 \lambda_{\min}(-A) \quad (15)$$

Thus, the augmented observer ensures that  $\bar{e}(t) \rightarrow \mathbf{0}$  as  $t \rightarrow \infty$ .

On the other hand, the Lyapunov function satisfies that

$$\dot{V}_1(t) \leq \lambda_{\max}(P) \|\bar{e}\|^2 \quad (16)$$

Thus, one has

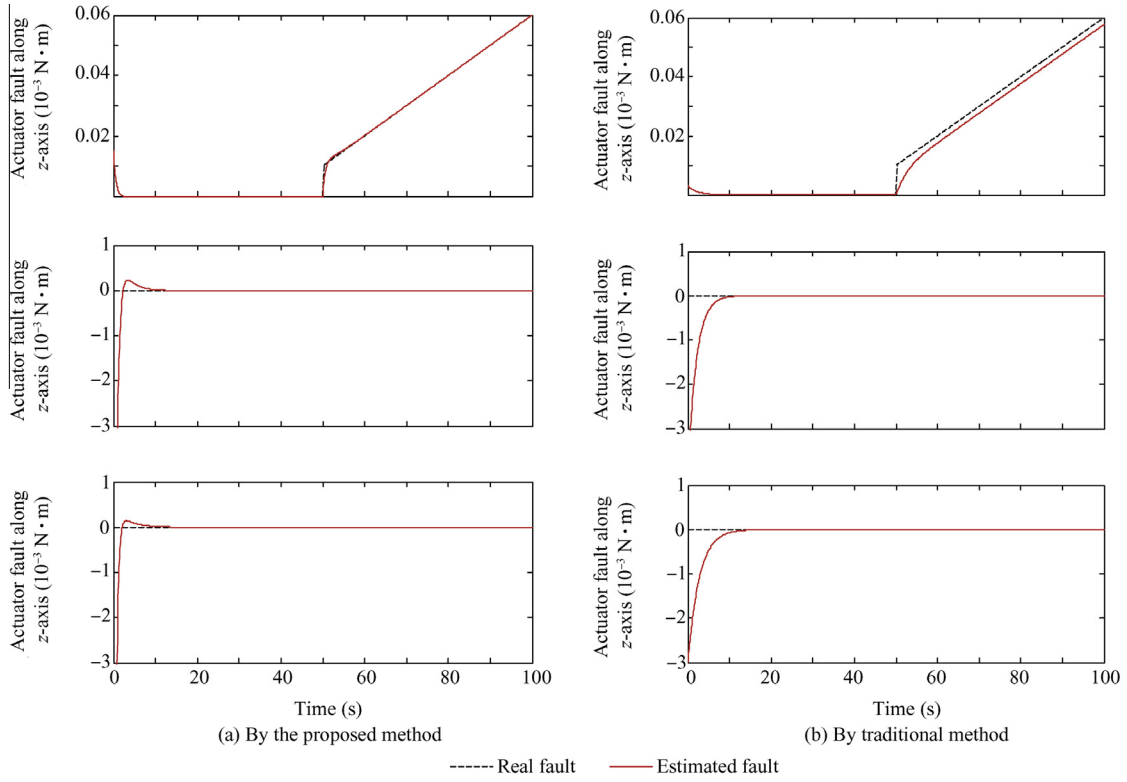
$$\frac{-\lambda_{\min}(-A) \|\bar{e}\|^2 V_1(t)}{\lambda_{\max}(P) \|\bar{e}\|^2} \geq -\lambda_{\min}(-A) \|\bar{e}\|^2 \quad (17)$$

Substituting Eq. (15) into Eq. (17) gives

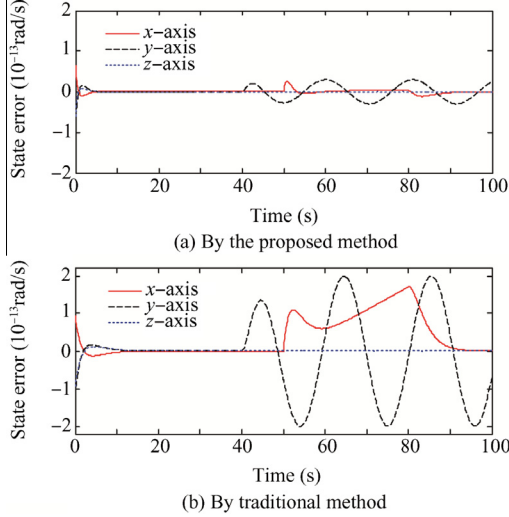
$$\dot{V}_1(t) \leq \frac{-\lambda_{\min}(-A)}{\lambda_{\max}(P)} V_1(t) \quad (18)$$

Integrating Eq. (18), one can obtain  $V_1(t) \leq V_1(0) \cdot \exp(-\mu t)$  where  $\mu = \frac{\lambda_{\min}(-A)}{\lambda_{\max}(P)} > 0$ .

Since  $\lambda_{\min}(P) \|\bar{e}\|^2 \leq V_1(t)$ , we have



**Fig. 2** Faults and their estimates in Case 1 by different methods.



**Fig. 3** State estimation errors in Case 2 by different methods.

$$\begin{aligned} \|\bar{\mathbf{e}}(t)\| &\leq \sqrt{\frac{V_1(0)}{\lambda_{\min}(\mathbf{P})}} \exp\left(-\frac{\mu}{2}t\right) \\ &\leq \sqrt{\frac{\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{P})}} \|\bar{\mathbf{e}}(0)\| \exp\left(-\frac{\mu}{2}t\right) \end{aligned} \quad (19)$$

Therefore, the augmented observer is asymptotically stable and the estimated error of state and fault converges to zero exponentially at a rate greater than  $\exp\left(-\frac{\mu}{2}t\right)$ .  $\square$

#### 4. Robust augmented observer design: the general case

In this section, the general case is considered. The nonlinear dynamic system is subject to disturbances and  $\mathbf{f}^{(q)}$  is not zero but assumed to be bounded.

For the system given in Eq. (1), let  $\xi_i = \mathbf{f}^{(i)}$  ( $i = 1, 2, \dots, q-1$ ) and define the augmented state vector as in Eq. (6), then we have an augmented system as

$$\begin{cases} \dot{\bar{\mathbf{x}}}(t) = \bar{\mathbf{A}}\bar{\mathbf{x}}(t) + \bar{\Phi}(\bar{\mathbf{x}}) + \bar{\mathbf{B}}\mathbf{u}(t) + \bar{\mathbf{E}}\mathbf{d}(t) + \bar{\mathbf{Q}}\mathbf{f}^{(q)}(t) \\ \mathbf{y}(t) = \bar{\mathbf{C}}\bar{\mathbf{x}}(t) \end{cases} \quad (20)$$

where

$$\begin{cases} \bar{\mathbf{E}} = [\mathbf{E}^T, \mathbf{0}, \dots, \mathbf{0}]^T \in \mathbf{R}^{\bar{n} \times l} \\ \bar{\mathbf{Q}} = [\mathbf{0}, \dots, \mathbf{0}, \mathbf{I}]^T \in \mathbf{R}^{\bar{n} \times k} \end{cases}$$

and the other symbols are the same as those defined in Eq. (6).

**Theorem 2.** For the given constant  $\gamma$  and  $\delta$ , if there exist matrices  $\mathbf{P} > \mathbf{0}$  and  $\mathbf{F}$  such that the following condition holds:

$$\begin{cases} \bar{\mathbf{E}} = \begin{bmatrix} \mathbf{A} + \mathbf{I} & \mathbf{P}\bar{\mathbf{T}}\bar{\mathbf{E}} & \mathbf{P}\bar{\mathbf{T}}\bar{\mathbf{Q}} \\ * & -\delta^2\mathbf{I} & \mathbf{0} \\ * & * & -\delta^2\mathbf{I} \end{bmatrix} < \mathbf{0} \\ \mathbf{A} = \mathbf{F}^T\mathbf{P} + \mathbf{P}\mathbf{F} + \gamma^2\mathbf{P}\bar{\mathbf{T}}\bar{\mathbf{T}}^T\mathbf{P} + \mathbf{I} \end{cases} \quad (21)$$

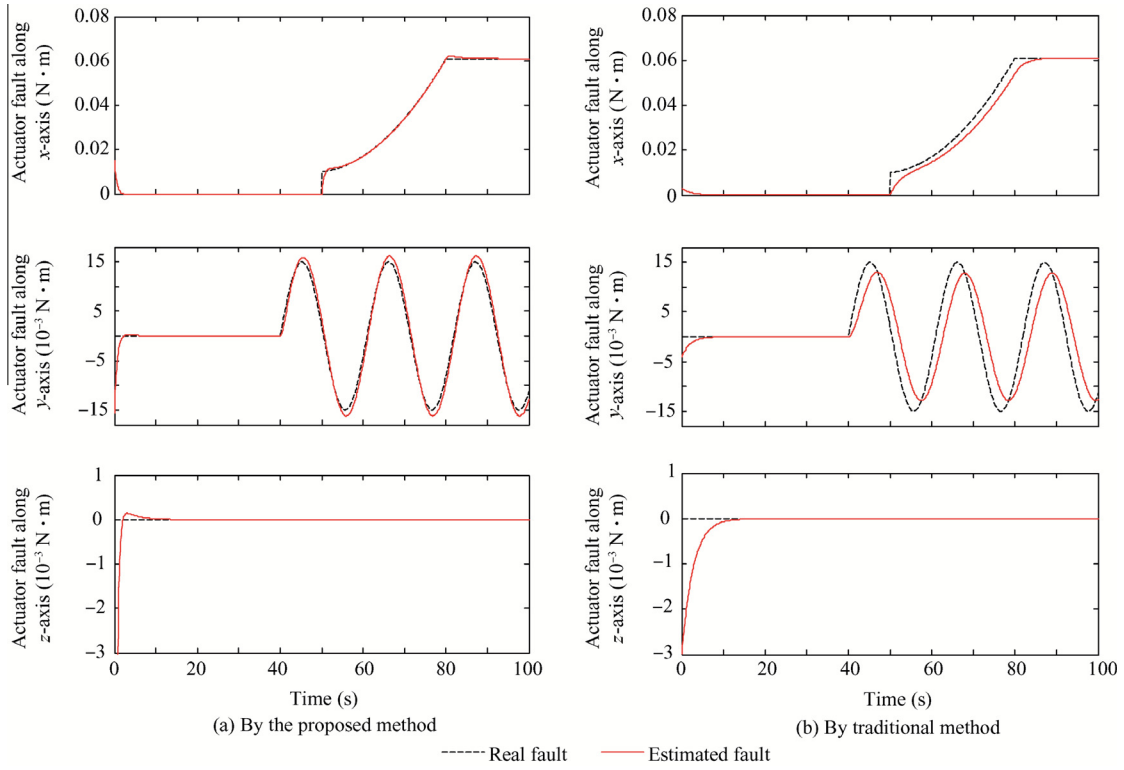


Fig. 4 Faults and their estimates in Case 2 by different methods.

then the observer in the form of Eq. (7) is robustly stable, that is, the estimated error of state and fault is uniformly bounded.

**Proof.** According to the observer Eq. (7) and system Eq. (20), the dynamics of the augmented state error can be derived as

$$\begin{aligned} \dot{\bar{e}}(t) &= \dot{\hat{x}}(t) - \dot{\bar{x}}(t) = \mathbf{T}\dot{\hat{x}}(t) - \dot{\bar{z}}(t) \\ &= \mathbf{F}\bar{e}(t) + \mathbf{T}(\bar{\Phi}(\mathbf{x}) - \bar{\Phi}(\hat{\mathbf{x}})) + \mathbf{T}\mathbf{E}d(t) + \mathbf{T}\bar{\mathbf{Q}}\mathbf{f}^{(q)}(t) \end{aligned} \quad (22)$$

Choose the following Lyapunov function:

$$V_2(t) = \bar{e}^T(t)\mathbf{P}\bar{e}(t) \quad (23)$$

The time derivative of it reads

$$\begin{aligned} \dot{V}_2(t) &= \dot{\bar{e}}^T(t)\mathbf{P}\bar{e}(t) + \bar{e}^T(t)\mathbf{P}\dot{\bar{e}}(t) \\ &= \bar{e}^T(t)(\mathbf{F}^T\mathbf{P} + \mathbf{P}\mathbf{F})\bar{e}(t) + 2\bar{e}^T\mathbf{P}\mathbf{T}(\bar{\Phi}(\mathbf{x}) - \bar{\Phi}(\hat{\mathbf{x}})) \\ &\quad + 2\bar{e}^T\mathbf{P}\mathbf{T}(\bar{\mathbf{E}}d + \bar{\mathbf{Q}}\mathbf{f}^{(q)}) \end{aligned} \quad (24)$$

Considering that  $\|\bar{\Phi}(\mathbf{x}) - \bar{\Phi}(\hat{\mathbf{x}})\| \leq \gamma\|\bar{\mathbf{x}} - \hat{\mathbf{x}}\| = \gamma\|\bar{e}\|$ , we obtain

$$\begin{aligned} \dot{V}_2(t) &\leq \bar{e}^T(t)(\mathbf{F}^T\mathbf{P} + \mathbf{P}\mathbf{F})\bar{e}(t) + \gamma^2\bar{e}^T(t)\mathbf{P}\mathbf{T}\mathbf{T}^T\mathbf{P}\bar{e}(t) \\ &\quad + \bar{e}^T(t)\bar{e}(t) + 2\bar{e}^T\mathbf{P}\mathbf{T}(\bar{\mathbf{E}}d + \bar{\mathbf{Q}}\mathbf{f}^{(q)}) \\ &= \bar{e}^T(t)\Delta\bar{e}(t) + 2\bar{e}^T\mathbf{P}\mathbf{T}(\bar{\mathbf{E}}d + \bar{\mathbf{Q}}\mathbf{f}^{(q)}) \end{aligned} \quad (25)$$

Define

$$J = \int_0^T (\dot{V}_2(t) + \bar{e}^T(t)\bar{e}(t) - \delta^2\mathbf{v}^T(t)\mathbf{v}(t))dt \quad (26)$$

where  $T > 0$  is the integral time and  $\mathbf{v}^T(\mathbf{x}, \mathbf{u}, t) = [\mathbf{d}^T(t), (\mathbf{f}^{(q)})^T]^T$  represents uncertainties vector.

Then it is clear that

$$\begin{aligned} J &\leq \int_0^T [\bar{e}^T(t)\Delta\bar{e}(t) + 2\bar{e}^T\mathbf{P}\mathbf{T}(\bar{\mathbf{E}}d + \bar{\mathbf{Q}}\mathbf{f}^{(q)}) + \bar{e}^T(t)\bar{e}(t) \\ &\quad - \delta^2(\mathbf{d}^T(t)d(t) + (\mathbf{f}^{(q)})^T\mathbf{f}^{(q)})] dt \\ &= \int_0^T (\xi^T(t)\Xi\xi(t)) dt \leq \int_0^T (-\|\xi\|^2\lambda_{\min}(-\Xi)) dt \end{aligned} \quad (27)$$

where  $\xi^T(\mathbf{x}, \mathbf{u}, t) = [\bar{e}^T(t), \mathbf{d}^T(t), (\mathbf{f}^{(q)})^T]^T$ .

Under the zero initial condition, we have

$$\int_0^T (\bar{e}^T(t)\bar{e}(t) - \delta^2\mathbf{v}^T(t)\mathbf{v}(t)) dt = J - V_2(T) < 0 \quad (28)$$

Therefore,

$$\int_0^T \bar{e}^T(t)\bar{e}(t) dt < \delta^2 \int_0^T \mathbf{v}^T(t)\mathbf{v}(t) dt \quad (29)$$

□

**Remark 1.** If there are no disturbances and  $\mathbf{f}^{(q)} = \mathbf{0}$ , Eq. (25) reduces to Eq. (14). Since the matrix  $\Delta$  is a negative matrix according to Schur Complement Lemma, one can see that it is just the result which has been addressed in the ideal case. Thus, the observer designed in the ideal case serves as a particular case of robust augmented observer design.



**Remark 2.** To obtain the augmented observer discussed, how to calculate the corresponding matrices  $F$ ,  $T$ ,  $G$  and  $N$  is an important problem. In the following section, the solution of the above theorem is achieved by transferring inequality (21) to an LMI with the required transformation. Thus, the problem can be solved easily from the standard scientific computing software.

According to Schur Complement Lemma,  $\Xi < 0$  in inequality (21) can be rewritten into the following matrix inequality form:

$$\begin{cases} \Xi = \begin{bmatrix} \Lambda & \gamma PT & PTE & PT\bar{Q} \\ * & -I & \mathbf{0} & \mathbf{0} \\ * & * & -\delta^2 I & \mathbf{0} \\ * & * & * & -\delta^2 I \end{bmatrix} < \mathbf{0} \\ \Lambda = F^T P + PF + 2I \end{cases} \quad (30)$$

Substituting  $T = I - N\bar{C}$  and  $F = \bar{A} - N\bar{C}\bar{A} - G\bar{C}$  into Eq. (30), and letting  $X = PN$  and  $Y = PG$ , we have

$$\begin{cases} \Xi = \begin{bmatrix} \Lambda & \gamma P - \gamma X\bar{C} & P\bar{E} - X\bar{C}\bar{E} & P\bar{Q} - X\bar{C}\bar{Q} \\ * & -I & \mathbf{0} & \mathbf{0} \\ * & * & -\delta^2 I & \mathbf{0} \\ * & * & * & -\delta^2 I \end{bmatrix} < \mathbf{0} \\ \Lambda = \bar{A}^T P + P\bar{A} - \bar{A}^T \bar{C}^T X^T - X\bar{C}\bar{A} - \bar{C}^T Y^T - Y\bar{C} + 2I \end{cases} \quad (31)$$

This matrix inequality can be solved by using MATLAB LMI toolbox with  $X$ ,  $Y$  and  $P$  as the matrix variables. Once  $X$ ,  $Y$  and  $P$  are obtained, one can get  $N = XP^{-1}$  and  $G = YP^{-1}$ . Furthermore,  $T$  and  $F$  can be determined by using Eqs. (8) and (9) and then the observer is obtained.

**Remark 3.** In order to compare the proposed method with the traditional method, a system without disturbance is considered and the fault is assumed to be in the form of Eq. (3).

Generally speaking, constant fault is only considered in traditional augmented observer. That is, it is assumed that  $f(t) = \mathbf{0}$ . The corresponding algorithm can be viewed as our designed observer Eq. (7) for the following system.

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{\Phi}(x) + \bar{B}u(t) + \bar{Q}f^{(1)}(t) \\ y(t) = \bar{C}\bar{x}(t) \end{cases} \quad (32)$$

where  $\bar{x} = [x^T, f^T]^T \in \mathbf{R}^{\bar{n}}$ ;  $\bar{\Phi}(x) = [\Phi^T(x), \mathbf{0}]^T \in \mathbf{R}^{\bar{n}}$ ;  $\bar{B} = [B^T, \mathbf{0}]^T \in \mathbf{R}^{\bar{n} \times m}$ ;  $\bar{C} = [C, \mathbf{0}] \in \mathbf{R}^{\bar{n} \times \bar{n}}$ ;  $\bar{n} = n + q$ ;  $\bar{A} = \begin{bmatrix} A & L \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \in \mathbf{R}^{\bar{n} \times \bar{n}}$ ;  $\bar{Q} = [\mathbf{0}, I]^T \in \mathbf{R}^{\bar{n} \times k}$ .

According to the discussion of Section 4, the observer can be designed such that the estimated error of state and fault is uniformly bounded. That is,  $\int_0^T \bar{e}^T(t)\bar{e}(t) dt < \delta^2 \int_0^T v^T(t)v(t) dt$ , where  $v^T(x, u, t) = [\mathbf{0}, (f^{(1)})^T]^T$ .

However, as designed in Section 3, our proposed augmented observer can achieve an unbiased estimation of the state and the fault. **Case 1** in simulation part is carried out to verify the above analysis and show the effectiveness.

## 5. Application

### 5.1. Mathematical model of satellite attitude control system

In this section, the effectiveness of the proposed estimation method is illustrated by considering a satellite attitude control system. The dynamics model with actuator faults can be given in state space formulation as Ref.<sup>28</sup>.

$$\begin{cases} \dot{x}(t) = \Phi(x) + Bu(t) + Ed(t) + Lf(t) \\ y(t) = Cx(t) \end{cases} \quad (33)$$

where  $x = [\omega_x, \omega_y, \omega_z]^T$  is the angular velocity vector;  $u(t) = [u_x, u_y, u_z]^T$  is the control torque of actuator;  $d(t) = [T_{dx}, T_{dy}, T_{dz}]^T$  is the unknown disturbance torque due to the space environment, such as solar radiation;  $f(t) = [f_x(t), f_y(t), f_z(t)]^T$  is the fault vector, with  $f_x(t)$ ,  $f_y(t)$  and  $f_z(t)$  representing the actuator fault of  $x$ -axis,  $y$ -axis and  $z$ -axis, respectively;  $y(t)$  is the measurement of the gyro;  $\Phi(x) = \left[ \frac{I_y - I_z}{I_x} \omega_y \omega_z, \frac{I_z - I_x}{I_y} \omega_z \omega_x, \frac{I_x - I_y}{I_z} \omega_x \omega_y \right]^T$  is the nonlinear term of the satellite dynamics and  $I_x$ ,  $I_y$ ,  $I_z$  represent the principle axis moments of inertia; the corresponding matrices are  $B = L = E = \text{diag}\{1/I_x, 1/I_y, 1/I_z\}$ ,  $C = d = I_{3 \times 3}$ .

The satellite is assumed to be running in a small angle maneuver, which means that the nonlinear vector function  $\Phi(x)$  is locally Lipschitz nonlinear and satisfies the condition Eq. (2). The moment of inertia matrix is assumed as  $I_m = \text{diag}(930, 800, 1070)$  kg · m<sup>2</sup>. The orbital angular velocity  $\omega_0 = 0.001$  rad/s and initial attitude angle velocity  $\omega(0) = [1, 1, 1]^T \times 10^{-5}$  rad/s. The satellite is equipped with three actuators dispatched at each principle axis and the maximum control torque of each actuator is  $u_{\max} = 0.1$  N · m. Since solar radiation is periodic, we assume that the disturbance torque as  $d(t) = [1.4 \times 10^{-5}, 1.4 \times 10^{-5}, 1.4 \times 10^{-5}]^T \times \sin(\omega_0 t)$  N · m.

### 5.2. Simulation results

In this simulation, two fault cases are considered to illustrate the performance of the designed augmented observer. The first case concerns with the single-fault in ideal situation and the second case deals with the multiple-fault in general situation.

**Case 1.** Assume that  $x$ -axis actuator suffers a ramp-wise fault in the following form. That is, friction torque suddenly increases at 50 s and continued to increase.

$$f_x(t) = \begin{cases} 0 & t < 50 \text{ s} \\ 0.001(t - 50) + 0.01 & t \geq 50 \text{ s} \end{cases}$$

It can be seen that the second times derivatives of the fault is zero piecewise. Therefore, a two-step augmentation (the augmented system in the form of Eq. (20) with  $q = 2$ ) is carried out to illustrate the performance of the proposed method.

The curves of the state estimation errors generated by the proposed method are given in Fig. 1(a). In order to show that the proposed method is superior to the conventional method, a traditional augmented observer is also designed. The corresponding simulation results are given in Fig. 1(b). The

trajectories of the faults and their estimates are exhibited in Fig. 2. It can be seen that the tracking performance is desired. From Fig. 1(a), it is shown that the state estimation errors converge to zero. From Fig. 2(a), one can see that the fault is estimated successfully. Therefore not only the  $x$ -axis actuator fault is detected, but also the accurate fault information is provided. It can be seen that the state estimation errors in Fig. 1(b) are bounded but not converge to zero. It is not surprising, because the traditional observer serves as one-step augmented observer. This observer can only achieve the unbiased estimation for the fault whose first times derivatives is zero. The estimation performance shown in Fig. 2(b) is obviously less satisfactory than that shown in Fig. 2(a).

**Case 2.** It is supposed that the  $x$ -axis actuator and  $y$ -axis actuator are prone to faults simultaneously, and  $z$ -axis actuator is fault free. The faults are considered in the following form. Friction torque of  $x$ -axis actuator rapidly increased after 50 s and stabilized at a certain value after 80 s. Friction torque of  $y$ -axis actuator increased periodically after 40 s.

$$f_x(t) = \begin{cases} 0 & t < 50 \text{ s} \\ 5 \times 10^{-5}(t-50)^2 + 2 \times 10^{-4} \\ \quad \times (t-50) + 0.01 & 50 \text{ s} \leq t < 80 \text{ s} \\ 0.061 & t \geq 80 \text{ s} \end{cases}$$

$$f_y(t) = \begin{cases} 0 & t < 40 \text{ s} \\ 1.5 \times 10^{-2} \sin(0.3(t-40)) & t \geq 40 \text{ s} \end{cases}$$

We still use the observer designed in Case 1. The curves of the state estimation errors and the estimated faults by the proposed method and traditional method are shown in Figs. 3 and 4, respectively. Since the disturbances exist and the second times derivatives of the faults are not zero, the state estimation errors in Fig. 3(a) are not zero but bounded. It can be seen from Fig. 4(a) that the two faults are estimated satisfactorily. Therefore, both the faults of  $x$ -axis actuator and  $y$ -axis actuator are detected and identified successfully by our proposed method, which means that the designed augmented observer has the ability to diagnose multiple faults simultaneously. From Fig. 3(b) and Fig. 4(b), it can be seen that the two faults can also be detected and estimated, but the estimation performance is less accurate than the results of our proposed method obviously.

From the above simulation results, it can be concluded that for the ramp-wise fault in Case 1, an unbiased estimation of the fault can be achieved using our proposed method, but the traditional method can only achieve a biased estimation. As for the two rapidly-varying faults in Case 2, estimation of two simultaneous faults can be both achieved using our proposed and traditional methods, but our method can enhance the performances of rapidly-varying faults estimation. Thus, our proposed augmented observer outperforms the traditional augmented observer.

## 6. Conclusions

- (1) In this paper, an augmented observer is presented to simultaneously estimate the states and actuator faults for nonlinear Lipschitz systems. Both of an ideal case

and a more general case are considered with detailed theoretical analyses. The design of the observer only adopts the original coefficient matrices. Based on LMIs techniques, the observer parameters are conveniently computed. Compared with the conventional method, the proposed augmented observer can improve the performances of fault estimation. The effectiveness is illustrated by a satellite attitude control system. It is shown that not only single fault but also multiple rapidly-varying faults can be estimated successfully.

- (2) Further research work includes two aspects. The first one is that although the robust nonlinear augmented observer is designed in this paper, disturbances should be further tackled using perfect or approximate decoupling strategy. Since only dynamics model of satellite attitude is considered, extension of the system model by adding kinematics model has more research significance, which should be investigated to further verify the proposed method.

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