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Eternally accelerating universe without event horizon

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Abstract

Increasing astronomical evidence indicates that the expansion of the universe is accelerating. By simply solving Einstein equations we show in this Letter that a wide class of generic quintessence models leading to eternal acceleration is associated with spacetime static metrics which do not exhibit future event horizons, and therefore do not pose the same problems for string theory as asymptotic de Sitter spaces.

String theory is now considered by most theorists as the fundamental framework to successfully deal with all small-scale phenomena, including quantum-gravity effects. However, it has recently been noted [1,2] that as it comes to the foundations of cosmology—and particularly modern precision cosmology, string theory does not appear to prove as successful. The recent growing astronomical evidence that the expansion of the universe has recently begun to accelerate [3] is actually posing rather stringent challenges to string theory. If the today dominating vacuum dark energy is all absorbed into the form of a cosmological constant (i.e., if the universe is entering an eternal de Sitter phase), then it has been noticed that no de Sitter space backgrounds can be accommodated within the framework of the controllable mathematics of string theory [4,5]. The reason essentially is that in de Sitter space there necessarily is a future event horizon and hence the space will contain regions that are inaccessible to light probes, which does not allow introducing the S-matrix or S-vector description required in string theory [1,4,5].

In addition, it has been also argued [2,6] that in case that the accelerating expansion of the universe be originated by a quintessence, slow-varying field with sufficiently negative pressure $p = \omega\rho$, if the state-equation parameter ω is kept constant in future evolution, the universe will inexorably accelerate forever, eventually showing an event horizon which again leads to the above-alluded problem for string theory. However, while the incompatibility of de Sitter space with string theory appears fully unavoidable, the case for quintessence seems still to offer some room for a peaceful coexistence between string theory and cosmology [7,8]. Actually, whereas the existence of an event horizon has been long proved for de Sitter space, nobody has hitherto mathematically shown (though it has been generally assumed) that an eternally accelerating universe driven by a quintessence field be associated with spacetime metrics that inexorably show event horizons. The aim of this Letter is to investigate whether an eternally accelerating universe would necessarily show a future event horizon. This research will be here carried out by simply obtaining static,

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spherically symmetric solutions of the Einstein equations that correspond to the vacuum dark energy associated with a perfect fluid which is made of a homogeneous, slowly-varying quintessence field for different constant state equations. Our final conclusion is that for the entire range of possible negative pressures (other than $p=-\rho/3$ and $p=-\rho$) which lead to an accelerating universe, there exist static spacetime metrics which do not exhibit event horizons.

We start by introducing a metrical ansatz with Lorentzian signature given by

$$ds^2=-A(r)\,dt^2+B(r)\,dr^2+r^2\,d\Omega_2^2, \tag{1}$$

where $A(r)$ and $B(r)$ are the metric tensor components which depend only on the radial coordinate r, and $d\Omega_2^2$ is the metric on the unit two-sphere. For a generic state equation of the quintessence field $p=\omega\rho$ and spatial curvature $\kappa=+1$, the Einstein equations can be written as [9]

$$8\pi G\omega\rho=\frac{1}{B}\left(\frac{1}{r^2}+\frac{A'}{rA}\right)-\frac{1}{r^2}, \tag{2}$$

$$8\pi G\omega\rho=\frac{1}{4B}\left(\frac{2A''}{A}-\frac{(A')^2}{A^2}-\frac{2B'}{rB}-\frac{A'B'}{AB}+\frac{2A'}{rA}\right), \tag{3}$$

$$8\pi G\rho=-\frac{1}{B}\left(\frac{1}{r^2}-\frac{B'}{rB}\right)+\frac{1}{r^2}, \tag{4}$$

in which the prime denotes derivative with respect to the radial coordinate r. To these equations we have to add [9] the gravitational equations for the energy-momentum tensor, $T^k_{i;k}=0$, which in the present case lead to the general expression for the energy density ρ

$$\alpha\rho=A(r)^{-(1+\omega)/(2\omega)}, \tag{5}$$

with α an arbitrary integration constant. From Eqs. (2) and (4) we can further derive

$$\left(\frac{1+3\omega}{2\omega}\right)\int\frac{dr\,(1-B)}{rB}A^{(1+\omega)/(2\omega)}=\frac{2\pi(\omega-1)}{\alpha}-\frac{A^{(1+\omega)/(2\omega)}}{B}-K_1, \tag{6}$$

where K_1 is an integration constant.

We solve in what follows the above field equations, restricting to cases within the interval of ω-values which lead to an accelerating behaviour for the expanding universe, including the extreme situations $\omega=-1/3$ and $\omega=-1$. The latter case describes the upper end point of quintessence models and can be easily solved (note that all dependence on $A(r)$ in Eqs. (5) and (6) is ruled out for $\omega=-1$) to produce for $K_1=-1$ the well-known de Sitter solution

$$A(r)=B(r)^{-1}=1-\frac{8\pi Gr^2}{3\alpha}, \tag{7}$$

where one can isolate the cosmological constant to be given by $\Lambda=8\pi G/\alpha$. Of course, this corresponds to an eternally accelerating universe which will eventually show an event (cosmological) horizon at $r=\sqrt{3/\Lambda}$ and, therefore, keeps the kind of fundamental physics problems which were alluded previously [1–6]. The other extreme case, $\omega=-1/3$, corresponds to the onset of the accelerating regime for a universe without any ordinary (observable + dark) matter content. It already has been considered for any value of κ by Chernin et al. [10] who denoted quintessence for a constant state equation $\omega=-1/3$ as *Einstein quintessence* because it actually describes an Einstein static universe [11]. A general solution can be in this case readily obtained because the integral term in the right-hand-side of Eq. (6) vanishes for $\omega=-1/3$. Thus, from the Einstein equations we obtain

$$\frac{8\pi Gr^2}{3\alpha}+\frac{1}{AB}+K_1=0,\quad B=c_0\left(\frac{A'r^2}{\sqrt{A}}\right)^2, \tag{8}$$

with c_0 again an integration constant, whose general solution reads

$$A(r)=K_3-\frac{8\pi GK_2}{3K_1\alpha}\left(1+\frac{3K_1\alpha}{8\pi Gr^2}\right)^{1/2}, \tag{9}$$

$$B(r)=-\frac{1}{(K_1+\frac{8\pi Gr^2}{3\alpha})A(r)}, \tag{10}$$

where K_1, K_2 and K_3 are arbitrary constants, unless by the condition $K_1K_3=-1$. This solution possesses a singularity at $r=0$.

In the solution given by the metric tensor components (9) and (10) one can generally distinguish the following three situations.

(i) If $K_1<0, K_3>0$ (for any value of K_2), then there will be an apparent singularity at $r_{\rm h}=\sqrt{3\alpha|K_1|/(8\pi G)}$. This does not represent a conventional event horizon since, while the metric for $r>r_{\rm h}$ has a Kleinian signature $(--++)$, the metric for $r<r_{\rm h}$ is complex.

(ii) If $K_1 > 0, K_2 < 0$ and $K_3 < 0$, there will be a kind of event horizon at

$$r_{\rm h} = \left(\frac{3\alpha K_1 K_3^2}{8\pi G K_2^2} - \frac{8\pi G}{3\alpha K_1}\right)^{-1},$$

which marks the transition from a Kleinian metric for $r < r_{\rm h}$ to an Euclidean (Riemannian) metric for $r > r_{\rm h}$.

(iii) Finally, when $K_1, K_2 > 0$ and $K_3 < 0$ there is no coordinate singularity (no event horizon) and the metric keeps an Euclidean signature everywhere.

Of much greater interest for the aim of the present study are the cases which correspond to an accelerating universe $-1/3 > \omega > -1$ for which cases the deceleration parameter [12]

$$q_0 = \frac{1}{2}\left[\Omega_{\rm M} + \Omega_\phi(1+3\omega)\right] \tag{11}$$

(where $\Omega_{\rm M}$ and Ω_ϕ are the specific energy densities for ordinary (observable + dark) matter and the quintessential field) becomes negative even for moderate $\Omega_{\rm M} > 0$. Because for this regime none of the terms and metric tensor components present in Eq. (6) are canceled, it is very difficult to obtain a general solution of the Einstein equations. None the less, starting with the general expressions

$$\frac{A'}{A} + \frac{B'}{B} = \frac{8\pi G(1+\omega)r}{\alpha} A^{-(1+\omega)/(2\omega)} B, \tag{12}$$

$$B = 1 + \frac{[(A')^{-2\omega/(1+\omega)} A r^2]'}{A(A')^{-(3\omega+1)/(\omega+1)}(A'r + \frac{2(3\omega+1)A}{\omega+1})}, \tag{13}$$

which are obtained from the field equations, one can still obtain some simple solutions for any particular constant values of the quintessential state equation parameter ω, covering the entire range of possible accelerating universes, except the extreme Einstein quintessence ($\omega = -1/3$) and de Sitter ($\omega = -1$) cases. Actually, inserting Eqs. (12) and (13) in the field equations, we can also derive a general differential equation for the metric component $A(r)$ for any constant value of ω within the accelerating regime $-1/3 > \omega > -1$

$$\left(\frac{A' A^{-(1+\omega)/(2\omega)}}{r^{(1+2\omega)/\omega}}\right)' + \left(\frac{A^{(\omega-1)/(2\omega)}}{r^{(1+3\omega)/\omega}}\right)' + \frac{\alpha(A'r^2)'}{8\pi G\omega r^{(1+6\omega)/\omega}} = 0. \tag{14}$$

A simple solution for $A(r)$ for any ω in the accelerating range $-1 < \omega < -1/3$ can now be obtained, and hence using Eq. (13) the corresponding solution for $B(r)$ can also be derived. These metric components can finally be worked out to read:

$$A(r) = K r^{4\omega/(1+\omega)}, \tag{15}$$

$$B = \frac{\omega^2 + 6\omega + 1}{(1+\omega)^2}, \tag{16}$$

where the component $B \equiv B(\omega)$ is a constant which is negative definite for all ω in the whole accelerating range $-1 < \omega < -1/3$. The parameter K appearing in Eq. (15) is also a constant coefficient which depends on ω and the quintessence vacuum energy density as follows:

$$K \equiv K(\omega, \alpha) = \left(\frac{2\pi G(\omega^2 + 6\omega + 1)}{\omega\alpha}\right)^{2\omega/(1+\omega)}. \tag{17}$$

All the solutions given by Eqs. (15)–(17) show:

(i) a curvature singularity at $r = 0$,
(ii) a Kleinian definite signature, $(--++)$,
(iii) *no* event horizon.

The latter property appears to prevent any of the shortcomings that one would expect when defining the S-matrix and/or the proper observables in a theory described by a finite-dimensional Hilbert space if the universe is accelerating [1–6]. This would circumvent any challenging questions for string theories, unless we consider the vacuum dark energy to be originating from a cosmological constant so that the spacetime is given by the de Sitter solution [4,5]. Clearly, the price to be paid for this rather comfortable property is to have a static spacetime metric with a Kleinian signature where the metric component g_{rr} is constant. Of course, one can always continue our general solution

$$ds^2 = -K(\omega,\alpha) r^{4\omega/(1+\omega)}\, dt^2 + \frac{(\omega^2+6\omega+1)\, dr^2}{(1+\omega)^2} + r^2\, d\Omega_2^2, \tag{18}$$

into a metric with Lorentzian signature by introducing suitable coordinate rotations, such as

$$\sqrt{B}\, r \to \bar{r}, \qquad \theta \to \sqrt{B}\,\bar{\theta}, \qquad \phi \to -i\sqrt{B}\,\bar{\phi},$$

to obtain

$$ds^2 = -K(\omega,\alpha)\bar{r}^{\,4\omega/(1+\omega)}\,dt^2 + d\bar{r}^{\,2} + \bar{r}^{\,2}\big(d\bar{\theta}^2 + \sinh^2\big(\sqrt{-B}\,\bar{\theta}\big)\,d\bar{\phi}^2\big),$$

which obviously shows a breakdown of the original spherical symmetry. A similar breakdown can be obtained using the coordinate change $\Omega_2 \to \overline{\Omega}_2 + \sqrt{-B}\ln r$, which now converts metric (18) into

$$ds^2 = -K(\omega,\alpha)r^{4\omega/(1+\omega)}\,dt^2 + 2\sqrt{-B}\,r\,d\overline{\Omega}_2\,dr + r^2\,d\overline{\Omega}\,{}^2_2.$$

The main result of the present Letter is the realization that, contrary to what has been currently believed, cosmological spacetimes which are accelerating due to the presence of quintessential vacuum energy, are described by static metrics that show no event horizon. This result is made possible due to the Kleinian character of the metric signature. Actually, one may argue that such spacetimes possess nonchronal regions, as it can be checked by the coordinate change

$$r = \ell\tan^2\left(\frac{\psi}{2}\right), \quad \ell = K^{-(1+\omega)/(4\omega)}, \tag{19}$$

with the new coordinate being half-periodic, running from $\psi = 0$ $(r = 0)$ to $\psi = \pi$ $(r = \infty)$. In terms of this coordinate, metric (18) can be written as

$$ds^2 = -\tan^{8\omega/(1+\omega)}\left(\frac{\psi}{2}\right)dt^2 + B\ell^2\frac{\tan^2(\psi/2)\,d\psi^2}{\cos^4(\psi/2)} + \ell^2\tan^4\left(\frac{\psi}{2}\right)d\Omega_2^2. \tag{20}$$

Let us now consider a world line for matter defined by $\theta, \phi = \text{const}$, $t = -\beta\phi$, where β is a real constant. In this case, metric (20) reduces to

$$ds^2 = -\left[\beta^2\tan^{8\omega/(1+\omega)}\left(\frac{\psi}{2}\right) - B\ell^2\frac{\tan^2(\psi/2)}{\cos^4(\psi/2)}\right]d\psi^2.$$

Thus, since B is a negative definite constant in the accelerating range $-1 < \omega < -1/3$ and provided the parameter β is real, this element of line is timelike for such a range. It follows that an observer moving on the world line will have an increasingly negative time coordinate and, even though she cannot reach the point $\psi = \pi$, her evolution will be confined to follow a path within a half-closed timelike curve. It appears therefore that the price to be paid for having an accelerating universe without future event horizon, and hence for having quintessence cosmological models that can peacefully coexist with string theory, is to allow for spacetimes with nonchronal regions which evolves along half-closed timelike curves in such models.

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