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Complexity in a monopoly market with a general demand and quadratic cost function

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Abstract

In this paper we extend the work done by Askar, 2013. (Askar, S.S., 2013. On complex dynamics of monopoly market, Economic Modelling, 31, 586-589). The equilibrium state of a bounded rational monopolist model is studied. It is assumed that the entire demand has a general non-linear form and the cost function is quadratic. The equilibrium of the model is equal to the level of price that maximizes profits, as can be seen in the classical microeconomic theory. However, complex dynamics can arise and the stability of equilibrium state is discussed. The complex dynamics, bifurcations and chaos are displayed by computing numerically Lyapunov numbers and sensitive dependence on initial conditions.

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Keywords: Monopoly, Difference Equation; Equilibrium, Stability, Sensitive dependence on initial conditions, Chaotic Behavior.

1. Introduction

In recent years, many researchers have demonstrated that economic agents may not be fully rational. Even if one tries to do more efforts to perform things correctly, it is important to utilize simple rules previously tested in the past (Kahneman *et al*,1986; Naimzada and Ricchiuti, 2008). Efforts have been made to model bounded rationality to

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different economic areas: oligopoly games (Agiza, Elsadany, 2003, Bischi *et al*, 2007); financial markets (Hommes, 2006); macroeconomic models such as multiplier-accelerator framework (Westerhoff,2006). In particular, difference equations have been employed extensively to represent these economic phenomenons (Elaydi, 2005; Sedaghat, 2003).

Oligopolistic market structures have a distinguishing feature that is the output, pricing and other decisions of one firm affect, and are affected by, the similar decision made by other firms in the market. Indeed, game theory is one of the important tools in the economists' analytical kit for analyzing the strategic behavior of this market (Gibbons, 1992, Webb, 2007). Various empirical works have shown that difference equations have been extensively used to simulate the behaviour of monopolistic markets (Abraham et al., 1997; Elaydi, 2005; Sedaghat, 2003).

The canonical approach of the monopoly theory is essentially static and the monopolist has full rationality: both perfect computational ability and large informational set in such a way that she can determine both quantity and price to maximize profits. However, in the real market producers do not know the entire demand function, though it is possible that they have a perfect knowledge of technology, represented by the cost function. Hence, it is more likely that firms employ some local estimate of the demand. This issue has been previously analyzed by Baumol and Quandt, 1964, Puu 1995, Naimzada and Ricchiuti, 2008, Askar, 2013. Naimzada and Ricchiuti evaluate a discrete time dynamic model with a cubic demand function without an inflexion point and linear cost function. Askar extends the work done by Naimzada and Ricchiuti with a general demand function.

In this paper, the equilibrium state of a bounded rational monopolist model is studied. It is assumed a general demand and quadratic cost functions and in this way we extend the work done by Askar, 2013. We show that complex dynamics can arise and the stability of equilibrium state is discussed. The complex dynamics, bifurcations and chaos are displayed by computing numerically Lyapunov numbers and sensitive dependence on initial conditions.

2. The model

The inverse demand function has a general form, it is downward sloping and concave:

$$p = a - bq^n, \ \mathrm{n} \in \mathbb{Z}, \ \mathrm{n} > 1 \tag{1}$$

where p indicates commodity price, q indicates the quantity demanded, and a and b are positive constants. The downward sloping is guaranteed if:

$$\frac{dp}{dq} = -nbq^{n-1} < 0 \tag{2}$$

that is if b>0.

The quantity produced, q, is positive and non-negative prices are achieved if

$$0 < q < \sqrt[n]{\frac{a}{b}} \tag{3}$$

We suppose that the cost function is quadratic

$$\mathrm{C(q)=cq^2} \tag{4}$$

Moreover, we assume the general principle of setting price above marginal cost, p - c > 0, for each non negative q; that is, a > c. The main aim of the firm is to maximize the following profit function:

$$\Pi(q) = (a - bq^n)q - cq^2 \tag{5}$$

This function is concave and given the following first order condition:

$$\frac{d\Pi}{dq} = a - 2cq - (n+1)bq^n = 0 \tag{6}$$

The marginal profit is strictly decreasing with range in the interval $(-\infty, a]$, therefore Eq. (6) has a unique solution q^* in this interval and the profit has a maximum at q^*. If $\Pi(q^*) > 0$ a positive equilibrium production is guaranteed. To achieve increasing profits, it is assumed that locally the monopoly firm, using a gradient mechanism, looks at how the variation of quantity affects the variation of profits. A positive (negative) variation of profits will induce the monopolist to change the quantity in the same (opposite) direction from that of the previous period. No changes will occur if profits are constant. This mechanism can be represented as follows:

$$q(t+1) = q(t) + k\frac{d\Pi}{dq(t)}, \text{ t=0,1,2,...} \tag{7}$$

where k>0 is the speed of adjustment to misalignments. Substituting Eq. (6) in (7), we obtain the following one-dimensional nonlinear difference equation:

$$q(t+1) = q(t) + k[a - 2cq(t) - (n+1)bq^n(t)] \tag{8}$$

3. Dynamical Analysis

3.1. Equilibria and Stabilty

If

$$f(q) = q + k[a - 2cq - (n+1)bq^n] \tag{9}$$

The fixed points of Eq. (8) are the solutions of the equation $f(q) = q$, and then the fixed point is the solution of Eq.(6)

$$q = q^* \tag{10}$$

Since

$$\frac{df}{dq}(q^*) = 1 + k[-n(n+1)b(q^*)^{n-1} - 2c] = 1 + k\frac{d^2\Pi}{\partial q^2} \tag{11}$$

The steady state is locally stable if:

$$\left|1 + k\Pi''(q^*)\right| < 1 \tag{12}$$

Or, equivalently,

$$0 < k < \frac{2}{-\Pi''(q^*)} \tag{13}$$

It follows that:

Proposition. *The map Eq. (8) has a unique steady state which is exactly the quantity that maximizes profits. It is locally stable if* $0 < k < \dfrac{2}{-\Pi''(q^*)}$

3.2. Numerical Simulations

The previous result indicates that a limited reaction of the monopolist to changes in profits can stabilize the quantity produced. On the other hand turbulences in the market are generated by an overreaction. To shed some light on what really happens in the market we employ a numerical analysis. Fixing the other parameters of the model as follows:

$$a = 4, b = 0.6, c = 0.5 \tag{14}$$

Then,

$$n = 4 \Rightarrow q^* = 1, k^* = \frac{2}{-\Pi''(1)} \simeq 0.1538 \tag{15}$$

and

$$\text{n}= 5 \Rightarrow q^* \simeq 0.966,\ k^* \simeq 0.119 \tag{16}$$

The dynamic map (8) satisfies the canonical conditions required for the flip bifurcation (Abraham *et al*., 1997) and there is a period doubling bifurcation if $k = k^*$. When $k < k^*$ the fixed point is attracting, when $k > k^*$ it is repelling. Therefore, there is a change in the nature of dynamics when $k = k^*$, a unique asymptotically stable period two-cycle arises.

We graphically show how the behavior of the map Eq. (8) changes for different values of the reaction coefficient, k. (Kulenovic, M., Merino, O., 2002). In Fig. 1, we show the map Eq. (8) when k = 0.1. From Eq. (13), the steady state is asymptotically stable. In Fig. 2, we show the particular set of parameters that determines a period two-cycle, actually, with k = 0.16. Further growth of k leads the attractor to follow a typical route of flip bifurcations in complex price dynamics: a sequence of flip bifurcations generate a sequence of attracting cycles in period 2^n, which are followed by the creation of a chaotic attractor. In Fig. 3, a cycle of period four is shown. To clarify the dynamics depending on k, we have reported a bifurcation diagram in Fig. 4. It shows different values of quantity for different values of k, particularly between 0 and 0.3. It is easily illustrated that we move from stability through a sequence of a period doubling bifurcations to chaos. In figure 4 are represented also the Lyapunov numbers of the orbit of 0.01, for k = 0.24, versus the number of iterations of the map (8). If the Lyapunov number is greater of 1, one has evidence for chaos. To demonstrate the sensitivity to initial conditions of Eq.(6), we compute two orbits (100 iterations of the map) with initial points q_0 and q_0+ 0.0001 respectively. The results are shown in Fig.5. At the beginning the time series are indistinguishable; but after a number of iterations, the difference between them builds up rapidly. In Fig. 6, we have reported a bifurcation diagram of the map Eq. (6) for n = 5 and the Lyapunov numbers of the orbit of 0.01, for n = 5 and k = 0.18. In Fig. 7, we show the sensitivity to initial conditions of Eq.(6), for n = 5, k = 0.18.

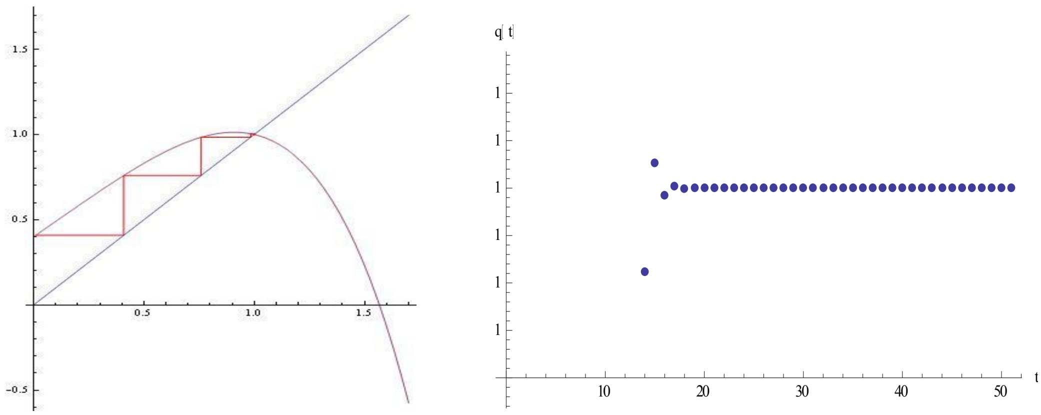

Fig.1. Convergence for for a =4,b=0.6, c= 0.5, n = 4 and k=0.1

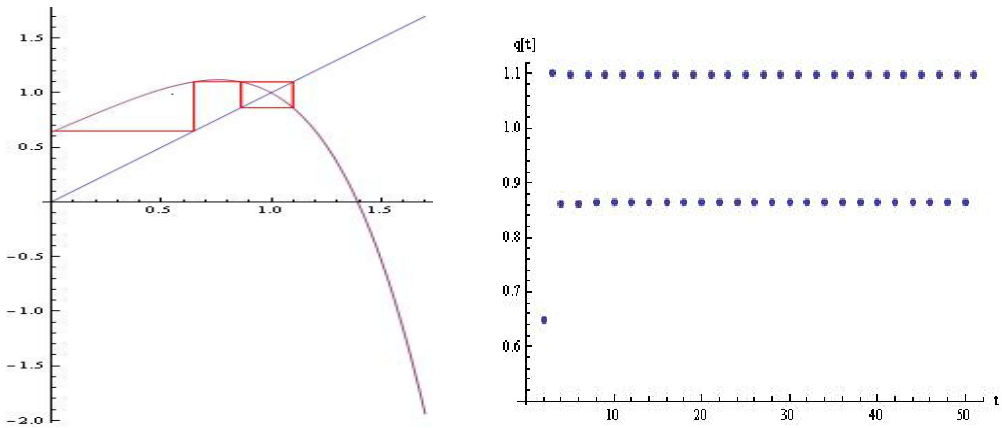

Fig. 2. Cycle of period 2, for a =4,b=0.6, c= 0.5 and k= 0.16

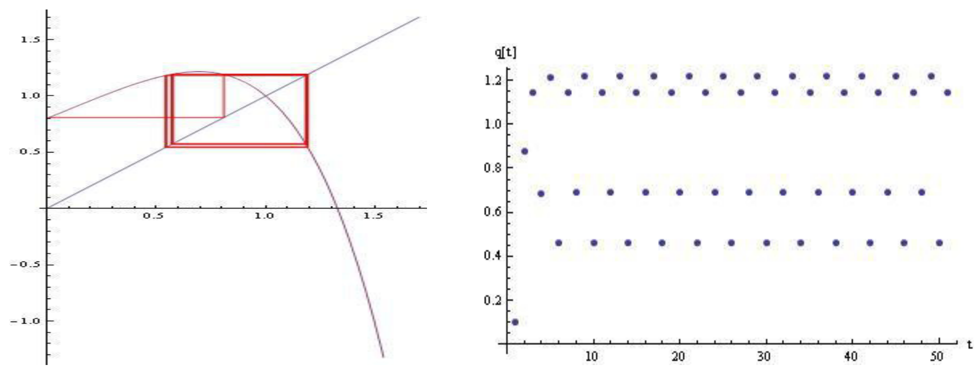

Fig. 3. Cycle of period 4, for a =4,b=0.6, c= 0.5 and k= 0.2

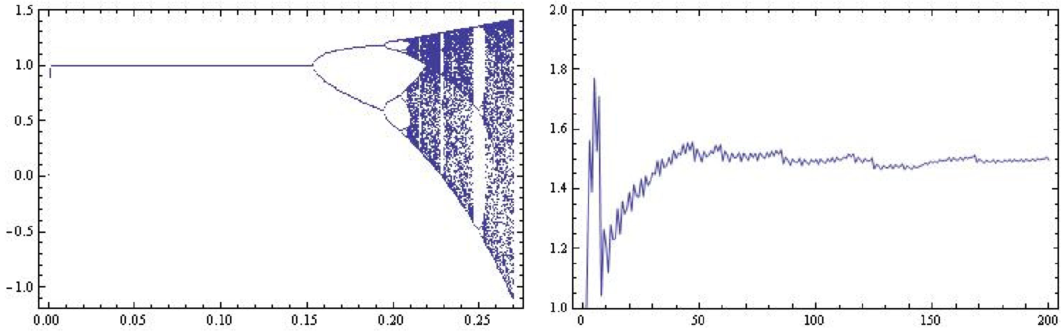

Fig.4. For n=4, bifurcation diagram with respect to the parameter k against variable q, for q_0=0.01 and 550 iterations of the map (8) (left) and Lyapunov numbers of the orbit of 0.01, for k =0.24, versus the number of iterations of the map (8) (right).

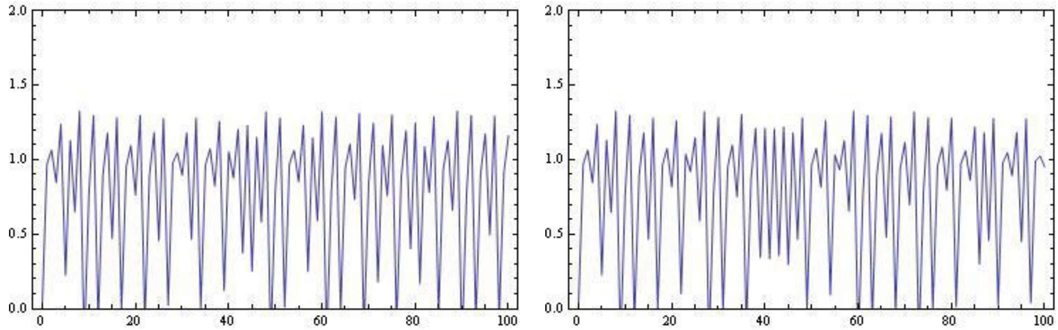

Fig.5. For n = 4, sensitive dependence on initials conditions: q plotted against the time, parameter value k=0.24 and initial condition q_0 = 0.01 (left), q_0 = 0.0101 (right).

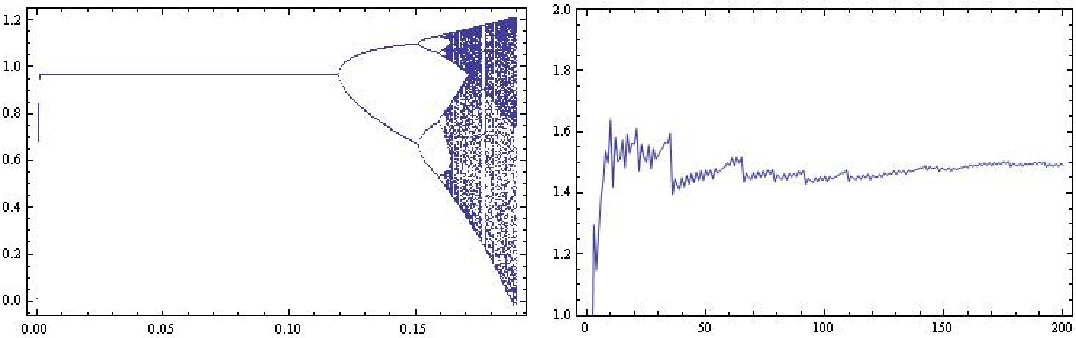

Fig.6 For n = 5, bifurcation diagram with respect to the parameter k against variable q, with q_0=0.01 and 550 iterations of the map (8) (left) and Lyapunov numbers of the orbit of 0.01, for k =0.18, versus the number of iterations of the map (8) (right).

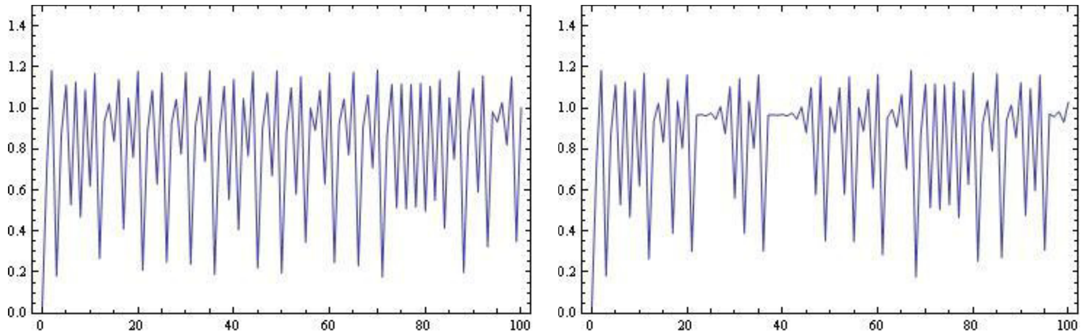

Fig.7. For n = 5, sensitive dependence on initials conditions: q plotted against the time, parameter value k=0.18 and initial condition q_0 = 0.01 (left), q_0 = 0.0101 (right).

4. Conclusion

In this paper, we have analyzed the effects on the equilibrium of a monopoly when the monopolist has bounded rationality. Particularly, the monopolist, not knowing the entire demand function, employs a rule of thumb to produce the quantity that guarantees the largest profits. The steady state of the map is exactly the level of production that maximizes profits, as can be seen in the classic microeconomic theory. However, complex dynamics can arise. For some values a parameter there is a locally stable equilibrium which is the value that maximizes the profit function. Increasing these values, the equilibrium becomes unstable, through period-doubling bifurcation. The complex dynamics, bifurcations and chaos are displayed by computing numerically Lyapunov numbers and sensitive dependence on initial conditions. We achieve these results employing a discrete time dynamical model such as that used by Askar 20013; however, we use a quadratic cost function.

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