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ScienceDirect
Procedia Materials Science 3 (2014) 1423 – 1428
Procedia Materials Science
www.elsevier.com/locate/procedia

20th European Conference on Fracture (ECF20)

On modeling plasticity-damage couplings in polycrystalline materials

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Abstract

At present, modeling of the plastic response of porous solids is done using stress-based plastic potentials. To gain understanding of the combined effects of all invariants for general three-dimensional loadings, a strain-rate based approach appears more appropriate. In this paper, for the first time strain rate-based potentials for porous solids with Tresca and von Mises matrices are obtained. The dilatational response is investigated for general 3-D conditions for both compressive and tensile states using rigorous upscaling methods. It is demonstrated that the presence of voids induces dependence on all invariants, the noteworthy result being the key role played by the plastic flow of the matrix on the dilatational response. If the matrix obeys the von Mises criterion, the shape of the cross-sections of the porous solid with the octahedral plane deviates slightly from a circle, and changes very little as the absolute value of the mean strain rate increases. However, if the matrix behavior is described by Tresca's criterion, the shape of the cross-sections evolves from a regular hexagon to a smooth triangle with rounded corners. Furthermore, it is revealed that the couplings between invariants are very specific and depend strongly on the particularities of the plastic flow of the matrix.

Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering

Keywords: strain-rate potentials; porous Tresca solid; porous Mises solid; coupled effects of invariants

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doi:10.1016/j.mspro.2014.06.230

1. Introduction

Using the plastic work equivalence principle, Ziegler (1977) has shown that a strain-rate potential can be associated to any convex stress-potential. Thus, to describe plastic flow a strain-rate potential (SRP) can be used instead of a stress-based potential. Up to now, SRPs have been developed only for fully-dense materials. However, all engineering materials contain defects (cracks, voids). To capture the characteristics of the plastic flow of porous metallic materials stress-based potentials have been developed. Most of these models are based on the hypothesis that the matrix obeys von Mises yield criterion (e.g. Gurson, 1977). Recently, using micromechanical considerations Cazacu et al. (2013, 2014) developed axisymmetric analytical yield criteria for porous materials with von Mises and Tresca matrix, respectively. These stress-based models account for the combined effects of the sign of the mean stress and the third-invariant of the stress deviator. Most importantly, it was explained the role of the J_3 on void evolution. However, to gain understanding of the combined effects of all invariants for general three-dimensional loadings, a strain-rate based approach appears more appropriate. In this paper, for the first time three-dimensional (3-D) strain-rate potentials for porous solids with von Mises and Tresca matrix are obtained and their properties investigated for both compressive and tensile loadings. After a brief presentation of the modeling framework, we present the strain-rate potentials for porous von Mises and Tresca solids (Section 3). To fully assess the properties of the respective SRPs, the shape of their cross-sections with the octahedral plane are analyzed. For a porous solid with von Mises matrix, the noteworthy result is that the shape of the cross-section changes little with the mean strain rate and the most pronounced influence of J_3 occurs for axisymmetric states. On the other hand, if the matrix obeys Tresca's criterion, the shapes of the cross-section of the SRP change dramatically, evolving from a hexagon to a triangle with rounded

2. Modeling framework

The kinematic homogenization approach of Hill-Mandel offers a rigorous framework for the development of plastic potentials for porous solids. If the matrix is rigid-plastic, there exists a strain-rate potential $\Pi(\mathbf{D}, f)$ such that the stress at any point in the porous solid is given by:

$$\mathbf{\Sigma} = \partial\Pi(\mathbf{D}, f)/\partial\mathbf{D} \quad \text{with} \quad \Pi(\mathbf{D}, f) = \inf_{\mathbf{d}\in K(\mathbf{D})} \langle \pi(\mathbf{d}) \rangle_{\Omega}, \tag{1}$$

where Ω is a representative volume element (RVE) composed of the matrix and a traction-free void, while < > denotes the average value over Ω; f is the porosity, $\pi(\mathbf{d})$ is the matrix's plastic dissipation with $\mathbf{d}$ being the local plastic strain rate tensor. Minimization is done over $K(\mathbf{D})$, which is the set of incompressible velocity fields compatible with homogeneous strain-rate boundary conditions. In this paper, the limit analysis is conducted for general 3-D conditions for both tensile and compressive states, i.e.,

$$\mathbf{D} = D_1\mathbf{e}_1 \otimes \mathbf{e}_1 + D_2\mathbf{e}_2 \otimes \mathbf{e}_2 + D_3\mathbf{e}_3 \otimes \mathbf{e}_3, \tag{2}$$

with D_1, D_2, D_3 being the eigenvalues (unordered) of $\mathbf{D}$ and ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) its eigenvectors. We further assume that the voids are spherical and randomly distributed in the matrix. For this void geometry an appropriate RVE is a hollow sphere. Let a denote its inner radius and b its outer radius. We use the trial velocity field $\mathbf{v}$, deduced by Rice and Tracey (1969) ,

$$\mathbf{v} = \frac{b^3}{r^2} D_m \mathbf{e}_r + \left[D'_1(\mathbf{e}_1 \otimes \mathbf{e}_1) + D'_2(\mathbf{e}_2 \otimes \mathbf{e}_2) + D'_3(\mathbf{e}_3 \otimes \mathbf{e}_3) \right]\mathbf{x}, \tag{3}$$

where $\mathbf{x}$ is the Cartesian position vector, $\mathbf{e}_r$ is the radial unit vector, and r is the radial coordinate.
For the given void geometry and distribution, the strain-rate potential of the porous solid with either Tresca or von Mises matrix ought to be isotropic. It follows that $\Pi(\mathbf{D}, f)$ should depend on the stress tensor $\mathbf{D}$ only through its invariants, i.e. $D_m = (D_1+D_2+D_3)/3$ and the second and third-invariant of $\mathbf{D}'$, the deviator of $\mathbf{D}$, respectively, i.e.,

$$\Pi(\mathbf{D},f)=\Pi(D_m,J_{2D},J_{3D}),\ \text{with}\ J_{2D}=\sqrt{(D_1'^2+D_2'^2+D_3'^2)/2}\ ,J_{3D}=D_1'\ D_2'\ D_3', \tag{4}$$

and $D_i' = D_i - D_m$, i = 1...3. Of particular interest is the analysis of the shape of the cross-sections of either SRP with the deviatoric planes D_m= constant. For this purpose, it is convenient to introduce the system ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$), which are related to the principal frame ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) by the following relations:

$$\mathbf{e}_x=(\mathbf{e}_1+\mathbf{e}_2+\mathbf{e}_3)/\sqrt{3},\ \ \mathbf{e}_y=-(\mathbf{e}_1+\mathbf{e}_2)/\sqrt{2},\ \ \mathbf{e}_z=(2\mathbf{e}_3-\mathbf{e}_1-\mathbf{e}_2)/\sqrt{6}. \tag{5}$$

Thus, any point P(D_1, D_2, D_3) belonging to the intersection of the SRP locus with any deviatoric plane is characterized by two polar-type coordinates, (R,γ):

$$R=|OP|=\sqrt{D_1'^2+D_2'^2+D_3'^2}=\sqrt{2J_{2D}},\tan(\gamma)=D_z/D_y=\sqrt{3}D_3'/(D_2'-D_1'). \tag{6}$$

For an isotropic porous solid, it is sufficient to determine the shape of the cross-section, i.e. R=R(γ), only in the sector $-\pi/6\le\gamma\le\pi/6$, the shape in all the other sectors being obtained by symmetry arguments. In particular, the sub-sector $-\pi/6\le\gamma\le 0$ corresponds to states on the SRP for which ($D_2'\ge 0, D_3'\le 0,\ D_1'\le 0$) so the third-invariant $J_{3D}>0$ while the sub-sector $0\le\gamma\le\pi/6$ corresponds to states for which $(D_2'\ge 0,\ D_3'\ge 0, D_1'\le 0)$ so $J_{3D}<0$. Axisymmetric states correspond to either γ = -π/6 or γ = π/6.

3. Strain-rate potentials for porous solids with von Mises and Tresca matrices containing spherical voids

3.1. 3-D strain-rate potential for a porous solid with von Mises matrix

An upper-bound estimate of the exact plastic potential of porous solid with von Mises matrix is:

$$\Pi_{Mises}(\mathbf{D})=\frac{\sigma_T}{V}\int_\Omega\sqrt{(2/3)\mathbf{d}:\mathbf{d}}\,dV=\frac{2\sigma_T}{V}\int_\Omega\sqrt{(R^2/6)+D_m^2(b/r)^6-4RD_m(b/r)^3F(\gamma,x_i^2)/(r^2/\sqrt{6})}\,dV \tag{7}$$

where $F(\gamma,x_1^2,x_2^2,x_3^2)=\sqrt{3}\left(x_2^2-x_1^2\right)\cos\gamma+\left(2x_3^2-x_1^2-x_2^2\right)\sin\gamma,\ V=4\pi b^3/3$, and $\mathbf{d}=\left(\nabla\mathbf{v}+\nabla\mathbf{v}^T\right)/2$ with $\mathbf{v}$ given by Eq. (3). Very recently Cazacu et al. (2013) have shown that for axisymmetric states the above integral can be calculated explicitly, without making the approximations considered by Gurson (1977). However, for general 3-D states, the integration can be done only numerically. To verify the accuracy of the numerical procedure, for axisymmetric loadings, the integral estimated numerically was compared to the exact results and the differences are negligible (less than 10^{-7}). As an example, in Fig. 1(a) is shown a 3-D isosurface of the von Mises porous solid for a porosity f = 1% for both tensile (D_m = tr ($\mathbf{D}$) > 0) and compressive (D_m < 0) states. Specifically, this convex surface contains all the points (D_m, R, γ) that produce the same plastic dissipation $\Pi_{Mises}(\mathbf{D},f)=9.21\cdot 10^{-3}$ for the porous solid. First, let us note that the presence of voids induces a strong influence of the mean strain rate D_m on the plastic dissipation, the SRP for f =1% is closed on the hydrostatic axis. The cross-sections of the 3-D isosurface with several deviatoric planes D_m = constant are shown in Fig.1(b). While the intersection with the plane D_m = 0, is a circle (von Mises behavior), the cross-sections with all the other deviatoric planes D_m = constant deviate slightly from a circle. It is worth noting the centro-symmetry of the SRP (Fig. 1(c)) and that the most influence of the parameter γ (or J_{3D}) occurs for axisymmetric states.

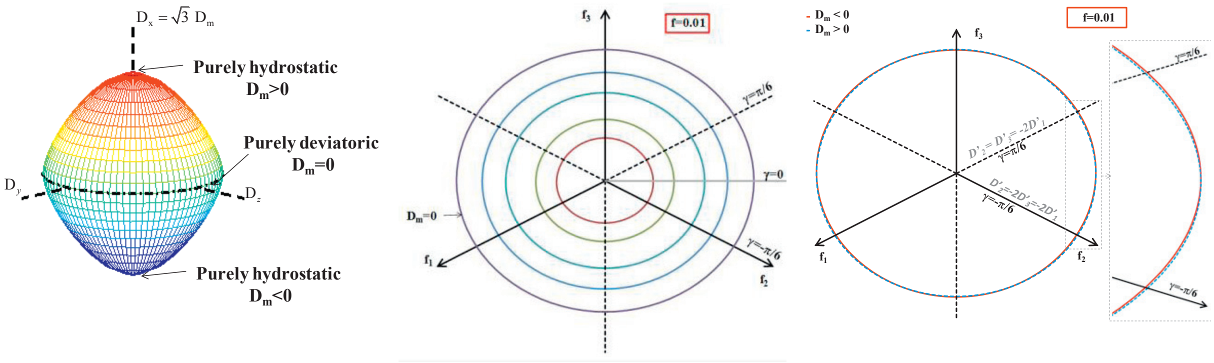
3.2. Three-dimensional strain-rate potential for a porous solid with von Tresca matrix

An upper-bound estimate of the overall plastic potential of porous solid with Tresca matrix is:

$$\Pi_{Tresca}(\mathbf{D},f)=\frac{\sigma_T}{V}\int_\Omega\sigma_T\left(|d_1|+|d_2|+|d_3|\right)\,dV\ , \tag{8}$$

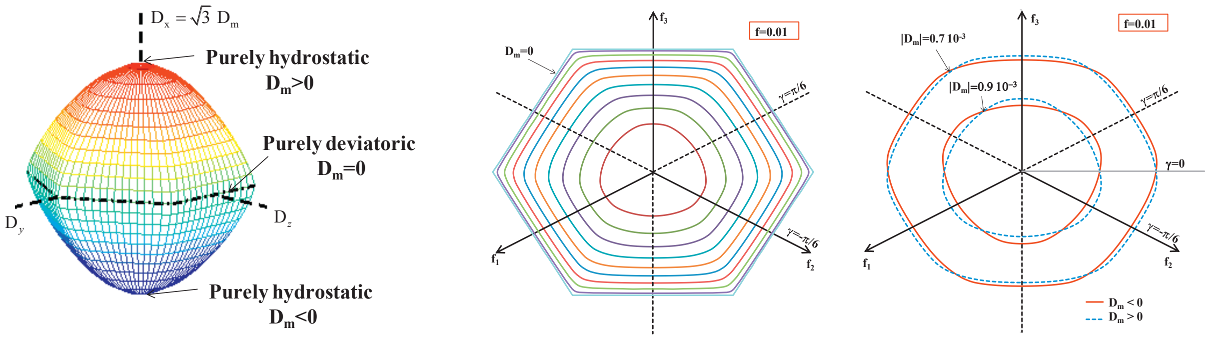
where d_1, d_2, d_3 are the principal values (unordered) of $\mathbf{d}$. Note that a major difficulty in obtaining the SRP is that

$\Pi_{Tresca}(\mathbf{D}, f)$ depends on the sign of each of the principal values of **d**. This is a direct consequence of the Tresca's criterion being dependent on the third-invariant of the stress deviator. Only for axisymmetric loadings, the signs of d_1, d_2, d_3 can be determined analytically. For these loadings, very recently Cazacu et al. (2014) showed that the integral of Eq. (8) could be calculated explicitly, without any approximation. For general 3-D states, $\Pi_{Tresca}(\mathbf{D},f)$ can be estimated only numerically. As an example, in Fig.2(a) is shown a normalized ($\sigma_T = 1$) 3-D isosurface $\Pi_{Tresca}(\mathbf{D},f) = 9.21\ 10^{-3}$ of the porous Tresca solid corresponding to a porosity f = 1% for states characterized by ($D_m > 0$) and ($D_m < 0$), respectively. Specifically, this convex surface contains all states (D_m, R, γ) that produce the same plastic dissipation $\Pi_{Tresca}(\mathbf{D},f) = 9.21\ 10^{-3}$ for the porous solid. It is very interesting to note the very strong influence of D_m on the plastic behavior of the porous Tresca material, the shape of the cross-sections changing drastically with the level of D_m. Due to the presence of voids, all cross-sections are smoothed out, their shape evolving from a hexagon ($D_m = 0$) to a triangle with rounded corners (the innermost cross-section corresponding to $D_m = 9.10^{-4} s^{-1}$). Note also that the SRP is centro-symmetric. i.e. $\Pi_{Tresca}(D_m, R, \gamma, f) = \Pi_{Tresca}(-D_m, R, -\gamma, f)$. To illustrate this remarkable property, in Fig. 2(c) are shown the cross-sections of the 3-D isosurface with the planes $D_m = 7\ 10^{-4}\ s^{-1}$ and $D_m = 9\ 10^{-4}\ s^{-1}$, respectively (interrupted lines) as well as these with the planes $D_m = -7\ 10^{-4}\ s^{-1}$ and $D_m = -9\ 10^{-4}\ s^{-1}$, respectively (solid lines).



(a) (b) (c)

Fig. 1. (a) Representation of the 3-D isosurface of the porous Mises SRP for both tensile and compressive states; (b). Cross-sections with deviatoric planes D_m =constant >0, outer one corresponds to D_m=0 and inner one to $D_m=0.9\ 10^{-3}\ s^{-1}$; (c) Cross-sections with $D_m = 6\ 10^{-4}\ s^{-1}$ (interrupted line) and $D_m = -6\ 10^{-4}\ s^{-1}$ (solid line).



(a) (b) (c)

Fig. 2. (a) Representation of the 3D isosurface of the porous Tresca SRP . (b). Cross-sections with deviatoric planes D_m =constant >0 (c) Cross-with $D_m = 7\ 10^{-4}\ s^{-1}$, $D_m = 9\ 10^{-4}\ s^{-1}$ (interrupted line) and $D_m = -7\ 10^{-4}\ s^{-1}$, $D_m = -9\ 10^{-4}\ s^{-1}$ (solid line) showing the centro-symmetry of the SRP.

The symmetry of all cross-sections with respect to the origin is clearly seen. For example, for states corresponding to $J_{3D} > 0$ ($-\pi/6 < \gamma < 0$) to produce the same plastic dissipation, R (or J_{2D}) must be higher for compressive states ($D_m<0$ -interrupted line) than for tensile states ($D_m>0$-solid line). The reverse holds true for loadings corresponding to $J_{3D} < 0$ ($0 < \gamma < \pi/6$).

3.3. Discussion on the importance of the plastic flow of the matrix on the dilatational response

The most widely used plastic potential for isotropic porous solids containing randomly distributed spherical voids was proposed by Gurson (1977). In his analysis, Gurson (1977) assumed that the coupled effects between the mean strain rate D_m and **D'** (i.e. the term $D_m R(\gamma)$ in Eq. (7)) can be neglected, and the corresponding SRP is :

$$\Pi_{Gurson}(\mathbf{D},f) = 2|D_m|\left[\frac{\sqrt{1+6D_m^2/R^2}-\sqrt{f^2+6D_m^2/R^2}}{(D_m/R)\sqrt{6}} + \ln\left(\frac{1}{f}\cdot\frac{(D_m/R)\sqrt{6}+\sqrt{f^2+6D_m^2/R^2}}{(D_m/R)\sqrt{6}+\sqrt{1+6D_m^2/R^2}}\right)\right] \quad (9)$$

Note that $\Pi_{Gurson}(\mathbf{D},f)$ is the exact dual of the classic stress-based potential of Gurson (1977). For axisymmetric states, Cazacu et al. (2014) have shown that if the same simplifying hypothesis considered by Gurson (1977) is made when evaluating the local plastic dissipation associated to Tresca's criterion, the truncated expression of the SRP at which one arrives coincides with that given by Eq. (9). This means that neglecting couplings between the mean strain rate and **D'**, the specificities of the plastic flow of the matrix are erased. In Fig.3 we compare the SRP for a porous Mises material obtained by Gurson (i.e. Eq. (9)) with the SRP for a porous Mises material given by Eq. (7), and the SRP for a porous Tresca material given by Eq. (8). Since Gurson's SRP was obtained by truncating the overall plastic dissipation, it is necessarily interior to the $\Pi_{Mises}(\mathbf{D},f)$ (Eq. (7)). Irrespective of the level of D_m, Tresca's SRP is exterior to the surfaces corresponding to von Mises matrix. This means that Gurson's SRP is the most dissipative of the three SRP's, since in order to reach the same value of the plastic dissipation, the norm of the loading, $R(\gamma)$, is lower than for a porous Mises (Eq. (7)) or porous Tresca (Eq. (8)). On the other hand, Tresca's SRP is the least dissipative potential. The noteworthy result revealed is the very strong influence of the plastic flow of the matrix on the response of a porous solid. If the matrix obeys the von Mises criterion the shape of the cross-sections of the porous solid changes very little as D_m increases, but if the matrix behavior is described by Tresca's criterion the shape of the cross-section evolves from a hexagon to a triangle with rounded corners. The difference between the response of a porous solid with von Mises matrix and that with a Tresca matrix becomes more important with increasing D_m, leading to a strong difference in void evolution (see the estimate of the differences in the rates of void growth for very high hydrostatic loadings reported by Rice and Tracey, 1969).

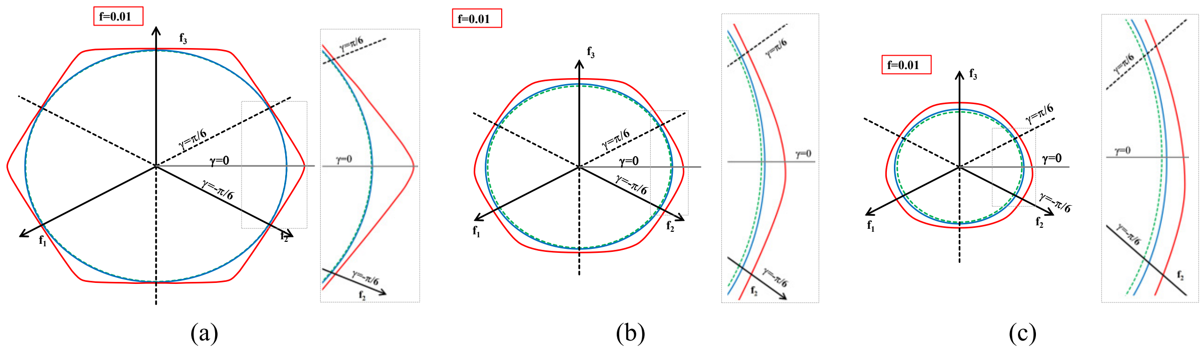


Fig. 3. (a) Comparison between the shapes of the cross-sections of Gurson's SRP (interrupted line), the exact SRP for a porous von Mises material (blue solid line), and the SRP for a porous Tresca material (red solid line) for: (a) $D_m= 2\ 10^{-4}\ s^{-1}$; (b) $D_m= 6\ 10^{-4}\ s^{-1}$; (c) $D_m= 8\ 10^{-4}\ s^{-1}$.

4. Conclusions

The aim of this paper was to investigate the properties of the 3-D plastic potentials for porous solids with Tresca and von Mises matrices, respectively. It has been shown that if the matrix is described by the von Mises criterion:

- The plastic response depends on all three invariants of the strain-rate tensor **D**.
- The corresponding 3-D surface is smooth and symmetric with respect to the origin.
- Its cross-sections with the deviatoric planes D_m = constant ($\neq 0$) have three-fold symmetry with respect to the origin, and deviate slightly from a circle.The strongest effect of the third-invariant is for axisymmetric states i.e. between $R(\gamma = \pi/6)$ and $R(\gamma = -\pi/6)$.

Therefore, the most influence of the parameter γ (or J_{3D}) on void evolution occurs for axisymmetric states. It is very worth noting that the same conclusions concerning the influence of the third-invariant on void evolution were drawn by Rice and Tracey (1969) for the case of large hydrostatic stresses.

It has been shown that if the plastic behavior of the matrix is described by Tresca's criterion:

- The 3-D surface is centro-symmetric and displays a strong coupling between all invariants.
- The shapes of the cross-sections with deviatoric planes are strongly dependent on the level of D_m. As the absolute value of D_m increases, the shape changes from a regular hexagon ($D_m = 0$) to a triangle with rounded corners.
- For D_m=0 (i.e. Tresca behavior) the maximum of $R(\gamma)$ is at $\gamma = 0$ (J_{3D} =0), the minima being for axisymmetric states. For D_m >0, the maximum of $R(\gamma)$ is not at $\gamma = 0$ anymore , but shifts toward the axisymmetric case corresponding to $\gamma = -\pi/6$ ($D_1 = D_3 < D_2$ and $J_{3D} > 0$); on the other hand, the minimum of $R(\gamma)$ is always obtained for $\gamma = \pi/6$ (axisymmetric state with $J_{3D} < 0$). The reverse holds true for $D_m < 0$.
- While in the case of the porous Mises solid, the most pronounced difference in the response is between the axisymmetric states, for a porous Tresca solid no general conclusions can be drawn because the specific expression of $R(\gamma)$ depends both on the level of porosity and the level of D_m .

The strain-rate potentials for porous solids with Tresca and von Mises matrices obtained in this work were compared to the exact conjugate in the strain-rate space of the classic Gurson (1977) stress-based potential. It was shown that only for three states, all three strain-rate potentials coincide. These states are: purely hydrostatic loading (**D'** = 0); and axisymmetric purely deviatoric states (two principal values of **D** equal and D_m= 0). In contrast to the exact porous von Mises SRP (Eq. (7)), the Gurson's SRP does not involve any dependence on J_{3D}. Therefore, its cross-sections with any deviatoric plane (irrespective of D_m or the porosity level) is always a circle. The exact SRP for a porous von Mises solid and for a porous Tresca solid (Eq. (8)) are centro-symmetric i.e. they are invariant to the transformation $(D_m, R, \gamma) \rightarrow (-D_m, R, -\gamma)$. In contrast, Gurson's SRP involves only dependence of D_m and J_{2D} (or R), it is insensitive to the sign of the mean strain rate (compressive or tensile states). Most importantly, irrespective of the level of D_m , Gurson's SRP is the most dissipative of the three SRP's, since in order to reach the same value of the plastic dissipation, the norm of the loading, $R(\gamma)$, is lower than that of the exact porous Mises or the porous Tresca SRP. On the other hand, Tresca's SRP is the least dissipative potential.

References

Cazacu, O., Revil-Baudard, B., Lebensohn, R. A., Garajeu, M., 2014. On the Combined Effect of Pressure and Third Invariant on Yielding of Porous Solids with von Mises Matrix. J. Appl. Mech. 80, 64501.

Cazacu, O., Revil-Baudard, B., Chandola N., Kondo, D., 2014. New Analytical Criterion for Porous Solids with Tresca Matrix Under Axisymmetric Loadings. . Int. J Solids Struct.51,861-874.

Gurson, A. L., 1977. Continuum Theory of Ductile Rupture by Void Nucleation and Growth. Part I: Yield Criteria and Flow Rules for Porous Ductile Media. J. Engng. Matl. Tech. Trans. ASME, Series H, 99, 2-15.

Rice, J.R., Tracey, D.M., 1969. On the Ductile Enlargement of Voids in Triaxial Stress Fields. J. Mech. Phys. Solids 17, 201-217.

Ziegler, H., 1977. An Introduction to Thermodynamics, North-Holland, Amsterdam.