Inflation in R^2 supergravity with non-minimal superpotentials

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ABSTRACT

We investigate the cosmological inflation in a class of supergravity models that are generalizations of non-supersymmetric R^2 models. Although such models have been extensively studied recently, especially after the launch of the PLANCK and BICEP2 data, the class of models that can be constructed has not been exhausted. In this note, working in a supergravity model that is a generalization of Cecotti's model, we show that the appearance of new superpotential terms, which are quadratic in the superfield Λ that couples to the Ricci supermultiplet, alters substantially the form of the scalar potential. The arising potential has the form of the Starobinsky potential times a factor that is exponential in the inflaton field and dominates for large inflaton values. We show that the well-known Starobinsky inflation scenario is maintained only for unnaturally small fine-tuned values of the coupling describing the Λ^2 superpotential terms. A welcome feature is the possible increase of the tensor to scalar ratio r, within the limits set by the new Planck and BICEP2 data.

1. Introduction

Models of inflation are constrained by observations of WMAP [1] and Planck [2] satellites. The spectral index is found in the range $n_s = 0.9608 \pm 0.0054$ while the tensor to scalar ratio is bounded from above $r < 0.111$. In addition the BICEP2 [3] experiment, claiming for the discovery of primordial gravitational waves resulting in a ratio $r = 0.16^{+0.06}_{-0.05}$, aroused the interest of both experimentalists and theorists. PLANCK satellite data [2] are in perfect agreement with the Starobinsky model of inflation [4] which predicts a tensor to scalar ratio in the range $r \simeq 0.004$, which is almost two orders of magnitude smaller than the claimed discovery of BICEP2 which points towards chaotic inflation [5]. In the meantime Planck collaboration released new data with increased precision [6], which are in agreement with the previous data, according to which $n_s = 0.968 \pm 0.006$ and $r < 0.11$. Also BICEP2 and Planck joint collaboration [7] established a robust upper bound $r < 0.12$, which is substantially lower than the value quoted in [3].

Much effort has been expended towards building inflationary models embedded in the framework of supergravity theories. Chaotic inflation [5] scenario can be incorporated in supergravity schemes [8,9] and more recently general chaotic inflationary su-

pergravity potentials have been studied [10]. Supergravity models that incorporate $R + R^2$ terms and reproduce Starobinsky's inflation predictions for r, n_s have received a lot of attention recently [11–35]. A class of supergravity models are described by no-scale Kähler potentials [36] and many of the proposed inflationary models have a no-scale structure [12,13,17,24,28,29,32,35]. It is worth noting that in this class of models there is the possibility of accommodating models interpolating between low ($r \sim 0.001$) and large values ($r \sim 0.1$) depending on the parameters. This can be also accomplished in attractor solutions that relate in a continuous manner the predictions of the Starobinsky model to those of the quadratic chaotic potential [37,38].

Among the possible theoretical schemes, incorporating the virtues of the Starobinsky R^2 model that lead to successful inflation, are higher derivative supergravity Lagrangians [39–43]. In these, besides the matter chiral and vector multiplets, additional chiral multiplets are unavoidably introduced. In the minimal scenario [40] one uses two multiplets that after eliminations of the auxiliary fields involved lead to a supergravity Lagrangian including R^2. This extends the Starobinsky model in a non-trivial manner in the sense that additional terms appear, in comparison with the non-supersymmetric theory, that only conditionally can sustain a successful inflationary scenario.

Our aim in this note is to further investigate natural extensions of some of these models that include superpotential couplings not

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considered in previous works. Such couplings lead to inflaton potentials that do not have the Starobinsky form, unless some of the parameters are fined tuned. The question that arises is under what conditions these supergravity generalizations lead to successful inflation and under which circumstances large values of the ratio r can be obtained saturating the upper bounds set on r by Planck and BICEP2.

This article is organized as follows:

In Section 2 we review the equivalence of R^2 supergravity to an ordinary supergravity theory, by generalizing the original Cecotti's Lagrangian [40], working in the framework of the Poincaré supergravity. In Section 3, working in the context of this framework, we discuss the properties of a class of models as far as cosmological inflation is concerned. Section 4 is devoted to a numerical treatment of the cosmological solutions and the consequences for the parameters describing the model under consideration. In Section 5 we discuss the cosmological predictions paying special emphasis on the tensor to scale ratio r. We end up with the conclusions.

2. Reviewing the R^2 supergravity model

It has been proven that R^2 gravity [44], upon supersymmetrization is equivalent to ordinary supergravity theory which includes two chiral multiplets with specific couplings [40]. In this note we start by considering the locally supersymmetric Lagrangian of SuperPoincaré algebra which involves two chiral multiplets Φ, Λ allowing for non-linear superpotential couplings of the field Λ that couples to the curvature multiplet. One can also start from a superconformal action, see for example [19,22], and then proceed to Poincaré supergravity action by a proper gauge fixing but this case is not considered in this work.

Our starting Lagrangian is therefore given by

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[-\Lambda \mathcal{R} - \frac{1}{8} \left(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R} \right) \Omega(\Phi, \bar{\Phi}) + W(\Phi, \Lambda) \right] + h.c. \tag{1}$$

which is described by the real kinetic function $\Omega(\Phi, \bar{\Phi})$, which is a function of $\Phi, \bar{\Phi}$, and a superpotential $W(\Phi, \Lambda)$ which is a holomorphic function of the supermultiplets Φ, Λ. Note that the curvature supermultiplet $\mathcal{R}$ couples to a chiral multiplet which we denote by Λ and this choice covers the most general case up to field redefinitions. The particular case $W(\Phi, \Lambda) = \Lambda \Phi$, in which Φ and Λ are coupled linearly to each other, is the one studied by Cecotti but evidently other options are available. The above Lagrangian can be cast in the form,

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[-\frac{1}{8} \left(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R} \right) \Omega'(\Phi, \Lambda, \bar{\Phi}) + W(\Phi, \Lambda) \right] + h.c. \tag{2}$$

which is also a supergravity Lagrangian with a redefined kinetic function

$$\Omega'(\Phi, \Lambda, \bar{\Phi}, \bar{\Lambda}) = \Omega(\Phi, \bar{\Phi}) - \frac{1}{2}(\Lambda + \bar{\Lambda}).$$

The superpotential can be brought to the following form by merely segregating the linear in Φ terms, which in general couples to a superfield which is a function of the chiral multiplet Λ,

$$W(\Phi, \Lambda) = g(\Lambda)\Phi + P(\Phi, \Lambda).$$

The function $g(\Lambda)$ is non-linear, in general, and its departure from linearity brings about new features that may affect the cosmological evolution as we shall see.

For our purposes it suffices to keep only the bosonic fields and thus the superfields can be expanded in the following way

$$\Phi = \varphi + \Theta\Theta F_\varphi, \quad \Lambda = \lambda + \Theta\Theta F_\lambda$$

leading to

$$g(\Lambda) = g(\lambda) + \Theta\Theta \frac{\partial g}{\partial \lambda} F_\lambda,$$

$$W(\Lambda, \Phi) = W(\lambda, \varphi) + \Theta\Theta \left(W_\lambda F_\lambda + W_\varphi F_\varphi \right).$$

An analogous treatment for the supergravity chiral multiplets $\mathcal{E}$ and $\mathcal{R}$, following standard notation, leads to

$$2\mathcal{E} = e\{1 - \Theta\Theta\bar{M}\}$$

and

$$\mathcal{R} = -\frac{1}{6} \left\{ M + \Theta\Theta \left[-\frac{1}{2}R + \frac{2}{3}M\bar{M} + \frac{1}{3}b_\mu b^\mu - iD_\mu b^\mu \right] \right\}.$$

Then the bosonic part of the off-shell Lagrangian is written as

$$e^{-1}\mathcal{L}_B = \frac{1}{6} \left[\Omega - \frac{1}{2}(\lambda + \bar{\lambda}) \right] \left(R + \frac{2}{3}M\bar{M} - \frac{2}{3}b_\mu b^\mu \right)$$
$$- \Omega_{\varphi\bar{\varphi}} \partial_\mu \varphi \partial^\mu \bar{\varphi} + \Omega_{\varphi\bar{\varphi}} F_\varphi \bar{F}_{\bar{\varphi}}$$
$$- \frac{i}{3} \left[\Omega_\varphi \partial_\mu \varphi - \Omega_{\bar{\varphi}} \partial_\mu \bar{\varphi} \right] b^\mu - \frac{i}{6}(\lambda - \bar{\lambda})\mathcal{D}_\mu b^\mu$$
$$- \left[(g(\lambda)\varphi + P)\bar{M} + h.c. \right]$$
$$+ \left\{ \left[g(\lambda) + P_\varphi - \frac{1}{3}M\Omega_\varphi \right] F_\varphi \right.$$
$$\left. + \left[\frac{\partial g}{\partial \lambda}\varphi + P_\lambda + \frac{1}{6}M \right] F_\lambda + h.c. \right\}.$$

The ordinary supergravity model is derived by solving the equations of the auxiliary fields $M, b_\mu, F_\lambda, F_\varphi$ in the usual manner. However in this frame (Jordan frame) the field λ has no kinetic term so alternatively one can use its equation of motion in eliminating the auxiliary fields. In doing this the equation of motion for the field λ yields

$$\left[\frac{\partial g}{\partial \lambda}\varphi + P_\lambda \right] \bar{M} + \frac{1}{12} \left[R + \frac{2}{3}M\bar{M} - \frac{2}{3}b_\mu b^\mu + 2iD_\mu b^\mu \right]$$
$$- \left(\frac{\partial g}{\partial \lambda} + P_{\varphi\lambda} \right) F_\varphi - \left(\frac{\partial^2 g}{\partial \lambda^2}\varphi + P_{\lambda\lambda} \right) F_\lambda = 0$$

while those stemming from F_λ and M yield respectively

$$\frac{\partial g}{\partial \lambda}\varphi + P_\lambda + \frac{1}{6}M = 0$$

and

$$\frac{1}{3}\Omega'\bar{M} - \Omega_\varphi F_\varphi + \frac{1}{2}F_\lambda - 3(\bar{g}\bar{\varphi} + \bar{P}) = 0.$$

Eliminating M, F_λ and substituting back in the Lagrangian the resulting expression for F_φ we get a supergravity model which includes R^2 term

$$\frac{1}{144} \frac{\Omega_{\varphi\bar{\varphi}}}{\left| \frac{\partial g}{\partial \lambda} + P_{\varphi\lambda} + 2\Omega_\varphi \left(\frac{\partial^2 g}{\partial \lambda^2}\varphi + P_{\lambda\lambda} \right) \right|^2} R^2. \tag{3}$$

Note that this term arises from the $F_\varphi \bar{F}_{\bar{\varphi}}$ term of the original off-shell Lagrangian.[1] The prefactor of R^2 in Eq. (3), in the general

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[1] In the case of non-linear $g(\lambda)$ the R^2 coefficient is different from the one appearing in [22]. The difference is due to the specific superconformal gauge fixing adopted in that work.

case, is a complicated function of the fields involved but it simplifies a great deal, becoming actually a constant, when $\Omega(\Phi, \bar{\Phi}) = \Phi\bar{\Phi}$, $g(\Lambda) = \Lambda$ and $P(\lambda, \Phi) = 0$, retrieving in this way Starobinsky's model. More general cases regarding mainly the field Φ have also been studied in [42] as far as the vacuum structure and the supersymmetry breaking is concerned. Recently in [43], besides the vacuum structure, the inflationary properties of generalized cases have been addressed.

We have thus seen that the Lagrangian given by Eq. (1) and (2) yields the "dual" prescription of R^2 supergravity where the R^2 term couples with two fields. This result can be generalized with the inclusion of additional chiral multiplets as well. The crucial point for the R^2 description is the existence of a field λ for which $\Omega'_{\lambda\bar{\lambda}} = 0$. In general we expect that R^2 terms will naturally arise from corrections which may involve, for instance, the dilaton and other moduli fields coming from string theory [45] and so identifying the field λ with one of these fields we actually consider a model with non-linear dependencies on this field. In this sense the study of models with nonlinear behavior, besides being interesting per se, it is useful in order to investigate the particular role these fields may play in the cosmological evolution.

3. No-scale supergravity with Λ^2 terms

As already outlined in the previous section we consider supergravity models described, in the Einstein frame, by the Kähler function[2]

$$K = -3ln\left(-\frac{\Omega'}{3}\right)$$

and a superpotential whose scalar component reads

$$W(\lambda, \varphi) = g(\lambda)\varphi + P(\lambda, \varphi).$$

For our considerations the relevant terms are the kinetic terms of the fields λ, φ

$$-\frac{1}{2}\Big[K_{\varphi\bar{\varphi}}\partial_\mu\varphi\partial^\mu\bar{\varphi} + K_{\varphi\bar{\lambda}}\partial_\mu\varphi\partial^\mu\bar{\lambda} + K_{\lambda\bar{\varphi}}\partial_\mu\lambda\partial^\mu\bar{\varphi} + K_{\lambda\bar{\lambda}}\partial_\mu\lambda\partial^\mu\bar{\lambda}\Big]$$

where

$$K_{\varphi\bar{\varphi}} = \frac{3}{\Omega'^2}\left(\Omega_\varphi\Omega_{\bar{\varphi}} - \Omega'\Omega_{\varphi\bar{\varphi}}\right), \ K_{\lambda\bar{\lambda}} = \frac{3}{4\Omega'^2},$$

$$K_{\lambda\bar{\varphi}} = -\frac{3}{2\Omega'^2}\Omega_{\bar{\varphi}}, \ K_{\varphi\bar{\lambda}} = -\frac{3}{2\Omega'^2}\Omega_\varphi,$$

and the scalar potential which, for the specific linear dependence of Ω' on λ and $\bar{\lambda}$, takes on the following form

$$\begin{aligned} V = \frac{9}{\Omega'^2}\Big\{ &\frac{1}{\Omega_{\varphi\bar{\varphi}}}\big[W_\varphi\bar{W}_{\bar{\varphi}} + 2\Omega_\varphi W_\lambda\bar{W}_{\bar{\varphi}} + 2\Omega_{\bar{\varphi}}W_\varphi\bar{W}_{\bar{\lambda}} \\ &+ 4\left(\Omega_\varphi\Omega_{\bar{\varphi}} - \Omega'\Omega_{\varphi\bar{\varphi}}\right)W_\lambda\bar{W}_{\bar{\lambda}}\big] \\ &- 6\big[W_\lambda\bar{W} + \bar{W}_{\bar{\lambda}}W\big]\Big\}. \end{aligned}$$

For its derivation we have used the fact that $\Omega'_\varphi = \Omega_\varphi$, $\Omega'_{\bar{\varphi}} = \Omega_{\bar{\varphi}}$ and $\Omega'_\lambda = \Omega'_{\bar{\lambda}} = -1/2$. Moreover we shall assume that

$$\Omega' = -3 + \varphi\bar{\varphi} - \zeta(\varphi\bar{\varphi})^2 - \frac{1}{2}(\lambda + \bar{\lambda})$$

or in other words the function Ω' includes quadratic in $\varphi\bar{\varphi}$ terms, specified by the constant ζ. Omitting the quadratic in $\varphi\bar{\varphi}$ term this
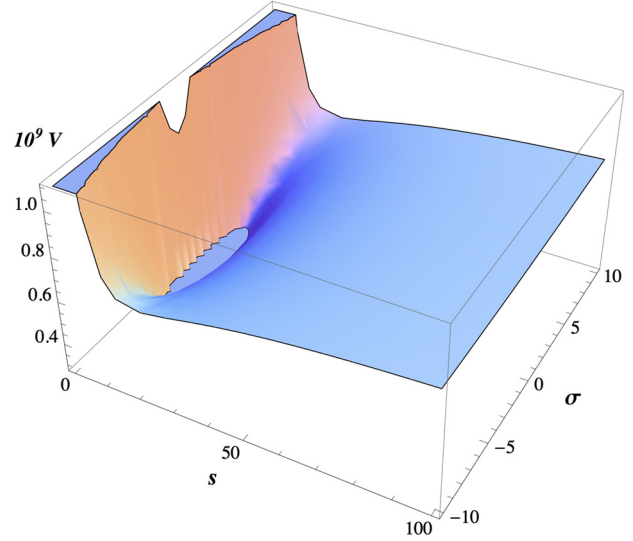

Fig. 1. 3D plot of the scalar potential of the Lagrangian (5) for values of parameters $d = -3 \times 10^{-5}$, $d_1 = 10^{-5}$, and $d_2 = 5 \times 10^{-10}$. The axes are along the real and the imaginary direction of the field $\lambda \equiv s + i\sigma$.

is of the no-scale type [36]. Such terms are introduced for the stabilization of the potential in the φ direction and have been also considered in [14]. For the superpotential we take $P(\phi, \lambda) = \alpha$, that is a constant, and also assume a nonlinear function of $g(\Lambda)$ which includes quadratic in λ terms. We shall see that these terms may play an important role and their presence upsets the cosmological inflation scenario. With these in mind the superpotential has the form

$$W(\Phi, \Lambda) = g(\Lambda)\Phi + \alpha, \quad g(\Lambda) = d + d_1\Lambda + d_2\Lambda^2,$$

and thus it is described by four arbitrary constants namely d, d_1, d_2 and α. Note that throughout this paper the reduced Planck mass, $M_P \equiv M_{Planck}/\sqrt{8\pi}$, is set to unity, $M_P = 1$.

At this point it would be useful to make contact with the findings of other authors who use the following notation for the Kähler function and the super potential [14],

$$K = -3\ln(T + \bar{T} - C\bar{C}), \ W = 3MC(T - 1). \tag{4}$$

Our model corresponds precisely to this model with the identifications

$$\Lambda = 6T - 3, \ \Phi = \sqrt{3}C$$

if the constants are taken as

$$\alpha = d_2 = 0, \ d = -3d_1 \equiv -\frac{\sqrt{3}}{2}M$$

and the superpotential part $P(\Phi, \Lambda)$ is taken zero. In the model under consideration for simplicity we take $\alpha = 0$, in which case there is a minimum at $\varphi = 0$ and the potential is concave in the φ direction. Denoting the real and imaginary parts of λ by s and σ respectively the relevant Lagrangian part takes on the form

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{3}{4}\frac{\partial_\mu s\partial^\mu s}{(s+3)^2} - \frac{3}{4}\frac{\partial_\mu\sigma\partial^\mu\sigma}{(s+3)^2} - \frac{9\,|g(s+i\sigma)|^2}{(s+3)^2} \tag{5}$$

where the last part of it is the scalar potential. Its form, as function of s and σ is displayed in Fig. 1. We can see, by taking d, d_1 and d_2 real, that the minimum of the potential is at $\sigma = 0$. Ignoring

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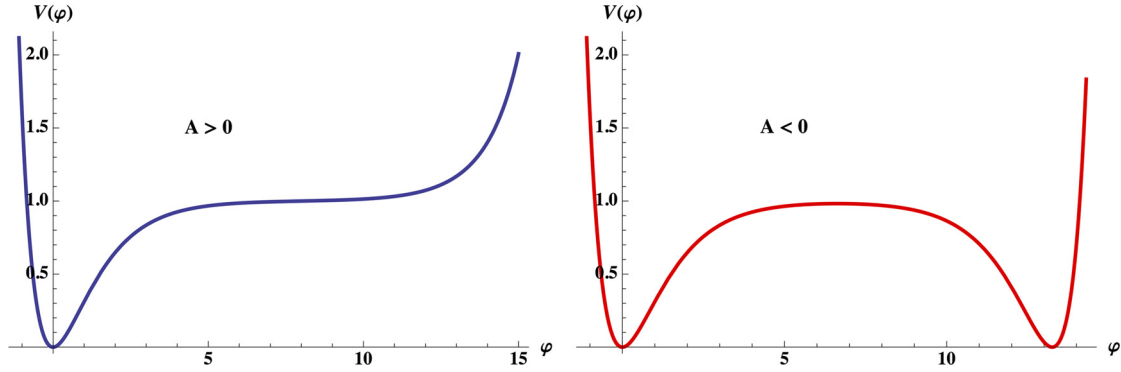[2]   In this work we follow closely the notation of Bagger and Wess [46].

Fig. 2. The general form of the scalar potential (9) as function of ϕ for the cases $A > 0$ (left panel) and $A < 0$ (right panel).

then the fluctuations of the field σ around its minimum [31] we have a single-field theory specified by the Lagrangian

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{3}{4}\frac{\partial_\mu s \partial^\mu s}{(s+3)^2} - \frac{9|g(s)|^2}{(s+3)^2} \qquad (6)$$

whose kinetic terms can appear canonical if we properly redefine the field s. In particular we define ϕ, not to be confused with the scalar component of the superfield Φ used so far, so that

$$s = -3 + (3+\ell)e^{\sqrt{\frac{2}{3}}\phi} \qquad (7)$$

with the constant ℓ defined by

$$\ell = \frac{-d_1 + \sqrt{d_1^2 - 4d\,d_2}}{2d_2}. \qquad (8)$$

With these definitions the field ϕ is canonically normalized and the potential receives a form reminiscent of the Starobinsky potential. In fact the potential as function of ϕ is given by

$$V(\phi) = \frac{3M^2}{4}(1 - e^{-\sqrt{\frac{2}{3}}\phi})^2 |1 + A(e^{\sqrt{\frac{2}{3}}\phi} - 1)|^2. \qquad (9)$$

In the expression above the constants M, A are related to those appearing in the superpotential by

$$M^2 = 12|d_1|^2(1 - 4ab), \quad A = \frac{6b - 1 + \sqrt{1 - 4ab}}{2\sqrt{1 - 4ab}} \qquad (10)$$

where a, b are the ratios

$$a = \frac{d}{d_1}, \quad b = \frac{d_2}{d_1}.$$

The parameter M defined above sets the scale of inflation.[3]

Therefore the model is described by the three parameters d_1, a, b but essentially only the combinations M, A enter into the potential and are relevant for cosmological considerations. Note that the case $d_2 = 0$, that is when the quadratic in Λ terms are absent in $g(\Lambda)$, corresponds to $A = 0$ and the potential receives the well-known form of the Starobinsky potential. In that limiting case the value of ℓ is $\ell = -a$, as can be shown by expanding $\sqrt{1 - 4ab} = 1 - 2ab$. The potential of Eq. (9) has a single minimum when $A > 0$, but exhibits two minima when $A < 0$ as shown in Fig. 2. In the following we shall consider the $A > 0$ case for which

the potential exhibits an almost flat region (plateau) for values of ϕ that are smaller than

$$\phi_i = \sqrt{\frac{3}{2}}\ln\left(1 + \frac{1}{A}\right). \qquad (11)$$

For larger values of ϕ the terms proportional to A in the potential dominate and the potential departs from its Starobinsky form exhibiting a rapid exponential behavior,

$$V(\phi) = \frac{3M^2}{4}A^2 e^{2\sqrt{\frac{2}{3}}\phi}.$$

The coefficient of ϕ in the exponent is too large to sustain successful inflation unless the value of A is small in which case there is a rather extended plateau for values of ϕ less than ϕ_i. Then in this region the potential resembles that of Starobinsky's and the inflaton, after a short time, falls into the plateau and slow-roll inflation starts. These qualitative features will be quantified numerically in the following sections. In particular we shall see that the features of the Starobinsky inflation are maintained if A is taken at per thousand level so that we have sufficient number of e-foldings, larger than $\sim$60, for values of ϕ belonging to the almost flat regime. This guarantees that in the slow-roll regime a pivot scale ϕ^* can be obtained for which the number of e-foldings left, $N(\phi^*)$, is in the range 50–60 as required. We have verified, by solving the pertinent equations numerically, that for typical initial values of the inflaton field, its value rapidly drops to ϕ_i, following then a slow-roll inflation a la Starobinsky and the motion of ϕ from its initial position to ϕ_i it actually plays little role. This will be shown in the following section where a numerical solution is presented.

4. Numerical treatment of the cosmological equations

In this section we shall quantify the statements claimed in the previous section by solving the cosmological equation numerically. The system of the pertinent differential equations are well known given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$3H^2 = \frac{\dot{\phi}^2}{2} + V(\phi) \qquad (12)$$

where $H = \dot{a}/a$ is the Hubble expansion rate. We solve (12) for typical initial values of the inflation field at the start of inflation $t = 0$. The cosmic scale factor we take $a(t=0) = 1$. Note that since the potential grows exponentially, with increasing ϕ, it approaches the Planck energy density scale for inflaton values ϕ_P dictated by, (see for instance [47]),

$$V(\phi_P) = 1 \qquad (13)$$

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[3] As we shall see for small A and moderate ϕ the potential (9) is that of Starobinsky having a plateau where slow-roll can be realized. The normalization of CMB anisotropies yields then $M \simeq 10^{-5}$.
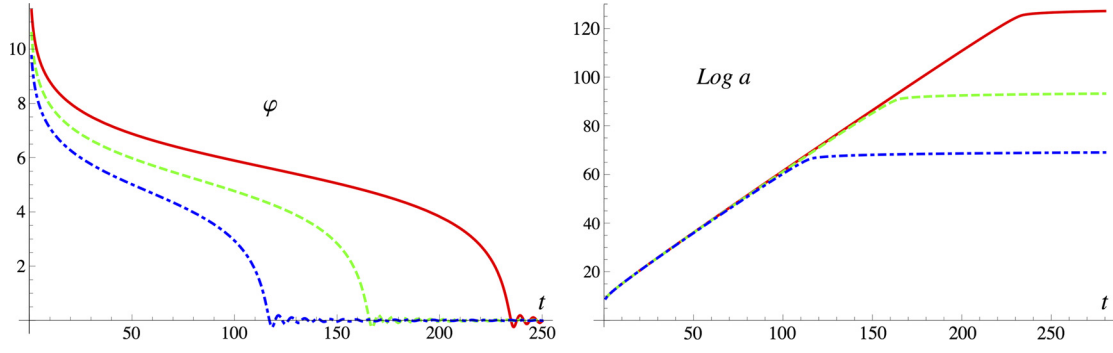
Fig. 3. Evolution of the inflaton ϕ (left) and the logarithm of the cosmic scale factor $\log a$ (right) with time. The time is taken in units of the scale $M = 10^{-5}$. The solid (red), dashed (green) and dash-dot (blue) lines correspond to values of $A = 10^{-4}, 2 \times 10^{-4}, 4 \times 10^{-4}$ respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
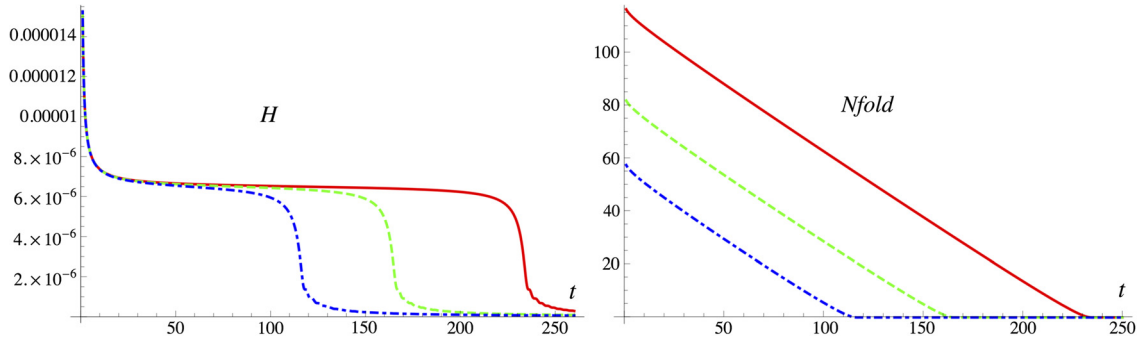

Fig. 4. Evolution of the Hubble rate H (left) and the number of e-foldings $N fold$ (right) with time. M, A are as in Fig. 3.

which entails to

$$\phi_P = \sqrt{\frac{3}{8}} \ln\left(\frac{4}{3 A^2 M^2}\right) \simeq 14.28 - 1.22 \ln A. \qquad (14)$$

The scale of the inflation M has been set equal to $M = 10^{-5}$. Therefore typical initial values at the beginning of inflation are given by

$$\phi \sim \phi_P \qquad \dot{\phi}_P^2/2 \sim V(\phi_P) = 1. \qquad (15)$$

For $A = 1$, $\phi_P \sim 14$ while for small $A = 10^{-4}$ the initial inflation value is $\phi_P \sim 25$.

The evolution for the inflaton ϕ and the cosmic scale factor for various values of the parameter A are displayed in Fig. 3. The solid (red), dashed (green) and dash-dot (blue) lines correspond to $A = 10^{-4}$, 2×10^{-4} and $A = 4 \times 10^{-4}$ respectively. From the left panel of this figure we see that after a sharp drop the inflaton follows a normal slow-roll evolution during which the Universe undergoes a de-Sitter expansion as shown on the right panel of this figure where the evolution of the cosmic scale factor is shown. The horizontal axis is the time in units of the inflation scale $M = 10^{-5}$. The exit from inflation occurs when $\log a$ starts becoming almost constant. Note the damped oscillatory behavior of ϕ as it drops within the minimum of the potential. In Fig. 4, on the left panel, the evolution of the Hubble rate is shown while on the right we show the number of e-foldings as function of time, $N(t) \equiv N(\phi(t))$. The parameters are as in Fig. 3. One observes that after a short drop-off H enters into the slow-roll era, during which it stays almost constant, exiting from it at a time, in each case displayed, that coincides with the time signaling departure of H from its constancy, as expected. The number of e-foldings $N(t)$ should be in the range is ~ 50–60 at a time t^* at which the inflation receives the pivot value ϕ^*. One notices that for the largest of the sample values chosen, $A = 4 \times 10^{-4}$, we can marginally obtain a pivot value

for which the number of the e-foldings left is in the aforementioned range. As a conclusion only small values of the A parameter are allowed which we have found to be bounded by $A \leq 5 \times 10^{-4}$. Note that for moderate to large values of the parameter A the potential is mainly exponential $\sim e^{\lambda \phi}$, with $\lambda = 2\sqrt{2/3}$, and such large values of λ in exponential potentials are known to be incompatible with slow-roll, unless $\lambda^2 < 2$. Therefore the exponential part by itself cannot sustain inflation and an extended plateau is required, on which the inflaton rolls after a rapid fall from the exponential region. This plateau must be extended enough to obtain the desired number of e-foldings which is quantified by the above upper bound on A.

Although this inflationary scenario resembles that of Starobinsky's model nevertheless both the slow-roll parameters ϵ, η depend on the additional parameter A which although small may affect the value of ϵ and hence the tensor to scalar ratio r. Whether large values for r can be obtained and if these can be consistent with the remaining observables, in particular the spectral index n_s will be investigated in the following section.

5. Slow-roll inflation

From the discussion of the previous section it becomes evident that only small values of A are allowed and all cosmological parameters depend on it. However small this might be it may have, in principle, a large impact on the slow-roll parameters and the number of e-foldings.

Starting from the latter, we have already seen that it puts an upper bound on the allowed value of $A < 5 \times 10^{-4}$ by the requirement that the number of N left until the exit from inflation is in the range 50–60. Its analytic form, for any value of ϕ in the range $\phi < \phi_i$, can be calculated analytically given by
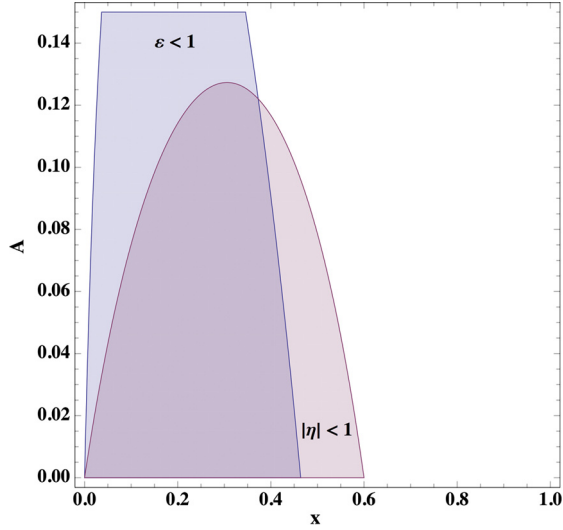
Fig. 5. The shadowed areas designate the region where slow-roll approximation holds. $\epsilon < 1$ (in blue) allows for large A values while $\eta < 1$ (in magenta) requires $A < .13$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$N(x) = -\frac{3}{4} ln\left[\frac{x\,(k+x_{end}^2)}{x_{end}\,(k+x^2)} \right]$$
$$+ \frac{3(1-k)}{4\sqrt{k}} \left(arctan\frac{x_{end}}{\sqrt{k}} - arctan\frac{x}{\sqrt{k}} \right). \qquad (16)$$

In this x stands for the more convenient variable $x = \exp\left(-\sqrt{\frac{3}{2}}\,\phi\right)$ through which the region $\phi = [0, +\infty]$ is mapped to $x = [1, 0]$. The constant k is $k \equiv \frac{A}{1-A}$. Eq. (16) holds for any $A < 1$ which is always the case when A is small. The value of x_{end} signaling the end of the inflation period is determined when at least one or both of ϵ, η become of order unity. In Fig. 5 we display the regions where slow-roll conditions hold in the x, A plane. The slow-roll region, where both ϵ, η are less than unity, allows for values $A < 0.13$. For each A the furthest point on the right of the allowed overlapping region marks the end of the inflation point x_{end}. For small A this is given analytically, to a good approximation, by

$$x_{end} \equiv e^{-\sqrt{\frac{2}{3}}\,\phi_{end}} \simeq \sqrt{3}/(2+\sqrt{3})\,(1-A) \simeq 0.5\,(1-A). \qquad (17)$$

Note that the above upper bound, $A < 0.13$, is much larger than the bound set by the requirement to have sufficient number of e-foldings and therefore for values $A < 5 \times 10^{-4}$ we are well within the slow-roll regime.

Concerning the slow roll parameter ϵ and in order to study its sensitivity on the parameter A it facilitates if we write potential of Eq. (9) as

$$V(\phi) = V_S(\phi)\,f(\phi) \qquad (18)$$

where V_S is the Starobinsky potential and $f(\phi)$ the additional factor that dominates for large values of ϕ and depends on the parameter A. Then in a straightforward manner one can show that the slow-roll parameter ϵ, as function of the inflaton field, or the field x defined before, is related to the corresponding value found for the Starobinsky potential, denoted by ϵ_s, by

$$\sqrt{\epsilon} = \sqrt{\epsilon_s} + \frac{2}{\sqrt{3}} \frac{1}{1 + (A^{-1}-1)\,x}. \qquad (19)$$

One observes that for the same value of the inflation field the ϵ parameter in the model studied here is larger than ϵ_s by amounts
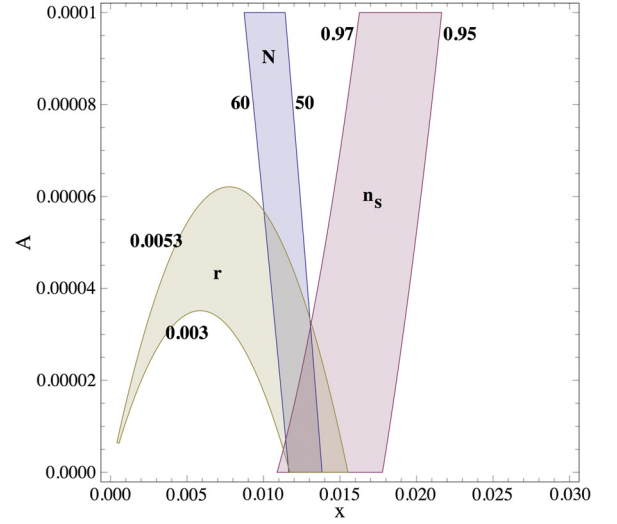

Fig. 6. Regions for N, r, n_s within the ranges designated at the borders of each region. The allowed r region by N and n_s predicts r in the range $0.003 < r < 0.0053$ although the N and n_s separately each allows for much higher values.

that are controlled by the parameter A as is evident from Eq. (19). This equation is valid provided we are in the slow-roll regime which entails to values of ϕ for which the additional term, appearing on the right hand side of Eq. (19), is much less than unity,

$$\phi << \sqrt{\frac{3}{2}}\,ln(A^{-1}-1). \qquad (20)$$

Therefore slow roll in the Starobinsky model yields slow-roll motion in the model under consideration in the regime specified by the inequality above. Note that since A has to be small the rhs of the equation above is very close to ϕ_i which signals departure from the simple Starobinsky potential. Although small in the regime of interest the additional term in (19) may not be small as compared to $\sqrt{\epsilon_s}$ and may significantly augment the value of the epsilon parameter tending to increase the ratio r of the tensor to scalar perturbations. The precise amount depends on the parameter A but the analysis has to take into account the remaining cosmological data, in particular the number of e-foldings, which specifies the pivot scale, and the value of the spectral index η_s as well.

In Fig. 6 we plot contour values for the ratio r the index η_s and the number of e-foldings, as functions of the field variable x. On the vertical axis are the values of the parameter A. Large values of r are allowed by N and n_s observational constraints separately (not shown in Fig. 6). However the intersection of N and n_s restricts the allowed region to values of A that are smaller than 4.0×10^{-5} and values of x that are around 0.013 as shown in the figure. Within this region the allowed values of r are $0.003 < r < 0.0053$. Note that the displayed region which is allowed by all data is well within the slow-roll regime. Therefore, although the model can yield larger values of r, as compared to the simple Starobinsky model, which are in agreement with the new data released by Planck and BICEP2, nevertheless it rather favors low values that cannot saturate the upper bound imposed on r by these experiments.

Concluding this section, the inclusion of quadratic in Λ terms in the superpotential introduces a new parameter in the potential that can result, in principle, to larger r values, in comparison with the linear (in Λ) supersymmetric Starobinsky model, but not sufficiently large to approach values of r close to the upper bound set by recent data of Planck and BICEP2 if all observational constraints are taken into account. The fact that one needs very small A values to maintain the good features of the Starobinsky model,

at least as far as the Planck satellite data are concerned, indicates that non-linear superpotential Λ terms are only allowed provided that they have small couplings.

6. Discussion – conclusions

In this note we generalize the supergravity Starobinsky models allowing for superpotential terms that are not linear in the superfield Λ that couples to the chiral Ricci multiplet. We exemplify the departure from the linearity by the appearance of quadratic in this field terms. The Kähler function is assumed to be of the no-scale type. The inclusion of such terms results to introducing additional parameters in the theory which however enter into the scalar potential in two combinations. One, M, sets the scale of the inflationary potential and the other, A, deforms the Starobinsky potential in a multiplicative way. In the limit $A = 0$, in which case the quadratic terms are absent, one recovers the simple Starobinsky model but for $A \neq 0$ the potential deviates from it increasing exponentially for large inflaton values. Successful inflation is achieved only if the parameter A is smaller than $\sim 5 \times 10^{-4}$, for natural initial values of the inflaton field. This constraint stems mainly from the requirement to have sufficient number of e-foldings left to the end of inflation. For such small values of A the inflaton starts its motion dropping rapidly to the Starobinsky plateau which is rather extended due to the smallness of the parameter A. The dependence of the slow-roll parameters ϵ, and hence of r, on the additional parameter A is investigated. Although in principle the tensor to scalar ratio can be theoretically much larger in comparison with the predictions of the Starobinsky model, nevertheless the imposition of the data concerning the spectral index n_s in combination with the required number of e-foldings suppresses the allowed values of A even more, by almost an order of magnitude, in regions where r cannot exceed $r \simeq 0.005$. This value is slightly larger than the one predicted in the Starobinsky model but in no case can saturate the new combined bound released by Planck and BICEP2 experiments if all data are observed. On the theoretical side, the smallness of the parameter A, as a result of the comparison with the observational data, indicates that the presence of non-linear in Λ terms within the superpotential is only possible provided that these are very small. This fine tuning suggests that the departure from the non-linearity is presumably due to quantum effects.

Acknowledgements

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