Radiative MHD heat and mass transfer nanofluid flow past a horizontal plate in a rotating system

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Abstract

Unsteady radiative MHD heat and mass transfer nanofluid flow through horizontal plate under the action of magnetic field have been investigated. To obtain the non-similar coupled nonlinear momentum, energy and concentration equations, usual non-dimensional variables have been used. The explicit finite difference method with stability and convergence analysis has been used to solve the obtained numerical solutions of the above problem. The stability and convergence analyses have been used for measuring the restriction of the useful parameters. The obtained numerical results have been presented graphically and discussed in details. Finally, a qualitative comparison of our results with published results has been explained in conclusion section.

Keywords: Nanofluid; MHD; Rotating System; Explicit Finite Difference Method.

1. Introduction

MHD radiative boundary layer nanofluid flow and heat transfer over a linearly stretched surface have attracted a lot of attention in the field of several industrial, scientific, and engineering applications in recent years. Day by day the applications of nanofluids are increasing. Nanofluids are frequently used in many engineering works, scientific works and industrial works. Nanofluids are used in microelectronics, fuel cells, hybrid-powered engines, pharmaceuticals process, heat exchangers, vehicle thermal management, nuclear reactor coolant, in grinding,
machining, in space technology, defense and ships, ceramic industries, plastic industries, biomedical technology etc. The word “nanofluid” was first invented by Choi [1] in order to develop advanced heat transfer fluids with substantially higher conductivities. The problem of three dimensional fluid flow due to stretching flat plate has been studied by Wang [2]. It has been examined by Putra et al. [3] that the water based nanofluid containing Al₂O₃ or CuO nanoparticles increased thermal conductivity two-to four folds. The effects of thermal radiation and magnetic field on unsteady stretching permeable sheet in presence of free stream velocity were investigated by Jangid and Tomer [4]. The problem of laminar boundary layer flow of a nanofluid past a stretching sheet has been investigated by Khan and Pop [5]. The boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition in presence of magnetic field and thermal radiation have been studied by Gbadeyan et al. [6].

In the present work, radiative heat and mass transfer flow past a horizontal plate in a rotating system. For solving the non dimensional coupled similar and non similar equations are solved by explicit finite difference method. Numerical results have presented for the range of various parameter.

2. Mathematical model of flow

Consider an MHD free convection and mass transfer flow of an electrically conducting viscous fluid through a plate \( y = 0 \) in a rotating system. The Cartesian coordinates \( x \), measured along the plate and \( y \) is the coordinate measured normal to the plate. The flow is assumed to be in the \( x \) direction.

\[ X = \frac{xU_0}{\nu}, \quad Y = \frac{yU_0}{\nu}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \quad W = \frac{w}{U_0}, \quad \tau = \frac{tU_0^2}{\nu}, \quad \Theta = \frac{T - T_x}{T_w - T_x}, \quad \Theta' = \frac{C - C_w}{C_w - C_x} \]

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\delta^2 U}{\partial Y^2} + 2R'W - MU
\]
\[
\frac{\partial W}{\partial \tau} + U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} = \frac{\partial^3 W}{\partial Y^2} - 2R'U - MW \\
\frac{\partial T}{\partial \tau} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \left(1 + R\right) \frac{\partial^2 T}{\partial Y^2} + N_b \left(\frac{\partial T}{\partial Y} \frac{\partial C}{\partial Y}\right) + N_f \left(\frac{\partial T}{\partial Y}\right)^2 \\
\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{L_e} \left[ \frac{\partial^2 C}{\partial Y^2} + \left(\frac{N_f}{N_b}\right) \frac{\partial^2 T}{\partial Y^2} \right]
\]

The non-dimensional boundary condition
\[U = 1, \ V = 0, \ W = 0, \ T = 1, \ C = 1, \text{ at } Y = 0\]
\[U = 0, \ V = 0, \ W = 0, \ T \to 0, \ C \to 0, \text{ as } Y \to \infty\]  

The non-dimensional quantities are
\[M = \frac{\beta B_0^2 U}{\rho U_0^2} \text{ (Magnetic parameter)}, \ R = \frac{16\sigma^* T_x^3}{3kk^*} \text{ (Radiation parameter)}, \ P_r = \frac{\nu}{\alpha} \text{ (Prandtl number)}, \ L_e = \frac{\nu}{D_B} \text{ (Lewis number)}, \ N_b = \frac{D_B (C_w - C_\infty)}{\nu} \text{ (Brownian parameter)}, \ N_f = \frac{D_f \tau}{T_w - T_\infty} \text{ (Thermophoresis parameter)}, \ R' = \frac{\Omega \nu}{U_0^2} \text{ (Rotational Parameter)}.

3. Numerical solutions

To solve the non-dimensional system by the explicit finite difference method, it is required a set of finite difference equation. To obtain the different equations for the region of the flow is divided into a grid or mesh of lines parallel to \(X\) and \(Y\) axes where \(X\)-axis is taken along the plate and \(Y\)-axis is normal to the plate.

![Fig. 2. Explicit finite difference system grid](image)

Here the plate of height \(X_{\text{max}} (=100)\) is considered i.e. \(X\) varies from 0 to 100 and assumed \(Y_{\text{max}} (=25)\) as corresponding to \(Y \to \infty\) i.e. \(Y\) varies from 0 to 25. Here \(m(=200)\) and \(n(=200)\) grid spacing in the \(X\) and \(Y\) directions respectively as shown Fig. 2. \(\Delta X, \Delta Y\) are constant mesh size along \(X\) and \(Y\) directions respectively and taken as follows, \(\Delta X = 1.00(0 \leq X \leq 100)\) and \(\Delta Y = 0.25(0 \leq Y \leq 25)\) with the smaller time-step, \(\Delta \tau = 0.005\).

Let \(U', \ W', \ T'\) and \(C'\) denote the values of \(U, \ W, \ T\) and \(C\) at the end of a time-step respectively. Using the explicit
finite difference approximation, the system of partial differential equations (1)-(5) and the boundary conditions (6), we obtain an appropriate set of finite difference equations,

\[ \frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta X} + \frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta Y} = 0 \] (7)

\[ \frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta \tau} + \frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta X} + \frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta Y} = \frac{U_{i+1,j}^n - 2U_{i,j}^n + U_{i-1,j}^n}{(\Delta Y)^2} + 2R'W_{i,j} - MU_{i,j} \] (8)

\[ \frac{W_{i,j}^{n+1} - W_{i,j}^n}{\Delta \tau} + \frac{W_{i,j}^{n+1} - W_{i,j}^n}{\Delta X} + \frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta Y} = \frac{W_{i+1,j}^n - 2W_{i,j}^n + W_{i-1,j}^n}{(\Delta Y)^2} - 2R'U_{i,j} - MW_{i,j} \] (9)

\[ \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta \tau} + \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta X} + \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta Y} = \left( \frac{1 + R}{Pr} \right) \left( \frac{T_{i,j}^{n+1} - 2T_{i,j}^n + T_{i,j-1}^n}{(\Delta Y)^2} \right) \]

\[ + N_b \left( \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta Y} - \frac{\overline{C}_{i,j}^{n+1} - \overline{C}_{i,j}^n}{\Delta Y} \right) + N_t \left( \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta Y} \right)^2 \] (10)

\[ \frac{\overline{C}_{i,j}^{n+1} - \overline{C}_{i,j}^n}{\Delta \tau} + \frac{\overline{C}_{i,j}^{n+1} - \overline{C}_{i,j}^n}{\Delta X} + \frac{\overline{C}_{i,j}^{n+1} - \overline{C}_{i,j}^n}{\Delta Y} = \]

\[ \frac{1}{Le} \left[ \left( \frac{\overline{C}_{i,j}^{n+1} - 2\overline{C}_{i,j}^n + \overline{C}_{i,j-1}^n}{(\Delta Y)^2} \right) + \left( \frac{N_t}{N_b} \right) \left( \frac{T_{i,j}^{n+1} - 2T_{i,j}^n + T_{i,j-1}^n}{(\Delta Y)^2} \right) \right] \] (11)

With initial and boundary conditions;

\[ U_{i,0}^n = 1, \quad V_{i,0}^n = 0, \quad W_{i,0}^n = 1, \quad T_{i,0}^n = 1, \quad \overline{C}_{i,0}^n = 1 \]

\[ U_{i,L}^n = 0, \quad V_{i,L}^n = 0, \quad W_{i,L}^n = 0, \quad T_{i,L}^n = 0, \quad \overline{C}_{i,L}^n = 0, \quad \text{where} \quad L \to \infty \] (12)

Here the subscripts \( i \) and \( j \) designate the grid points with \( X \) and \( Y \) coordinate respectively and the subscript \( n \) represents a value of time, \( \tau = n\Delta \tau \) where \( n = 0,1,2,3, \ldots \ldots \ldots \). The stability conditions of the method are not shown for brevity.

4. Results and discussion

In order to investigate the physical significance of the problem, the numerical values of primary velocity, secondary velocity, temperature and concentration within the boundary layer has been computed for different values of various parameters. To obtain the steady-state solutions, the calculation has been carried out up to non-dimensional time \( \tau = 5 \) to 80. It is observed that the numerical values of \( U, W, T \) and \( C \) however, show little changes after \( \tau = 40 \). Hence at \( \tau = 40 \) the solutions of all variables are steady-state solutions. The primary and secondary velocity is displayed in Fig. 3 for different values of Magnetic parameter. These results show that the primary velocity is increasing with the increase of Magnetic parameter while the secondary velocity is decreasing. In Fig. 4., the primary and secondary velocity are illustrated for various values of Radiation parameter. It is noted that the primary velocity is decreasing with the increase of Radiation parameter while the secondary velocity is also decreasing near the plate. But for secondary velocity, it has been shown the opposite effects after \( Y \geq 1.9 \) (approx.).
Fig. 3. (a) Primary velocity profile and (b) Secondary velocity profile for different values of Magnetic parameter $M$

Fig. 4. (a) Primary velocity profile and (b) Secondary velocity profile for different values of Rotational parameter $R'$
The temperature profile has been shown in Fig. 5-6 for various values of Radiation parameter, Prandtl number, Brownian parameter and Thermophoresis parameter respectively. These results show that the fluid temperature is increasing with the increase of Radiation parameter, Brownian parameter and Thermophoresis parameter. But opposite effect has been found with the increasing of Prandtl number. In Fig. 7, the concentration profile has been illustrated for various values of Lewis number and Thermophoresis parameter. It is noted that the concentration is decreasing with the increase of Lewis number and Thermophoresis parameter.

5. Conclusions

The finite difference solution of radiative MHD heat and mass transfer nanofluid flow past a horizontal plate in a rotating system is investigated. The results are compared with the work of Khan and Pop [5]. The accuracy of our results is qualitatively good in case of all parameters. The comparison has not been shown for brevity. Some
important findings of this study are given below:

1. For increasing the Brownian and Thermophoresis parameter, temperature distribution increases where as the concentration distribution decreases for increasing the Brownian parameter.

2. Temperature decreases for increasing of Prandtl number and concentration decreases for increasing of Lewis number.

References


